Transactions of the 15th International Conference on
Structural Mechanics in Reactor Technology (SMiRT-15)
Seoul, Korea, August 15-20, 1999

Topology of the Internal Ribbed Plates Cross Section

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ABSTRACT: Topology optimization is a tool for obtaining the optimal layout of a structure, in other words, the optimal topology of the structure. In this paper the algorithm for analysing the cross-section of the internal ribbed plates is presented. The problem for ribbed plates is to find the optimal ribs placing. The optimization process gives an answer on this question. Design for minimum compliance under the constraints of assumed mass is considered. In this case the three-dimensional domain of the internal ribbed plates is discussed as two-dimensional (cross-section ribs position). The material distribution in the cross section is finding out. Based on [1] the optimal topology is obtained within small numbers of steps in black and white (one - zero) mode distribution.

1 INTRODUCTION

This paper deals with the cross-section material distribution for internal ribbed plates (Fig. 1). These ribs are placed in regular intervals in one direction and they connected together two (upper and lower) thin facings. Former presented research (previous SMiRT Conferences) concerning ribbed shells is completed now by topology optimization of these structures. Recently different methods have been developed for optimal topology design in three-dimensions. In this contribution the cross section is considered and for this two-dimensional object the topology optimization procedure is employed. It is an open question right now, how to distribute ribs in optimal manner. The answer presented in this paper consists of two ways: in the first one for assumed ribs distribution topology procedure leads to confirmation of the assumption. The assumed volume of the material is satisfied, or sometimes it is a little bit smaller. The second way leads to not "full distribution" (what means that, there is not enough material for such ribs placing – in other words - mass constraint is too strong). In this case it is needed to change ribs placing and next to find out the topology for assumed mass constraints or to change the mass volume. It is only topology consideration and from the other hand one must check such changed structure, because of changing the mass volume and the shape.

Design for minimum compliance under the constraint of assumed mass is considered. The numerical implementation of the problem is done by FEM. The entire cross-section is mapping by finite element mesh, which is not changing during the process. This is very useful, because the same procedure is employed from the beginning to the end of the optimization process. Every element is identified with a unit cell of the structure. The density of the element mesh for the cross-section is the key for obtaining refined optimal topology where only black and white (one and zero) distribution is a final result of the process. Two steps of the topology optimization are usually presented here: The first one when at the obtained topology stage no more than a small number of elements have the density off the bounding one and zero levels. It is obvious that the density in these elements is close to one and they correspond to shades of grey in the figures. The mass constraint is satisfied. The second one (the next step): for only one and zero distribution, where the mass constraint is not fully satisfied sometimes. When the cross section is not divided into enough elements the postprocessing is needed. In such cases the postprocessing is usually carried out to get a smooth shape of the structure, but it is a tool which considers mainly geometrical aspects of the problem. Decisions about leaving or taking off the material from the cell (element) are making arbitrary and should be verified from equality point of view. It is mainly deals with the mass constraints, which should be kept, but sometimes with a small tolerance. To exceed the assumed mass volume in small range is acceptable proposition for the constraints. Presented method proposes using small adjustments in every optimization steps, what let to obtain refined optimal topology in small number of steps [1, 2] for the proper finite element mesh. In this paper because of the size of the computing problem the method will be shown using medium division into elements, but it is enough to shown the idea of the optimization procedure and to show how it works.

