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B_2 Stress Indices for Elbows and Straight Pipes Using Finite Element Analysis

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ABSTRACT

Plastic deformation in Class 1 nuclear power plant piping systems is governed by Equation (9) in NB-3652. In this equation, nominal stresses for straight pipes are modified by scalar multipliers called stress indices so that the equation can be applied to components such as elbows and tees as well as straight pipes. We computed the stress index for bending (B_2) in elbows using the equation $B_2=S_y/(M_{CL}/Z)$. Nonlinear finite element analyses were performed for a series of stainless steel elbows of different sizes and schedules, with flanges attached at different locations and with different levels of internal pressure to obtain load-deflection curves from which collapse limit loads and then collapse moments were determined. We discuss the application of our procedure to straight pipes, and offer modifications to the procedure that would produce a B_2 of 1.0 for straight pipes.

NOMENCLATURE

B_2 = primary stress index for bending
D_i = inside diameter of pipe
D_m = mean diameter of pipe
D_o = outside diameter of pipe
E-PP = elastic-perfectly plastic constitutive model
M_{CL} = collapse moment of component
ML = multilinear constitutive model
P_a = maximum allowable internal pressure
P_{CL} = collapse load of component
R = nominal bend radius of elbow
S = nominal stress
S_{EQV} = equivalent von Mises stress
S_m = allowable design stress intensity
S_y = yield stress
Z (Z_e) = elastic section modulus = $\pi(D_o^4 - D_i^4)/(32D_o) \approx \frac{\pi}{4}D_m^2 t$
Z_p = plastic section modulus = $(D_o^3 - D_i^3)/6 \approx D_m^2 t$
h = tR/r_m^2
r_m = mean pipe radius
t = nominal wall thickness of pipe

x = moment arm in elbow experiment
y = 0.4 (NB-3641.1)
σ = stress magnitude corresponding to a limit load

BACKGROUND

The basic idea in our work with elbows is taken from Appendix II in Section III of the ASME Code (1992) in which an experimental approach is given for the determination of a defined collapse limit load which, in turn, can be used to define a B stress index (Mello and Griffin, 1974; Rodabaugh et al., 1993; Wais, 1995). Using the collapse limit line to define a collapse load P_{CL}, we calculated B_2 using the formula $B_2 = \frac{\sigma}{S}$ where σ was interpreted to be S_y, $S = M_{CL}/Z$, M_{CL} is the collapse moment $M_{CL} = P_{CL}{*}x$, and Z is the elastic section modulus. For the elbow, we considered in-plane opening, in-plane closing and out-of-plane bending.

In our earlier research (Matzen and Yu, 1998), we determined the collapse limit load using Finite Element Analysis (FEA) instead of experimental data. A comprehensive study on 2", schedule 40, 90°, long radius, seamless, stainless steel elbows at room temperature was made using ANSYS (1994). In our finite element model, we used shell or solid elements for both the elbow and the straight pipe segments on each end of the elbow (needed to eliminate end effects.) The model was refined and verified as follows:

- We compared our FEA stresses to the stresses given in Table NB 3685.1-2 of the Code and found that they compared reasonably well.
- Using ANSYS 5.1 (1994) and ANSYS 5.3 (1997) we conducted a comprehensive study on finite element mesh densities and element types (Matzen and Yu, 1998). We initially investigated the use of both shell and solid elements with two mesh densities each. SHELL43 is a 4 node, warped plate element with 6 dofs/node. SOLID45 is an 8 node solid element with 3 dofs/node. Both elements have plasticity, stress stiffening, large deflection and large strain capabilities. We selected the SHELL43 element for the remainder of our analyses since, in our convergence study, there was little difference in the performance between the two elements, and they converged to approximately the same solution. (For higher schedule elbows, this result may be different.) Four elements along the elbow half-arc and eight elements around the half circumference were sufficient to produce accurate results.
- We investigated the effect of thickness variations between the elbow and pipe (the elbow is thicker) and found it to be small.

FINITE ELEMENT ANALYSIS

Finite Element Models and Material Models

For the elbow in-plane loading modes we took advantage of symmetry and used only a quarter of the model. For the out-of-plane loading mode and the reconciliation study between FEA and tests we used the whole model. The two elbow models are shown in Figures 1a and 1b.

