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Proposition of a Design Rule for Creep Buckling

A. Combescure

Lmt-Cachan/CEA-DMT-SEMT, France

Abstract

The prediction of creep buckling is a very difficult challenge because we have to face two difficulties in these types of problems. First, the buckling prediction of thin structures is very sensitive to a lot of parameters such as initial imperfections (shape and amplitudes) or boundary conditions. Second, variations and scatter of the creep behavior of a given material are rather wide.

The work presented in this paper is based on creep buckling of cylinders submitted to constant external pressure. It has been observed as well on experimental results as on theoretical model that the creep instability mechanism is very dangerous: the structure seems to have a stabilised deformation during a very long time but suddenly, at some critical time, the shell becomes unstable within a very short period of time. This phenomenon has been observed even with a very low level of membrane stress (10% of the yield stress). The explanation of this surprising effect is the combination of inevitable initial imperfections with creep deformations which grow as time passes and induce bending stresses increasing exponentially until failure. It is hence of interest to establish a rule to prevent a structure to collapse by this type of mechanism.

The paper presents the basic work done to establish a rule to avoid creep buckling. This rule is based on a simplified Shanley beam model. The model is presented and explained. The predictions of the simplified model are compared to finite element simulations on a wide range of parameters for a cylinder under constant external pressure. The model is also applied to the prediction of collapse time of two sets of experiments on cylinders subjected to external pressure: one of them is a thin cylinder (R/t=500), the other is a thicker one (R/t= 7.7). The experimental-theoretical comparison presented in the paper validate the methodology proposed to prevent creep buckling.

1 Introduction

The prediction of creep buckling is a very difficult task because of the accumulation of many uncertainties. For the creep characterisation of the material, one knows that the identification of the material parameters entering into the creep law is a very delicate task and that large uncertainties are often attached to the material parameters if the identification is not carried with sufficient care. For the buckling characterisation of thin shells the knowledge of the real geometry is of prime importance. This geometry has to be very well controlled if one wants to have a precise prediction of the failure loads. In the study of the fast-breeders safety the 316SPH steel designed not to creep at high temperatures can have to sustain temperature which are about 700°C in some accidental cases. Some of the structures are loaded by compressive stresses at these high temperatures and one can fear a loss of stability of some parts of the vessel. This type of situations also occur in the fuel tubes in accidental situations.

It has been observed as well on experimental results as on theoretical model that the creep instability mechanism is very dangerous: the structure seems to have a stabilised deformation during a very long time but suddenly, at some critical time, the shell becomes unstable within a very short period of time. The work presented in the paper frst gives the equations of a simple model which can be used to predict the buckling times of simple structures under constant load such as beams under constant axial compression or cylinder under uniform constant pressure. The model is based on a simple Shanley beam representation. The predictions of the model are then compared to to experimental tests on two typical configurations of cylindrical shells under external pressure: one of them are very thin (radius-to-thickness ratio R/h=500) and the other are much thicker (R/h= 7.6). In part 4 the comparison is done with finite element simulation on the same problem. Finally a general design procedure based on the simplified formula is proposed to design a gneral shell against creep buckling.

2 Theoretical model

21 introduction

The theoretical model is described in details in [1]. The model is shortly presented hereafter. We consider a cylinder subjected to both a uniform external pressure P_0 and to a zero axial load; the cylinder is therefore being subjected mainly to a constant compressive membrane hoop stress σ, given by the following equation:

$$\sigma = P_0 \frac{R}{h} \qquad (1)$$

This cylinder is then allowed to buckle. We shall call σ_E the elastic buckling hoop stress of this cylinder and P_E the corresponding applied load. The cylinder also has shape imperfections, and we shall suppose that these imperfection take the shape of the elastic buckling mode which was found to be associated with the circumferential Fourier mode m and axial mode I. The initial imperfection has a normalised amplitude of ξ_0. The cylinder will also be subjected to bending stresses due to this imperfection; the hoop bending stresses will be maximum in the mid plane of the cylinder. The stress state in this shell is mainly circumferential in the region of the mid plane. Since we will be seeking the amplification of the initial imperfections by creep, the creep law in the mid plane, is driving the deformation of the whole shell. We have also supposed that the evolution of the shell imperfection is not very different from the evolution of an inextensionnal mode m imperfection around the circumferential direction. Hence, we have replaced the problem by an imperfect ring model, exhibiting the same R/h ratio as the cylinder, an initial imperfection in the circumferential mode m, and the same material properties as the tested cylinder: this ring is subjected to a uniform external pressure, constant over time.