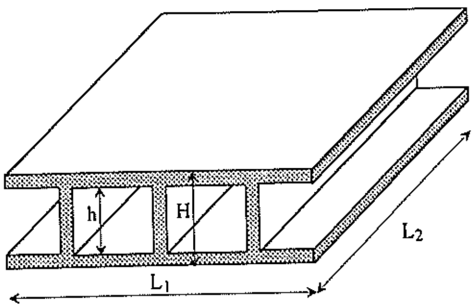


Fig. 1 The internally ribbed plate

2 PROBLEM FORMULATION AND FEM ALGORITHM

Topology optimization approach employed in this paper is based on [3], where the mean compliance of the structure is used to define the reverse of the stiffness. Designing is the process of maximizing the stiffness against applied loads. It corresponds to minimizing of the mean compliance. From the other hand the mean compliance is equal to the total strain energy of the structure.

$$2\,\Pi^{I}(u) = \Pi^{E}(u) \tag{1}$$

where the left and right sides are stated in the well known form:

$$\Pi^{E}(u) = \int_V \rho\, b^T u\, dV + \int_S t^T u\, dS$$
$$\Pi^{I} = \frac{1}{2}\int_V \varepsilon^T C \varepsilon\, dV \tag{2}$$

Matrix C is an elastic moduli matrix, which is a function of the Young's modulus. The Young's modulus is a function of the material density of the element j in every optimization step:

$$E_j(\rho_j) = E^0 \sum_{n=1}^{k} \left((\frac{\rho_j}{\rho^0})^n \right), \quad n \geq 1 \tag{3}$$

The sum sign is introduced here, because of better convergence of the solution. For every optimization step the Young's modulus is bringing up to date for every element separately. In this paper $k=3$. The problem is solved under the following mass constraint:

$$m_i \leq m_0 \tag{4}$$

where m_i is the mass at every *'i'* step in the optimization process and m_0 is an assumed mass of the structure. The structure has the mass amounting to $\alpha * m$ of the total mass:

$$m_0 = \alpha * m \qquad \alpha < 1 \tag{5}$$

Presented above the governing topology optimization equations are the based equations in his field, known in the literature, especially in [3]. Using them the algorithm for obtaing the topology of internal ribbed plates is presented. Because this bringing up to date follows the stresses what means that it follows the strain energy in elements, one can observe changing the Young's modulus in every element during the optimization process. The analysis is made on the element level. These elements are identified as a material points of the structure. The result of the procedure and the accuracy of the optimization depend on the finite element mesh. In elements where the energy decreases the density decreases too, but where the energy increases the density increases, too. This procedure let to separate these regions in the structure, which should have the density equal to zero, and from the other hand let to separate regions with the density equal to one. In the procedure the threshold function:

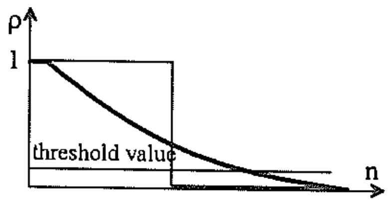


Fig.2 Idea of the optimization procedure

$$TF = 0.1 * \alpha * i \tag{6}$$

is employed [1]. It cuts the density values in every step. Starting at the first step from continues distribution of the density (the heavy line in Fig. 2) the procedure leads through the medium distribution to one zero distribution. The horizontal axis represents the number of elements. Normalized density is represented by the vertical axis.

3 EXAMPLES

As an example the cross-section of the ribbed plate is considered. The finite element mesh is shown in Fig. 3. The left mesh is 20x20 and the right one is 40x20 elements. The structure is loaded with the forces through the lower edge and clamped on its upper edge. This clamped edge is drawn symbolically. To solve the problem one must put boundary conditions and only in this means the upper edge is clamped and the material distribution is analysed assuming for this edge or for points on this edge (where ribs are introduced) displacements equal to zero. The results presented in this paper are subjected to qualitative analysis, the material data and dimensions are given as dimensionless. The cross-section has the mass amounting to $\alpha=0.30$ of the total mass ($m_0 = \alpha * m$) and $\alpha=0.15$ only in Fig. 11.