We developed a procedure to construct a curvilinear stress-strain curve using Code values of Young's modulus, 0.2% yield stress and ultimate stress based on the power law (Yu, 1998). The constructed true stress-strain curve, along with an elastic-perfectly plastic model is shown in Figure 2. Unless otherwise specified, the results in this paper are based on nominal geometric properties and the multilinear stress-strain curve described above.

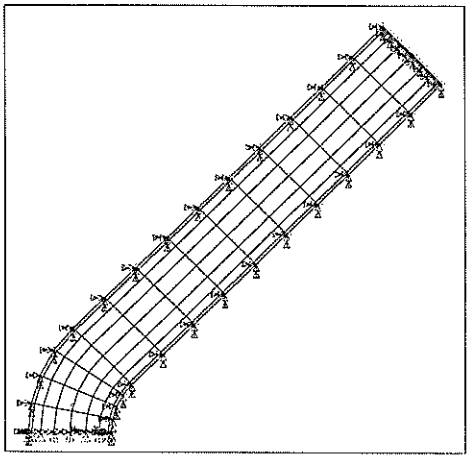

Figure 1a. FEA model used for in-plane analyses.

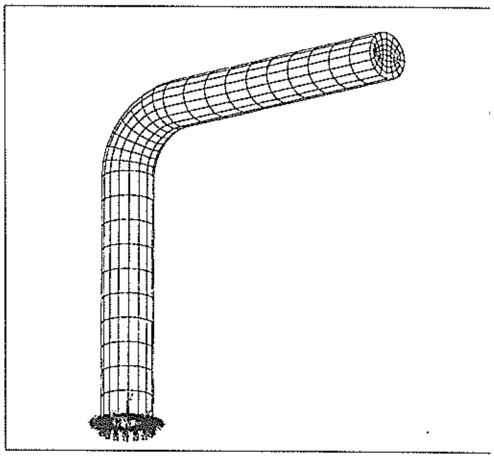

Figure 1b. FEA model used for out-of-plane analyses.

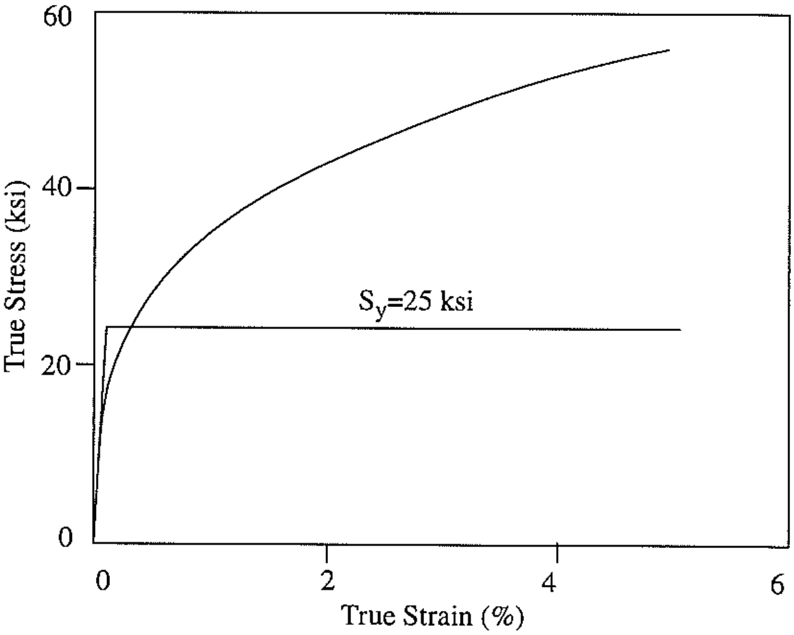

Figure 2 Code-based constitutive material models.

Tangent Length Study

When an elbow is loaded in its plane (in either opening or closing modes), the cross section ovalizes. The ovalization reaches a maximum in its mid-section and it becomes smaller towards the end sections of the elbow. In a piping system, elbows are connected to straight pipes. The ovalization of the elbow cross-section propagates some distance along straight pipes at both ends. To separate end effects from other effects, we attached straight pipe segments to both ends of the elbow. To determine the minimum tangent length, we considered all three modes of loading (in-plane opening, in-plane closing and out-of-plane load). Diem and Muller (1987) also investigated the effect of tangent length (on a much larger elbow), using in-plane opening and in-plane closing modes and found that a straight pipe with length five times the elbow diameter was enough to eliminate end effects.