22 GEOMETRY and LOADING

The basic geometry of the model is an imperfect ring with an initial imperfection in the shape of the buckling mode and of an amplitude ξ_0 $(\xi_0 = d_0 / h)$. In our case, the buckling mode of the tested cylinder is a Fourier mode m. We shall thus study one sector of the ring restricted by the angle π/m, due to symmetry conditions. The mean radius variations of the ring during creep deformations will be neglected. The ring is subjected to a circumferential membrane load Q (Q= h σ), constant over time. The membrane hoop stress associated with the external pressure shall be denoted by σ.Figure 1a shows the geometry of the imperfect ring.

23 MATERIAL

We assume that the ring exhibits an elasto-viscoplastic behavior. We also make the hypothesis that the material nonlinearities are concentrated in the sections at the ring's two extremities A and B (angles 0, and π/m). Next we evaluate the stresses in this section only on the two shell surfaces and make the simplified hypothesis that the stress is linear between the two extreme fibres. This model is represented in Figure 1b; it is called the Shanley model [2]. The creep law can be chosen as any type of usual creep law. In particular, equations (2) and (3) will be studied.

(2) $$\varepsilon^{vp} = a\sigma^{n} Log(1+bt)$$

(3) $$\varepsilon^{vp} = A\,\sigma^{N} t^{p}$$

In these equations ε^{vp} represents the creep strain, σ the stress in MPa, and t the time in hours. a, A, n, N, b, p are material constants. The plastic model used is that of perfect plasticity, with the yield stress being σ_y. The Young's modulus will be denoted E. We suppose that the section is hinged if one of the extreme fibre is plastified.

24 INSTABILITY MECHANISM

The instability mechanism applied is the following: when the initially imperfect ring is loaded by the external pressure P_0, it undergoes a constant axisymmetric displacement (w_0) which produces a hoop membrane stress σ as well as a mode m displacement field, which is supposed to be inextensional. The initial mode m imperfection amplitude ξ_0 increases to a value of ξ_1. If this does not lead to instantaneous failure by the formation of a plastic hinge or elastic instability, then the creep mechanism can take place. This leads to an increase in the deflection until the total normalised deflection ξ $(\xi = d\,/\,h)$ reaches a critical value ξ_{cr} associated with either elastic instability or the plastic hinge at the extremities. This model is of course simplified; it incorporates the hypothesis that when one fibre of the ring is plastified, the structure hinges. This is visualised in Figure 2. Figure 2 is, in fact, the basic diagram for the evaluation of creep instability. If a relatively high initial load is applied on a very perfect structure, elastic instability induced by the evolution of the initial imperfection by creep is likely; this is also the case when the material is essentially elastic. If the initial imperfection is greater, or the initial load P_0 lower, it can easily be imagined that the instability will be of the plastic hinge type.

25 ANALYTICAL SOLUTION

We recall in this section the results obtained in [1]. In the case of creep equation (2), the critical time t_{cr} is given by the following equation:

$$t_{cr} = \frac{1}{b}\left(\exp(I) - 1\right) \qquad (4)$$

$$I = \int_{\xi_1}^{\xi_{cr}} \frac{\alpha d\xi}{f(\xi)} \qquad (5)$$

In this equation, we have:

$$f(\xi) = (1.+6\xi)^n - sign(1.-6\xi)|1-6\xi|^n \qquad (6)$$

$$\xi_1 = \frac{\xi_0}{1-\dfrac{\sigma}{\sigma_E}}, \xi_{cr} = \frac{1}{6}\left(\frac{\sigma_y}{|\sigma|} - 1\right) \qquad (7)$$

and:

$$\alpha = \frac{12}{Ea\sigma^{n-1}}\left(\frac{\sigma}{\sigma_E}\left[1+\frac{1}{m^2}\right]-1\right) \qquad (8)$$

Along the same lines as explained in [1], it can be found, in the case of an Andrade type of creep law (equation (3)), that the critical time t_{cr} is given by:

$$t_{cr} = I^{\frac{1}{p}} \qquad (9)$$

where l is given by equation (5), and n and a are replaced by N and A respectively in equations (6) through (8).
It can be observed that we have now an explicit solution for the critical buckling time of the ring.