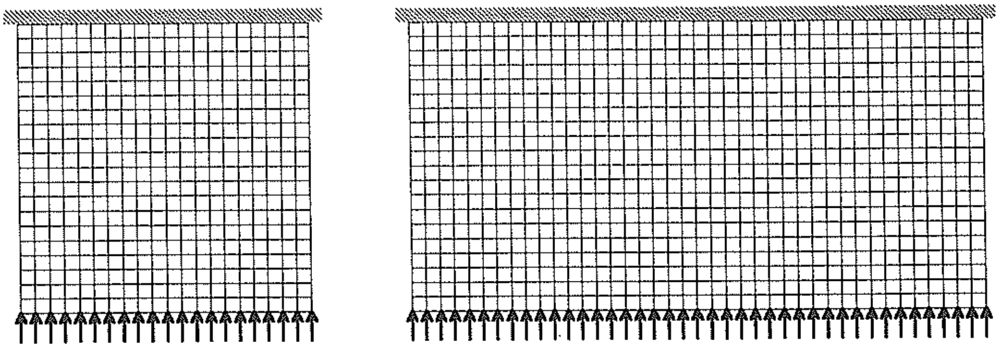


Fig. 3 FE mesh

Fig. 4 consists of four pictures (20x20 mesh). The first one (Fig. 4a) shows the initial mass distribution. The facings are assumed with the mass equal to one and it is one elements row on the upper and lower facing separately. Additionally in some places where ribs are planned, in some elements mass equal to one is assumed. In other elements the mass is assumed as $m_0=\alpha{*}m$ what should be marked by grey colour, but for simplicity this grey colour is neglected in the figures. The total mass can be described now as:

$$m_0 = \alpha{*}m(TE - AE) + AE \tag{7}$$

where TE is the total element number in the structure and AE is the number of elements where mass is assumed as equal to one. These elements are shown in discussed picture.

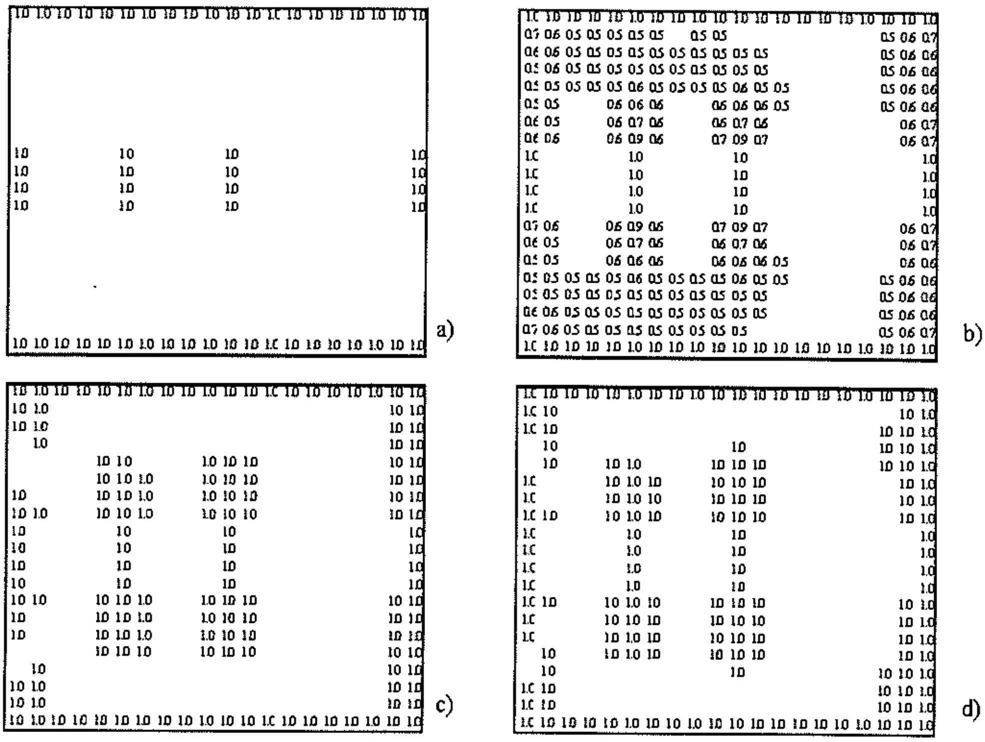
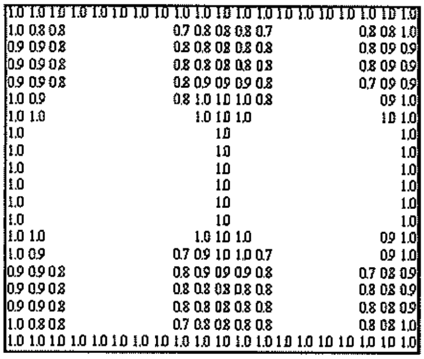
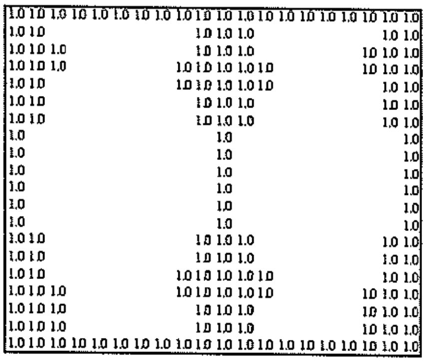


