



## Parametric Finite Element Analysis of Large Diameter Ratio Shell Intersections Subjected to Internal Pressure

G.E.O. Widera and Zhizhong Wei

*Marquette University, USA*

### ABSTRACT

Cylindrical shell intersections are configurations commonly used in many industries. However, for large diameter ratio, radial, thin shell intersections under internal pressure, design codes, such as the ASME Boiler and Pressure Vessel Code, are still in need of improvement. In this paper, the finite element modeling of these particular intersections is investigated and validated. Based on the present modeling, a parametric finite element study of the stresses in the intersection region is performed for various geometric parameters. Then, correlation equations are developed to predict the stress intensity factors. The equations are applicable to cylindrical shell intersections with diameter ratios from 1/3 to 1.0, diameter-thickness ratios of the vessel from 20 to 250, and thickness ratios from 1/3 to 3. Finally, the comparison of one of the present equations with an empirical equation based on 156 experimental data is conducted. As a result, it is found that the present correlation equations can serve as the basis for the improvement of the present design codes.

### NOMENCLATURE

$d$	= Mean diameter of nozzle
$D$	= Mean diameter of vessel
$t$	= Nozzle thickness
$T$	= Vessel thickness
$P$	= Internal pressure
$S_0$	= $PD/(2T)$ , nominal hoop stress in vessel
$S_{iv}$	= Stress (membrane plus bending) in the vessel in a direction tangent to the opening
$S_{in}$	= Stress (membrane plus bending) in the nozzle in a direction tangent to the opening
$S_{tmv}$	= Membrane stress in the vessel in a direction tangent to the opening
$S_{tmn}$	= Membrane stress in the nozzle in a direction tangent to the opening

Stress intensity=The difference between the maximum and minimum principal stresses

## INTRODUCTION

The sketch of a radial cylindrical shell intersection is shown in Fig. 1. These vessel / nozzle or run pipe / branch pipe intersections are configurations commonly used in many industries, such as pipeline transportation, nuclear and power engineering, chemical and petrochemical engineering, aerospace, etc. Under internal pressure or external force and moment loadings, points of weakness occur at the intersection due to the presence of stress concentrations. WRC Bulletin 368[1] provides designer with some formulas for calculating maximum stresses in both the vessel and the nozzle due to internal pressure, while WRC Bulletin 297[2] provides some guidelines for various external loadings. The equations and guidelines in both bulletins are limited to diameter ratios,  $d/D$ , less than 0.5. In recent years, some papers have been published regarding the stress intensity factors and flexibility factors for large diameter ratio ( $0.5 < d/D < 1.0$ ) shell intersections. However, due to the complexity of this problem, data in the literature are incomplete and inconsistent. Therefore, a parametric study of the stress intensity factors of large diameter ratio cylindrical shell intersections subject to internal pressure and external loadings is performed. The results for the internal pressure case are presented in this paper, while the results for the external loadings will be presented at a future date. Sixty-nine models, which cover all practical cases of thin wall cylindrical shell intersections, were investigated. The geometric parameters of these 69 models are shown in Table 1.

Table 1 Geometric parameters for finite element models

$t/T$	$D/T$	$d/D$
0.1 , 0.5 , 1 , 3	20 , 60 , 100 , 150 , 250, 500	0.333
0.1 , 0.5 , 1 , 3	20 , 60 , 100 , 150 , 250	0.5
0.1 , 0.5 , 1 , 3	20 , 60 , 100 , 150	0.75
0.1 , 0.5 , 1	20 , 60 , 100	1.0

## FINITE ELEMENT MODELING

Many prior investigations [3-4] have shown that the stress concentration at the shell intersection can be accurately predicted by finite element analysis if the element type and the mesh density in the vicinity of the intersection are properly chosen. To accomplish this, COSMOS/M [5], a commercial finite element (FE) package, was used to carry out the parametric study.