2.6 COMMENTS ON THE GENERAL SOLUTION

The limitations of this simple formula should be recalled. The critical time given by equations (4) or (9) is limited to thin shells submitted to small stresses which remain in the elastic range. The hypothesis that the creep strain is concentrated in the extreme fibres of the ring hinges has also been made. The application of this prediction equation to practical cases will provide indications about the effects of these simplifications on the predicted buckling times. Similar equations can be derived in the case of an imperfect beam subjected to a compressive axial load. The beam equations are the same (equations (4) and (9)) but we only have to replace $1+\frac{1}{m^2}$ by 1 in equation (8).

3 experimental-model comparisons

The model has been compared to two sets of experimental results.
Eleven thin cylinders under external pressure. The mean radius was 0.075m and the mean thickness was 0.146 microns, the nominal initial imperfection amplitudes ξ_0 lie between .08 to .3; the detailed description can be found in [3]. The creep law used for the analysis is given

by equation (2) with the following constants:

$$a = 5.67\,10^{-10}\;(MPa)^{-n} \quad n = 2.81 \quad b = 0.26\,(hours)^{-1}$$

The predicted results are always conservative but, for three cases, the conservatism on the buckling time is rather high. Table 1 below shows the experiments-model comparison.

SHELL N°	Radius /thick	Euler Pressure	Applied Pressure	Imperfection ξ_0	Experiment critical time	Computed time	Imp	Imp	Imp	Imp
	R/h	P_E (.0001MPa)	P_0 / P_E %	nominal %	$t_{exp}(hours)$	$t_{comp}(hours)$	ξ_{cr} %	maxi %	mini %	mean %
030191	426	333	78	18	210	25	10,5	18	8	13
210292	487	309	65	45	144	22	35,5	45	15	30
010692	452	268	74	18	912	267	15,4	18	12	15
030692	540	171	58	7	>2500	>10000		32	7	19.5
040692	560	149	81	18	72	46	16,4	40	10	25
050692	510	210	71	32	75	72	31,8	34	7	20.5
060692	556	157	86	8	72	68	8	35	7	21
070692	551	171	82	15	70	26	12	35	6	20.5
080692	556	164	91	3	36	67	2,5	30	3	16.5
210493	532	178	81	19	30	20	17	25	19	22
220493	528	182	77	26	75	28	21,2	45	26	35.5
MEAN value	518	208	77	19	154	58,3	15.5	32.5	11	21.75

Table 1: Experimental creep buckling test results and simplified predictions for thin cylinders

The thick cylinders had a radius of 4.46mm a thickness of .58mm and a length of 140mm. They buckle in circumferential mode 2. The initial ovalisation lie between .01 and .11. Their creep law is given by equation (3) with the following constants: $A = 7.039\,10^{-8}\;(MPa)^{-N}(hours)^{-p}, N = 2.04\,, p = 0.43$. Figure 3 gives for a series of experiment the comparison between the buckling experimental situations and the model prevision. One sees that the predictions can be considered as good. A systematic comparison can be seen in [3]. The models here again performs well. These experiments predictions tend to indicate that the model approximations remain sufficient to predict the creep buckling of cylinders under external pressure.

4 Computational-model comparisons

The analytical model was also compared to finite element predictions of creep buckling of cylinders under external uniform pressure constant in time. These simulations were nonlinear incremental simulations taking into account large displacements, creep and plasticity of the model as well as the follower pressures; in effect when the structure deforms due to pressure the bending moments, due to imperfection, increase significantly differ from the initial ones: this effect is crucial for the bending plastification of the structure. A whole series of computations on different cylinder types were compared to simplified formula predictions. The detailed comparison can be found in [1]. Figure 4 permits to compare the model predictions with the finite element simulation for one of the tested cases; the model is conservative. The simplified approach is found to always be conservative; furthermore, this conservatism is higher for thicker structures, as could be expected by the model's initial hypothesis. For a given initial imperfection-time limit couple, the critical stress determined using the simplified approach is about three quarter of the value predicted by the finite element approach.