Fig. 4 Cross-section topology, assumed topology (a), step 10^{th} (b), 11^{th} (c) and 217^{th} (d)

The second picture (Fig. 4b) shows the topology after optimization when not a small percentage of the elements is off the one and zero bounded values. In the third picture (Fig.4c, the next step to the second picture) only one-zero distribution is obtained, but the mass constraint is not fully satisfied (the total mass should be 159.2, but it is 148.0). Between these two topologies there is the optimal one. To find it out the threshold function is changed. Instead 0.1 coefficient 0.005 is introduced. The number of steps must increase in this case. For 217th step the topology is shown in Fig. 4d. Unfortunately two introduced ribs do not connected facings. There is to many assumed ribs at the beginning. For the right edge the procedure gives better solution than for left one.



a)



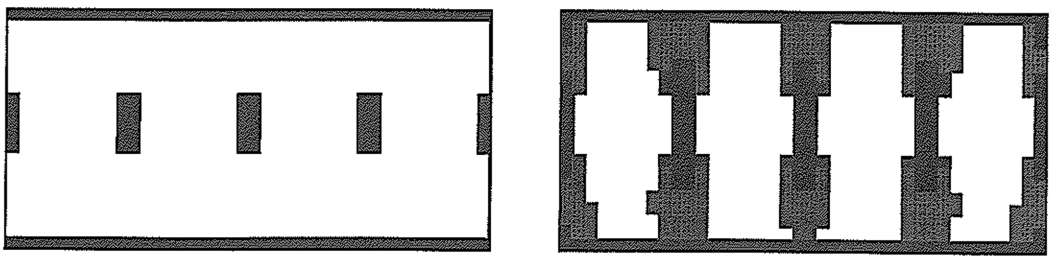
b)

Fig. 5 Topology for three initially assumed ribs for 10th step (a) and for 215th step (b)

Fig. 5 presents the topology for similar to Fig. 4 example, for which only on the left, right edges and almost in the middle ribs are introduced and six elements in each case have density equal to one (there are four in Fig. 4). Topology for 10th step is shown in Fig. 5a and for 215th step, for weaken 20 times the threshold function, in Fig. 5b. The optimal topology is obtained here within two processes. The first one gives the topology within small steps number, the second gives refined topology using the same, but weaken threshold function.

Generally the figures are presented in two modes. The first one uses numbers placed in elements (Fig.4 for example). This kind of presentation is better when density values are from wide range. The second presents the topology using black, white and sometimes shades of grey colours.