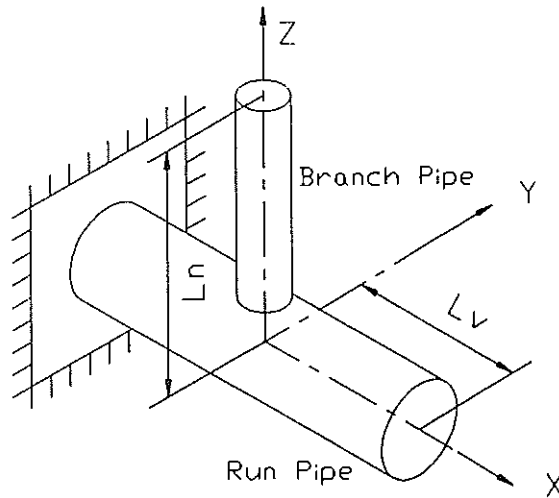


Fig. 1 Schematic of a shell intersection showing the boundary conditions

A cantilever model is adopted for this analysis. One end of the vessel is fixed, while the other end of the vessel and the end of the nozzle are free, see Fig. 1. Due to the symmetry, only one-half of the structure is considered. An 8 or 9-node quadrilateral isoparametric shell element is used. Six degrees of freedom (three translations and three rotations) are considered per node. The element can account for the effects of transverse shear and thus be used for the analysis of complicated shell structures. Further, it is assumed to be of constant thickness. Very fine meshes are used for the present FE models. The degrees of freedom for each FE model are between 20,000 to 35,000. For every model, the maximum aspect ratio of an element in the vicinity of the intersection is controlled so as to be less than 5. For most of the FE models, the element lengths in the vicinity of the vessel-nozzle intersection are less than  $1/3(RT)^{1/2}$  and  $1/3(rt)^{1/2}$ , respectively.

For the present FE models, the length of the vessel is  $4D$  and that of the nozzle is  $4d$ . In the vicinity of the intersection, the predicted stress at right free side of the vessel is almost identical with that at the left fixed side. This shows that the vessel lengths assumed for the present FE models are long enough so that the end conditions do not have an influence on the stress intensity at the intersection.

## VERIFICATION

Since test results for a series of four thin shell intersections carried out at Oak Ridge National Laboratory (ORNL) are available in the literature, the results for two of these models, ORNL models 1 and 2 [6] [7], served as the benchmarks for the present analysis. The parameters of two of the 69 FE models are exactly the same as those described in [6] and [7]. They are tabulated in Table 2. The comparison between the finite element results

and the experimental results for ORNL model 1 is presented in Figs. 2 and 3, while that for ORNL model 2 is presented in Table 3.

Table 2 Parameters for two finite element models

	Geometry	Material properties	Loading
ORNL-1	D= 10 in d= 5 in T=0.1 in t=0.05 in	E=30.0 Mpsi ν=0.3	Internal pressure: 50 psi.
ORNL-2	D=10 in d =10 in T=0.1 in t=0.1 in	E=30.0 Mpsi ν=0.3	Internal pressure: 60 psi.

Table 3 Comparison of the present FE results with ORNL-2 experimental results

Location		$S_{in} / S_0$	$S_{iv} / S_0$
Longitudinal plane, outside surface, 1/8 in. from mid-surface intersection	experimental	7.70	7.80
	present FEM	7.90	7.57
Longitudinal plane, inside surface, 1/8 in. from mid-surface intersection	experimental	5.07	6.07
	present FEM	5.53	5.67

These comparisons indicate that excellent agreement exists between the experimental and the present FE results. It is therefore believed that the finite element models are capable of providing accurate stress predictions at the shell intersections.

## RESULTS AND DISCUSSIONS

The present results show that the maximum stresses occur in the longitudinal plane. Many other investigations [8-11] reached the same conclusion. Therefore, only the stresses at longitudinal plane are collected. Note that in the longitudinal plane, axial stress at the intersection is small; only the stress component in the direction tangential to the opening is significant. Therefore, at this location, the stress intensity (the difference between the maximum and minimum principal stresses) is taken to be approximately equal to the stress component in the direction tangent to the opening (hoop stress). Four equations have been developed based on FE results. They are as follows:

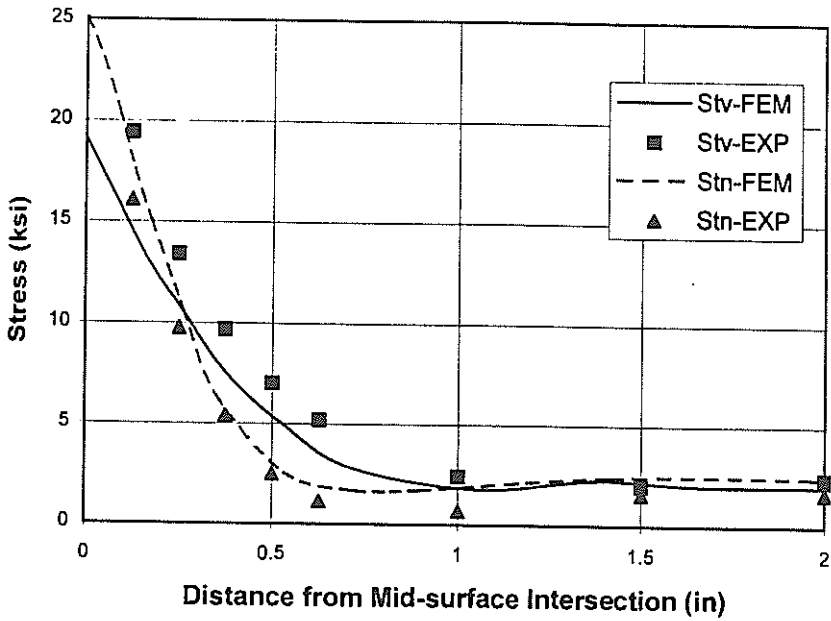


Fig. 2 Measured and predicted stresses on the outer surface at the longitudinal plane for ORNL-1

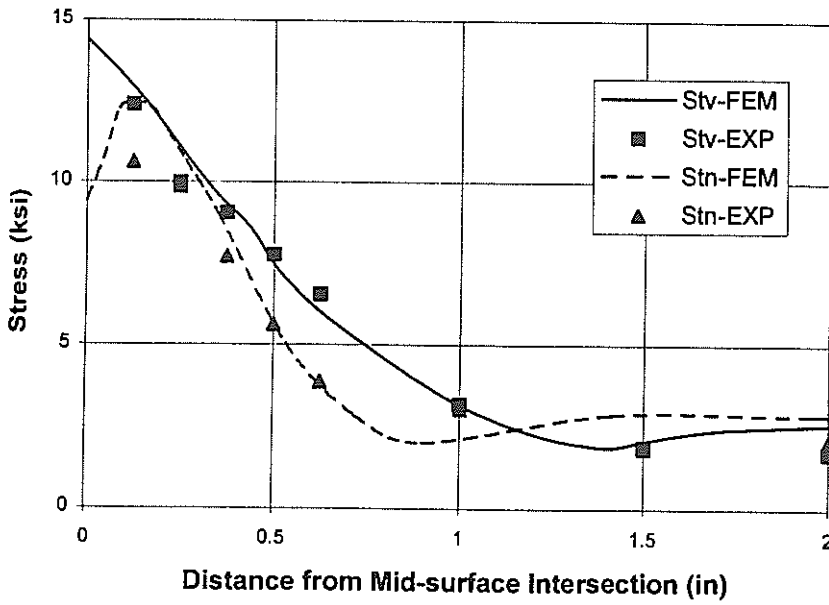


Fig. 3 Measured and predicted stresses on the inner surface at the longitudinal plane for ORNL-1

$$S_{tv}/S_0 = 1.115 - 1.599 (d/D)^{0.8} (D/T)^{0.3} (t/T)^{0.3} + 2.300 (d/D)^{0.8} (D/T)^{0.4} (t/T)^{-0.2} \quad (1)$$

$$S_{tmv}/S_0 = 1.061 + 3.123 (d/D)^{0.8} (D/T)^{0.3} (t/T)^{-0.1} - 2.291 (d/D)^{0.8} (D/T)^{0.2} (t/T)^{0.4} \quad (2)$$

$$S_{tn}/S_0 = 1.353 + 7.997 (d/D)^{0.9} (D/T)^{0.5} (t/T)^{0.0} - 7.044 (d/D)^{0.9} (D/T)^{0.5} (t/T)^{0.1} \quad (3)$$

$$S_{tmn}/S_0 = 1.255 - 0.032 (d/D)^{1.0} (D/T)^{0.6} (t/T)^{-2.0} + 0.674 (d/D)^{1.0} (D/T)^{0.5} (t/T)^{-1.0} \quad (4)$$