Using out-of-plane loading we performed nonlinear, large deformation FEAs on elbows with different lengths of straight pipe attached to them. We then compared B_2 stress indices for these out-of-plane loading cases to those from in-plane opening and in-plane closing loading cases with a tangent length of five times the elbow outer diameter. From the curve in Figure 3 which we plotted for out-of-plane bending of a 4" Sch. 40 elbow, we can see that the indices nearly stabilize when the tangent length is at least five times the elbow outer diameter. Because this result confirmed what we had concluded in our 2" elbow study, we used tangent lengths of five times the elbow outer diameter for the rest of the B_2 stress index studies. Comparing B_2 stress indices for all three loading modes, we found that the one for in-plane-closing mode was the largest. Thus for the rest of our analytical studies, we used only the in-plane closing mode.

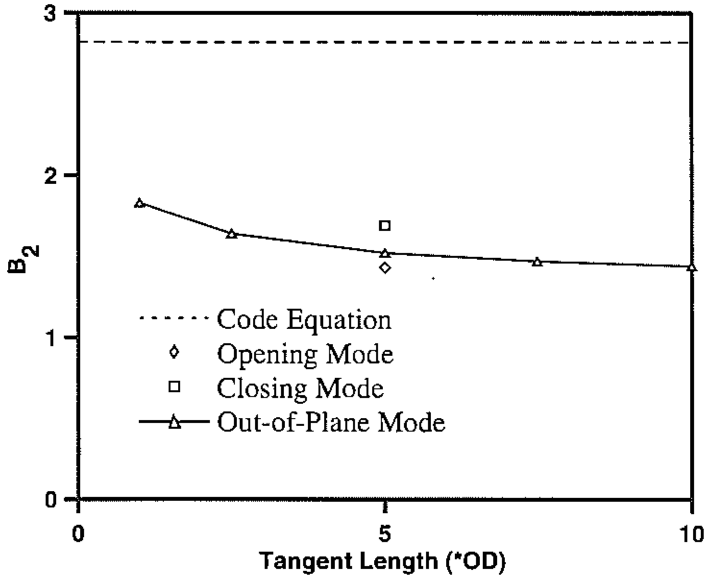

Figure 3 Effect of tangent length on B_2 for different loading modes (4", Sch. 40)

Effect of Flange Location

As mentioned earlier, a propagation length of five times the elbow outer diameter is needed to make the results of our study essentially independent of the end condition. However, the Code does allow flanges to be installed as close as one half elbow diameter from the end of the elbow. To investigate how the location of a flange might influence the B_2 stress indices, we kept the length of the straight segments constant at five diameters, but varied the location of the flange. We realize that this is not necessarily how elbows would be configured in a piping system, but varying one parameter at a time - the flange location in this case - allows us to determine the effect of changes in this parameter independently of other effects.

We used flange locations varying from zero to five times the elbow outer diameter on 4" Sch. 40, stainless steel elbows. A curve of B_2 vs. flange location is shown in Figure 4. From this curve, we can see that B_2 stress indices are reduced slightly as the flange is placed closer to the end of the elbow. This is reasonable since, when a flange is attached close to the elbow, it constrains the elbow cross-section ovalization, tending to make the elbow stiffer and the collapse moment larger, thus reducing the B_2 stress index.