Next figures deal with 40x20 element mesh. Fig 6a presents assumed initial mass distribution and Fig 6b presents distribution for 11th step, where two shades of grey are appropriate to the density equal to 0.8 and 0.9.



a) b)

Fig. 6 Initial topology (a) and topology for 11th step (b)

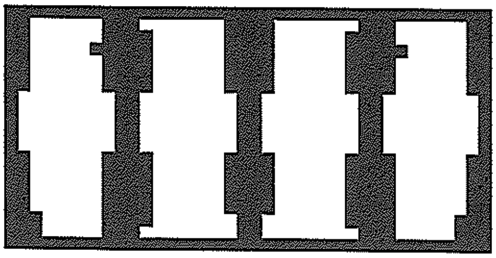


Fig. 7 Topology for 90th step

In Fig. 7 topology for the same as in Fig.6 example is shown, but the threshold function is weaken 8 times only and it is enough to obtain refined optimal topology within 90 steps. It is worth to mention that the mass constraint is satisfied exactly. Choosing the proper value of the threshold function is at the user's discretion and depends on his intuition, which is based on trial and error principle gathered by former researchers. Obtained topology has the vertical symmetry axis. On the upper facing it is loaded. The shape of the structure is proper to this loading and to boundary conditions.

Table 1 Facing deflection

step	deflection
8	9.46e-6
9	8.41e-6
10	6.04e-6
11	5.38e-6
12	8.42e-6
13	1.98e-3

Considered topology (Fig. 6b) satisfied minimum compliance (minimum strain energy) condition. In Table 1 as an example deflection of the point placed on the facing between two ribs for the succeeding steps is presented. The optimal topology is obtained for 11th step (Fig.6b), and for the same step the deflection (the strain energy) is minimum.

The next figure presents similar to Fig. 6 and Fig 7 example. The difference - assumed thickness of the ribs, they are thinner here. In Fig 8b 10th step is shown (for 11th only black and white distribution is obtained). This topology consists three shades of grey – one more than in Fig. 6b. Because this initial ribs placing is non symmetric the topology of 10th step is non symmetric too. The same can be observed in Fig. 8c. In this case small steps number is needed for optimal topology. For 11th step the mass constraint is not satisfied and it is a reason for presenting here 10th step. For 11th step only black and white distribution is obtained. The threshold function is weaken eight times only (quite like in Fig. 7), and final black and white

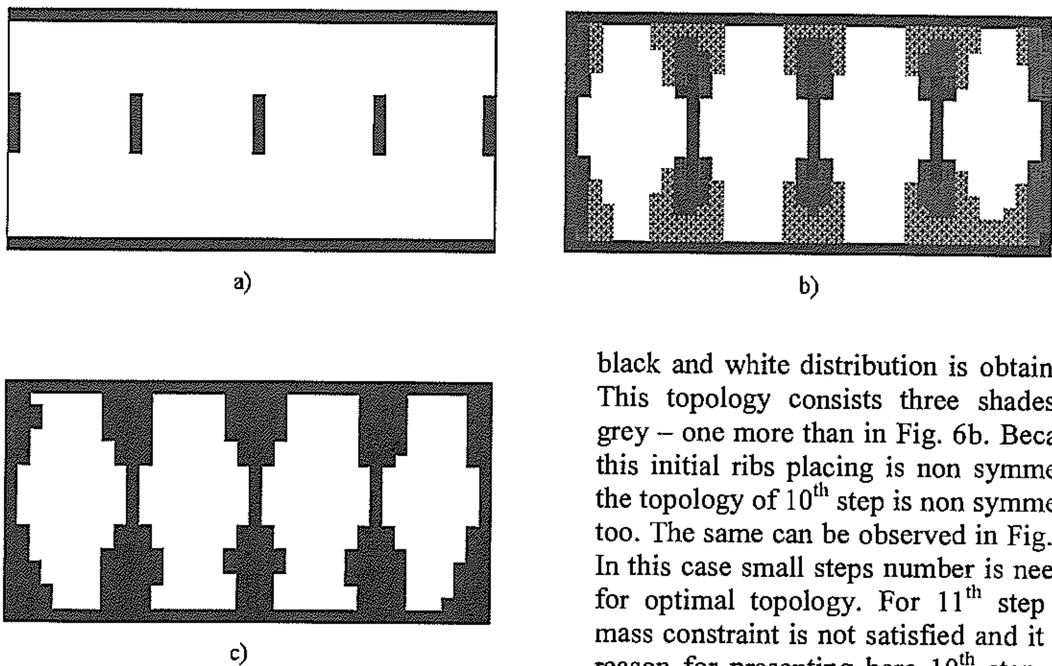
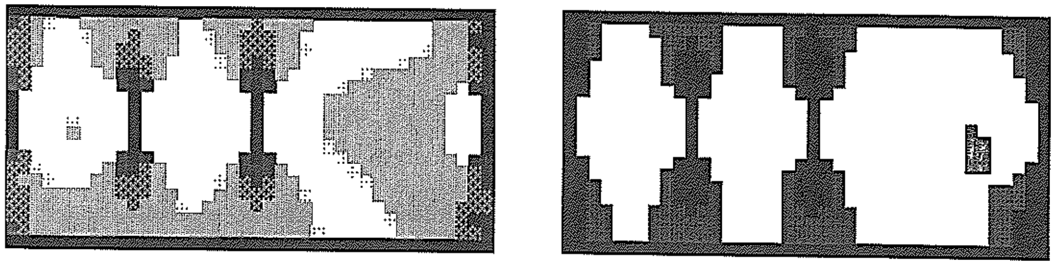