5 Design rule Application

Let us suppose that we have constructed a set of diagrams of the type given on figure 5 for a given steel (eg 316 SPH). The diagram are given for each temperature in the creep range and are depending on the P_E / P_L ratio. The design methodology can be now developped in the following way for a given loading:

a) determine the temperature at which your structure is working: if it is below the creep limit stop; else chose the diagrams with the temperature just above your temperature.

b) look for the yield stress at this temperature, and compute the elastic buckling load at this temperature P_E

c) compute the load P_L for which the structure plastifies, deduce the structure slenderness ratio P_E / P_L and choose the diagram having to the slenderness parameter just above.

d) if P_0 is the applied load compute the ratio $x=\frac{1.2\ P_0}{P_E}$

e) determine the nominal imperfection $y=\xi_0$

f) if the point M (x,y) is under the failure curve for the the failure time there shall be no creep buckling.

6 Conclusion

This paper has presented a design method based on a simple Shanley model, and validated by comparison on creep buckling experiments and on finite element simulations of creep buckling of cylinders under external pressure. The methodology has been applied to different type of steels and seems to be good enough. The application for 316SPH steel is also available and forms the basis of the design rule to prevent creep buckling. There is no safety margin on the failure times. The safety has been introduced by increasing the applied load by 20%. This choice can be discussed. The aim of this paper was to present the whole methodology to produce a creep buckling design rule: the design rule choices (over estimations of stresses, times, imperfections) are to be done by the relevant persons.

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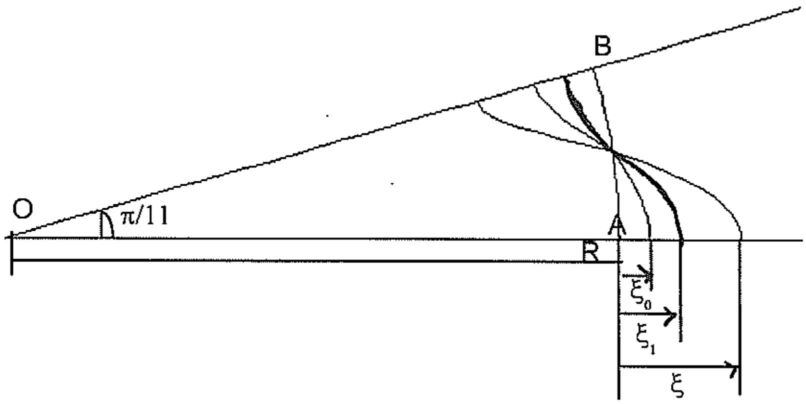