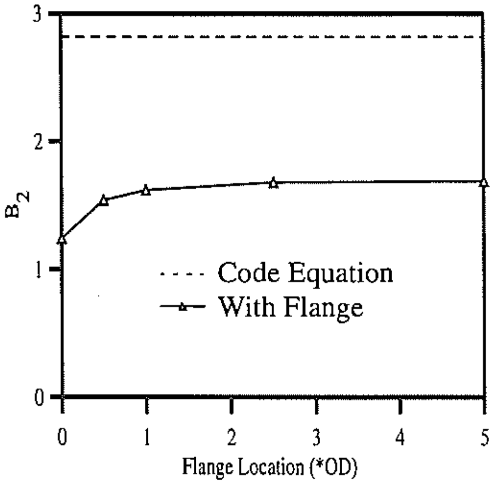

Figure 4 Effect of flange location on B_2 (4" Sch. 40)

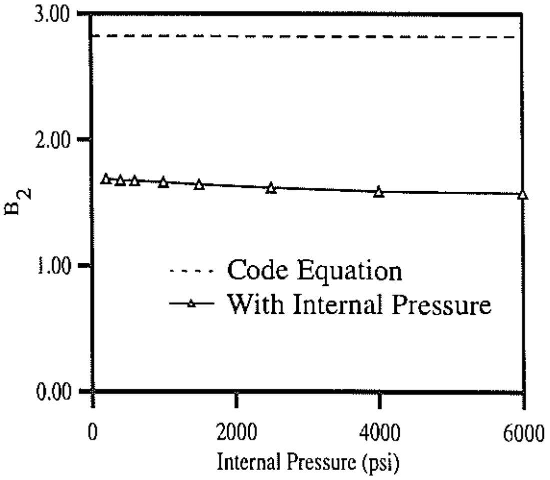

Figure 5 Effect of internal pressure on B_2 (4", Sch. 40)

Effect of Internal Pressure

Section NB-3641.1 of the Code gives the following equation for maximum allowable internal pressure for a straight pipe: $P_a = \frac{2S_m t}{D_o - 2yt}$. For 4", Sch. 40, stainless steel pipe, P_a is 2749 psi if we take S_m as 25 ksi.

We applied various levels of internal pressure (200 to 6000 psi) and obtained load-deflection curves by performing nonlinear large deformation analysis. Based on the load-deflection curves the B_2 stress indices were computed and are plotted vs. internal pressure in Figure 5. The B_2 stress indices decrease slightly with the increasing internal pressure. Again, this result may be different for thinner elbows.

Other Elbow Sizes and Schedules

As observed above, the B_2 stress indices we obtained using FEA are significantly lower than those computed from the Code equation. The previous results are for 4", Sch. 40, stainless steel elbows. We performed the same type of analyses on a wide range of elbow sizes and schedules. We have already observed for the 4" elbows that the addition of flanges and internal pressure tend to reduce B_2 stress indices from the case of remote flanges and no internal pressure. Only the closing mode was considered in this parametric study since from previous study, the in-plane closing mode always gave the largest stress indices.

Elbows of four sizes (2", 4", 6", 8") and five schedules (5, 10, 40, 80, 160) were used in the analyses. Results from these finite element analyses are shown as open circles in Figure 6. The tabulated values can be found in Yu and Matzen (1999). Also shown in the figure is the curve described by the Code equation, $B_2 = \frac{1.30}{h^{2/3}}$, and the Code minimum value of 1.0. Using regression analysis, we determined the two parameters in the Code Equation for elbow B_2 indices so that it would pass through the FEA data. The numerator was reduced from 1.30 to 0.80, while the exponent of 'h' changed only slightly from 2/3 to 0.657. Retaining the 2/3 exponent, the best fit curve for our results becomes: $B_2 = \frac{0.8}{h^{2/3}}$. The curve for this equation is also shown in the figure.

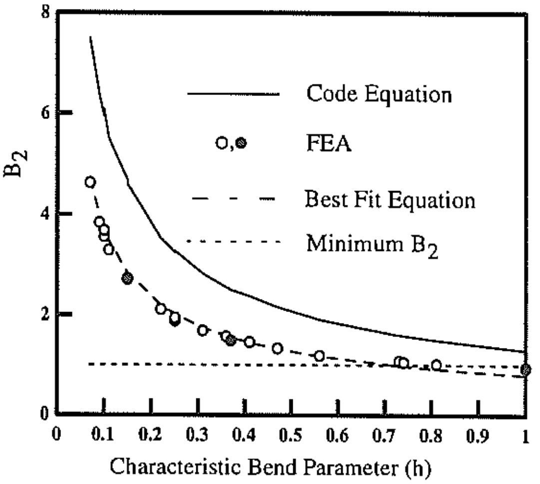

Figure 6 Comparison of B_2 stress indices from FEA and the Code equation.