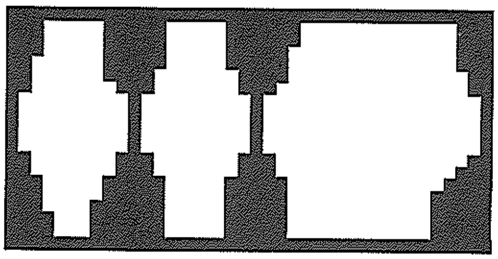
Fig. 8 Initial topology (a), topology for 10th step (b) and for weaken threshold function for 85th step (c)

topology is shown in Fig.8c.

The analysis for small ribs number is making now. Two next examples deal with the examples where only two and even one internal rib is left.



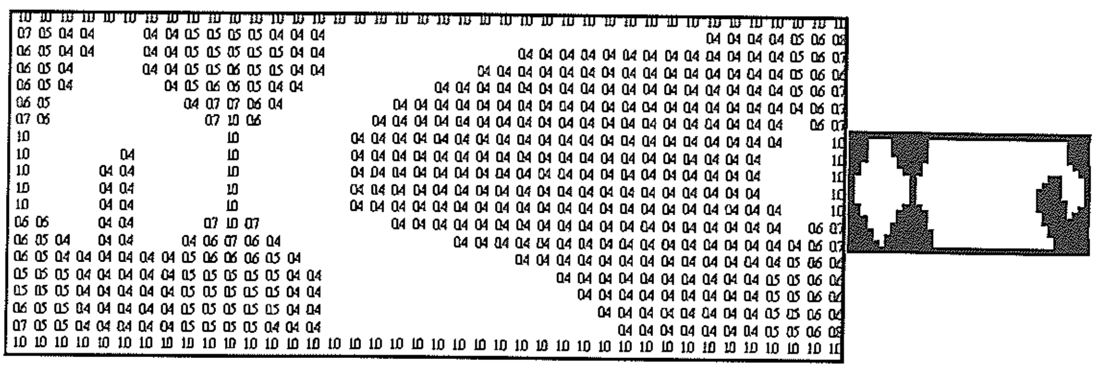
a) b)



c)

Fig. 9 Topology for 9th step (a), for 10th step (b), and for weaken threshold function for 81th step (c)