Equations (1) and (3) are for membrane plus bending stress in the vessel and nozzle, respectively; Equations (2) and (4) are for membrane stress in the vessel and nozzle, respectively. The applicable geometric parameter ranges are as follows:

$$0.333 \leq d/D \leq 1.0, \quad 20 \leq D/T \leq 250, \quad 0.333 \leq t/T \leq 3$$

In 1975, Decock [9] devised an empirical equation to predict the stress concentration factor (SCF) for pressure loading. His proposed equation was as follows:

$$SCF = [2 + 2(d/D)^{3/2}(t/T)^{1/2} + 1.25(d/D)(D/T)^{1/2}] / [1 + (t/T)^{3/2}(d/D)^{1/2}] \quad (5)$$

where SCF is defined as hoop stress at the inner crotch corner divided by  $(PD/2T)$ . The above equation was based on Decock's collection of 156 experimental results from steel and araldite models of branch junctions. Most of these 156 experimental results are for thick wall cylinder intersections. Only 10 out of 156 experimental results are for cylinder intersections with  $D/T > 60$ . Therefore, the applicability of Decock's equation for thin wall cylinder intersections ( $D/T > 60$ ) is open to question.

The comparison between Eq. (1) and Eq. (5) for  $t/T=0.5$  shell intersections is shown in Fig. 4. An examination of this figure indicates the following:

- a. Excellent agreement exists between these two equations in terms of the variation of the stress intensity factor as a function of  $d/D$
- b. For  $D/T=20$  shell intersections, these two equations yield almost exactly the same predictions. For shell intersections with  $D/T$  greater than 20, the predictions of Eq.(1) are a little higher than those of Eq.(5). Since there are only 10 out of 156 experimental results are for  $D/T > 60$ , it's believed that the predictions of Eq.(1) are better than those of Eq.(5) for  $D/T > 20$ .