Application of Procedure to Straight Pipes

The Code B_2 value for straight pipes is 1.0. The procedure described above, however, when applied to straight pipes leads to a value of less than one. To investigate the relationship between straight pipe and elbow B_2 values, we have performed several additional finite element analyses on both elbows and straight pipes. We considered three schedules — 5, 40 and 160 — for 2" pipe (corresponding to a range of bend parameters of 0.1462 to 0.9968), two types of constitutive models (Elastic-Perfectly Plastic (E-PP) and Multilinear (ML)), two alternatives for the nominal stress (one using the elastic section modulus, Z_e, and the other using the plastic section modulus, Z_p), and two alternatives for σ, the "stress magnitude corresponding to a limit load" (S_y and the equivalent von Mises stress, S_{EQV}, which is taken at the load for which collapse is obtained.) The results from this study are shown in Table 1.

Table 1: Summary of B_2 indices for 2" straight pipes and elbows.

Sch.	h	Code B_2 Index for Elbows		FEA B_2 Index for Straight Pipes			FEA B_2 Index for Elbows (In-plane, Closing Mode)			
		$\frac{1.30}{h^{2/3}}$		E-PP	ML		E-PP		ML	
				$\sigma = S_y = S_{EQV}$	$\sigma = S_y$	$\sigma = S_{EQV}$	$\sigma = S_y$	$\sigma = S_{EQV}$	$\sigma = S_y$	$\sigma = S_{EQV}$
5	0.1462	4.68	Z_e	0.76	0.89	0.82*	2.18 (2.87)**	1.82 (2.39)	**2.70***** (3.03)	1.82 (2.22)
			Z_p	1.00	1.17	1.07	2.86	2.39	3.54	2.38
40	0.3746	2.50	Z_e	0.73	0.86	0.81	1.19 (1.63)	1.19 (1.63)	**1.49** (1.73)	1.16 (1.43)
			Z_p	0.99	1.17	1.10	1.61	1.61	2.02	1.58
160	0.9968	1.30	Z_e	0.68	0.80	0.79	0.78 (1.15)	0.78 (1.15)	**0.94** (1.18)	0.91 (1.15)
			Z_p	0.99	1.17	1.16	1.14	1.14	1.38	1.33

* For the Sch. 5 piping, the two B_2 values for S_y and S_{EQV} are not the same because the maximum stress is below the yield stress at the point of Code-defined collapse.

** Values in parentheses are normalized with respect to the B_2 value for the straight pipe, e.g., 2.87 = 2.18/0.76.

*** The bold values are points from the original data set and are identified in Fig. 6 by solid dots.

The following observations can be made from this table: (1) if the plastic section modulus is used with the E-PP model, then the procedure comes close to producing a B_2 of 1.00 for straight pipes; (2) if the elbow stress indices using Z_e are divided by the corresponding indices for the straight pipe, then the procedure yields a B_2 for the straight pipe that is identically 1.0 (these values for elbows are shown in parentheses in the table); and (3) based on the FEA-Elbow-ML-S_{EQV}-Z_p. entries, the Code equation is apparently quite conservative for the schedule 5 elbow, but is unconservative for the schedule 160 elbow (this, of course, is based on the often conservative nominal geometric properties and Code-based constitutive model).

It is interesting to note that, when S_y is used in the determination of B_2, then it can be shown that the normalized B_2 (i.e. the one in parentheses in Table 1) is identically equal to the ratio of the collapse moment for the straight pipe to the collapse moment for the elbow. This does not occur when S_{EQV} is used because, in general, this stress quantity is different for each collapse load.

CONCLUSIONS

If the procedure for obtaining B_2 must be the same for elbows and straight pipes, then there appears to be different levels of conservatism in the Code equation for B_2 indices for elbows with different schedules. This result is based on large deformation, inelastic finite element analyses of stainless steel elbows and straight pipes at room temperature when in-plane, closing behavior of elbows is considered. Additional analytical work on carbon steel piping and on other bending modes, as well as correlation of FEA results with experimental results, will need to be carried out before a recommendation can be made on any changes to the ASME Code equation.

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