In Fig. 9 only two internal ribs are left. Especially step 9th is presented (Fig. 9a), because of its interesting topology. In the region where rib was not planned the procedure put the material. It is proper from the strain energy point of view. This picture consists the density values in the range from 0.4 to 1.0. There is no values equal to 0.9 and the first shade of grey to fit in 0.8 value. Most of "grey" elements have the density equal to 0.5 and only few 0.4. The next picture (Fig. 9b) consists density values 0.9 and 1.0. The shade of grey of 0.9 value is the same here as in Fig 9a is 0.8 value. Because there is no the rib in right part, the rib on the right edge is thicker than on the left edge. Some material is left without any connection with other material. To connect it the postprocessing is required or one can make the threshold function



a) b)

Fig. 10 Topology for 9th step (a) and for 10th step (b) smaller and can obtain smoother topology. In both cases the mass constraint is satisfied. In the case presented in Fig 9c 1.3% of the mass is lost. The ribs are distinctly thicker, because of initial assumption. If we compare Fig. 9b and Fig. 9c it is clear that this mass which have not connection in Fig. 9b is moved to lower part of the right rib. Besides that the topology is very similar in both pictures.

In Fig. 10 only one internal rib is left. For 9th step mass constraint is satisfied, for 10th 4% of mass is lost. When the threshold function is smaller this problem does not exist. It seems that the procedure leads to a new rib placed in the right part (Fig. 10b). The lost mass should be added to connect the rib to upper facing.

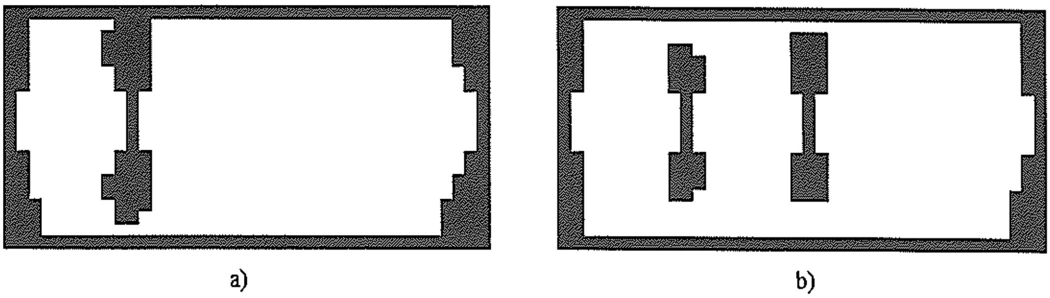


Fig. 11 Topology for α=0.15, one rib, 12th step (a) and two ribs, 13th step

In Fig. 11 the mass constraint is changed and assumed as $\alpha=0.15$. Topology is obtained for one rib (Fig. 11a) and for two ribs (Fig. 11b). In two cases the total mass volume is satisfied for presented steps number. The black and white distribution is obtained for 12 and 13 steps respectively. Unfortunately the mass constraint $(\alpha=0.15)$ is too strong and the ribs are not connected with facings.

4 CONCLUSION

In this paper the optimal topology for the cross-section of internal ribbed plate is considered Employed procedure demonstrated optimal black and white topology for the material density, which is chosen material property. The topology for various ribs placing and various mass constraints is presented. Satisfied refined topology is obtained for proper defined threshold function, which additionally let to save the mass constraint and let to get "smooth" topology for considered element mesh.

References

[1] Kutyłowski R., "Topology optimization – convergence problem", will appear *in Archives of Civil Engineering* (Poland) in 1999. The 6th Conference on Shell Structures, Theory and Applications – SSTA98, Gdańsk-Jurata, Poland, October 1998.

[2] Guedes, J.M., Taylor, J.E., "On the prediction of material properties and topology for optimal continuum structures". *Structural Optimization*, Vol. 14, 1997, pp.193-199.

[3] Ramm, E., Bletzinger, K.-U., Reitinger, R., Maute, K., "The challenge of structural optimization". *Advances in Structural Optimization,* 1994, pp. 27-52.