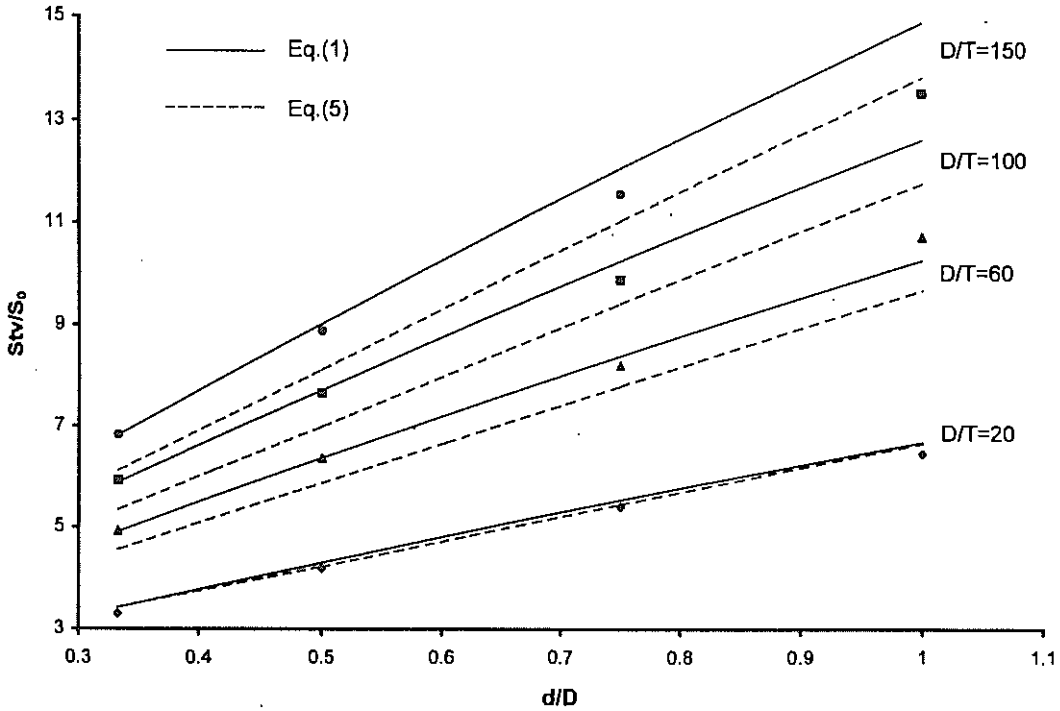


Fig. 4 Comparison of finite element data with Eqs. (1) and (5)

## CONCLUSIONS

Based on the research work described in this paper, the following conclusions can be reached:

1. The predictions employing the present finite element modeling are in excellent agreement with reliable experimental data. Thus, the present finite element models are capable of predicting the stress concentration factors of thin shell intersections.
2. The four correlation equations provide both membrane plus bending stress and membrane stress for both vessel and nozzle. The predictions of Eq.(1) are deemed to be more accurate than those of Decock's Eq.(5) for shell intersections with  $D/T > 20$ . It is expected that Eqs. (1) to (4) can serve as the basis of the improvement of design codes such as the ASME Boiler and Pressure Vessel code for the design of large diameter ratio thin shell intersections.

## ACKNOWLEDGEMENT

The support of the Pressure Vessel Research Council (PVRC) and the input received from the members of its Subcommittee on Shell Intersections are gratefully acknowledged.

## REFERENCES

1. Mokhtarian, K., Endicott, J.S., Stresses in Intersecting Cylinders Subjected to Pressure, Welding Research Council Bulletin 368, November 1991.
2. Mershon, J.L., Mokhtarian, K., Ranjan, G.V., Rodabaugh, E.C., Local Stresses in Cylindrical Shells due to External Loadings on Nozzles -- Supplement to WRC Bulletin No. 107, Welding Research Council Bulletin 297, August 1984
3. Widera, G.E.O, Hsu, S.Y., Stress Distribution in Vicinity of Oblique and Normal Shell-Shell Intersections, PVRC GRANT # 91-26, October 1994.
4. Natarajan, R., Widera, G.E.O., Afshari, P., "A Finite Element Model to Analyze Cylinder-Cylinder Intersections", Journal of Pressure Vessel Technology, November 1987, Vol.109, pp 411-420
5. Structure Research and Analysis Corporation, COSMOS/M User Guide and Command Reference, Version 1.71, June 1994.
6. Corum, J.M., Bolt,S.E., Greenstreet, W.L. and Gwaltney, R.C., Theoretical and Experimental Stress Analysis of ORNL Thin-Shell Cylinder-to-Cylinder Model No. 1, ORNL-4553, October 1972.
7. Gwaltney, R.C., Bolt,S.E. and Bryson, J.W., Theoretical and Experimental Stress Analysis of ORNL Thin-Shell Cylinder-to-Cylinder Model 2, ORNL-5021.
8. Xue, M.D., Hwang, K.C., Lu, W and Chen,W, "A Reinforcement Design Method Based on Analysis of Large Openings in Cylindrical Pressure Vessels", International Conference on Pressure Vessel Technology, Vol.2, 1996, pp197-205.
9. Decock,J, "Reinforcement Method of Openings in Cylindrical Pressure Vessels Subjected to Internal Pressure", Rep.No.MT104,Centre de Recherches Scientifiques et Techniques de L'Industrie des Fabrications Metalliques, February 1975.
10. Skopinsky, V.N., "Numerical Stress Analysis of Intersecting Cylindrical Shells", Journal of Pressure Vessel Technology, August 1993, Vol. 115, pp275-282.
11. Moffat,D.G, Mwenifumbo,J.A.M.,Xu,S.H.,Mistry,J.,"Effective Stress Factors for Piping Branch Junctions Due to Internal Pressure and External Moment Loads", Journal of Strain Analysis, Vol 26 No.2, 1991, pp85-101.