Figure 1a: the ring model

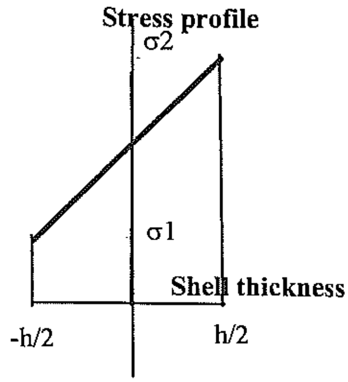

Figure 1b: stress profile at points A

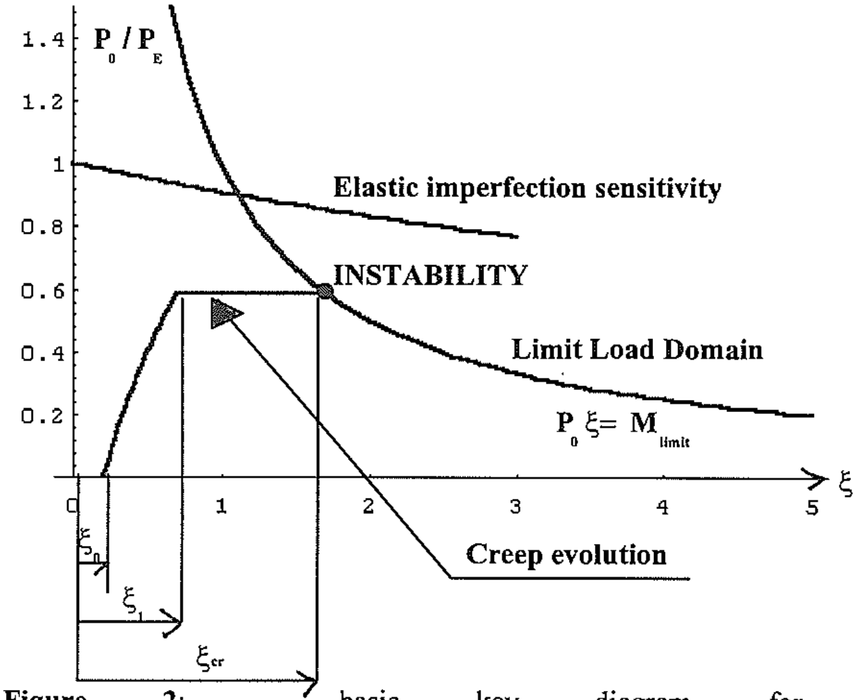

Figure 2: basic key diagram for creep buckling

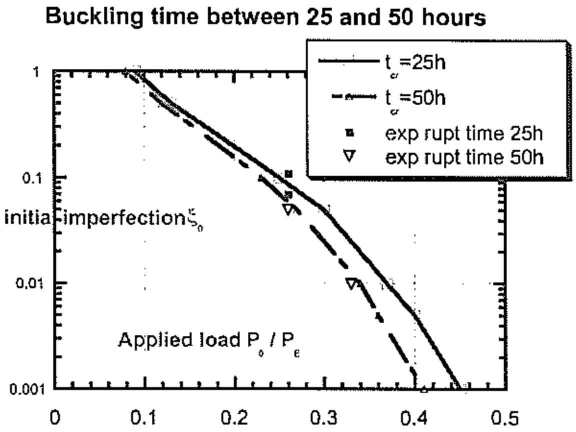

Figure 3: Model experiment failure time short buckling times thick cylinders

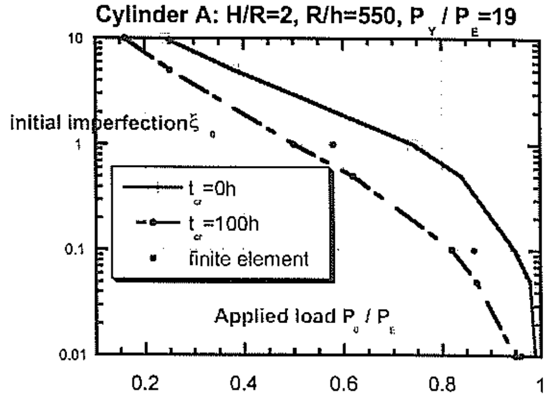

Figure 4: model-finite element comparison for Cylinder A failure time 100h

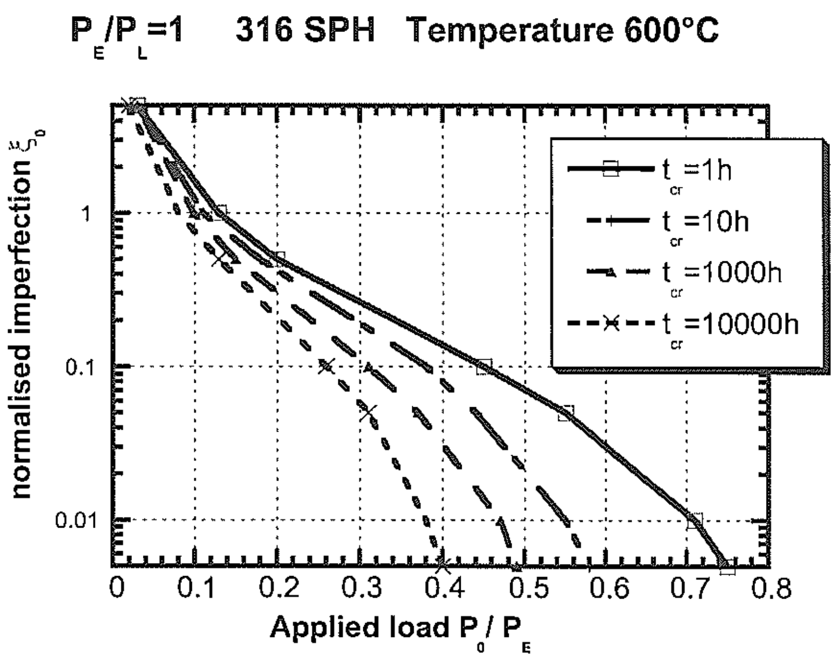

Figure 5: Basic design diagram for a 316SPH structure at 600°C having a PE/PL of 1.