



Limit Load of Elbows under Combined Internal Pressure and Bending Moment

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ABSTRACT : Analyses are performed to determine the effect of internal pressure on the limit moments of elbows of various pipe bend factors. Both opening and closing mode of bending moment is considered. Limit moments are evaluated from the moment vs. end rotation curves by twice elastic slope method. Moment - rotation curves are generated through non-linear finite element analysis. The results show that an elbow under opening bending moment is stiffer than that under closing bending moment. Internal pressure increases the limit moments of elbows up to a certain limit beyond which limit moments reduces with further increase in internal pressure. In case of straight pipes, increase in internal pressure consistently reduces the limit moments.

NOMENCLATURE

$h = tR_b / r_m^2$	Pipe bend factor
M_L	Limit moment
$m_L = M_L / 4r_m^2t\sigma_r$	Normalised limit moment
P	Internal pressure
$p = (Pr_m) / (2t\sigma_r)$	Normalised internal pressure
R_b	Mean bend radius of elbow
r_m	Mean radius of elbow cross section
t	Wall thickness of elbow cross section
σ_r	Material flow stress

INTRODUCTION

Pipe bends or elbows are commonly used components in a piping system. They are very flexible compared to the straight pipes. Pipe bend normally reduces the reaction forces and moments within the piping system and it becomes easier to satisfy the stress limits. Because of this increased flexibility, they are forced to accommodate large displacements arising from the differential thermal movements. However, care must be taken so that deformations of the bend remain predominantly elastic. Otherwise, the resistance to deformation may decrease rapidly leading to the failure of the system. It is, therefore, important to know how close a loaded bend is to its collapse load for the safe operation of the plant. At the collapse load, the deformation

of the elbow increases significantly without much increase in load. Different studies [1-2] had earlier been carried out to give the closed form solution of the limit load or the plastic collapse load of elbows. There are mainly two shortcomings in these studies : First, they are either true for pure internal pressure or for pure bending moment. In the actual piping system, the loading is usually the combined internal pressure and bending moment. Secondly, most of these expressions are valid for pipe bend factor, $h = tR_b / r_m^2 < 0.5$. In some cases, for example, primary heat transport piping system of nuclear power plant, the pipe bend factor exceeds 0.5 and thus the above expressions cannot be directly used. Recently, Shalaby and Younan [3,4] have undertaken the studies to address the first issue i.e. to determine the plastic collapse load of elbow under combined internal pressure and bending moment. However, the second issue is not addressed in their studies. The maximum pipe bend factor taken by them was 0.44. In the present study, collapse moment of the elbows have been evaluated for various combination of internal pressure and the pipe bend factor. The pipe bend factor varies from 0.24 to 0.6 in this study.

GEOMETRY

Fig.1 shows a 90° elbow. Geometrically, an elbow is characterised by three parameters, namely, mean cross sectional radius (r_m), wall thickness (t) and mean bend radius (R_b). Table 1 shows the different combinations of these parameters taken in the study. The mean bend radius is always kept as three times the mean cross sectional radius in the present study. The r_m/t varies from 5 to 12.5 and pipe bend factor, $h = tR_b / r_m^2$ varies from 0.24 to 0.6. In the present analyses, the elbow is connected with straight pipes of length equal to the six times the mean cross sectional radius. This is to allow the ovalisation effect to die down at the point where load is applied.

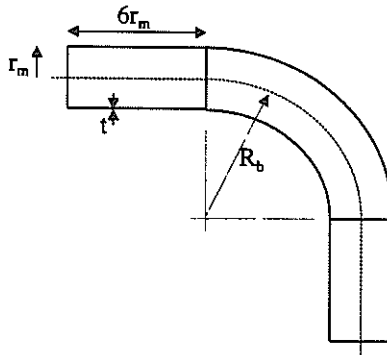


Fig.1 Geometry of a 90° elbow

LOADING

The load in the elbows is split in two components : a constant internal pressure and varying in-plane bending moment increasing in definite steps. The rationale behind this is that internal pressure rarely increase during service. Whereas, bending moment may increase significantly in an accidental condition. Thus it is of interest to predict collapse moment of an elbow for a constant internal pressure. Internal pressure is expressed in a non-dimensional form, $p =$

$(Pr_m) / (2t\sigma_f)$, where P is the applied internal pressure and σ_f is the material flow stress taken as the average of yield and ultimate strength. The value of 'p' are varied here from 0.0 i.e. pure bending moment to $p = 0.319$. Closed end condition is simulated by applying axial pressure of intensity $Pr_m / 2t$. There are two modes of in-plane bending moment : closing and opening. Closing mode bending moment tries to close the elbow and opening mode bending moment tries to open the elbow. Kussmaul [5] had reported significant difference in deflection behaviour of elbow for this two modes of bending moment. Consequently, in the present study, both opening and closing bending moments are considered separately. Bending moment has been simulated as triangularly varying face pressure. However, within an element, face pressure has been kept constant. The face pressure value is obtained as ' $M.c/I$ ' where ' M ' is the applied bending moment, ' I ' is the area moment of inertia of the elbow cross section and ' c ' is the vertical distance of the element face centre from the neutral axis. The application of bending moment in this way avoids the unwanted plastic deformation at the point of load application.

Table 1 Geometry of the Elbows

r_m (mm)	t (mm)	r_m / t	R_b / r_m	$h = tR_b / r_m^2$
250	20	12.50	3	0.240
250	28	8.93	3	0.336
250	35	7.14	3	0.420
250	40	6.25	3	0.480
250	45	5.55	3	0.540
250	50	5.00	3	0.600

FINITE ELEMENT MODEL

Twenty noded solid elements are used to model the elbow. Because of symmetry, only one fourth of the elbow is modelled. There are total 195 elements and 1508 nodes. Fifteen elements along the circumference, thirteen elements along the elbow - straight pipe axis and one element across the thickness are taken to model the elbow. The same mesh pattern is used for all the cases. Non-linear finite element analysis has been carried out to determine the limit moments of elbow for various geometric and loading combinations. Both geometric and material non-linearity are considered in the analysis. It has been seen in the course of this analysis that consideration of geometric non-linearity is very important to correctly capture the elbow deflection under various combinations of the closing and opening bending moment and internal pressure. True stress - true strain of a typical nuclear grade piping steel is used in the analysis. Five points are considered to define the stress - strain diagram of the material. Table 2 shows the material properties used in the analysis. General purpose finite element program NISA [6] is used for the analysis.

Table 2 Material properties used in the analysis

Yield stress (MPa)	270				
UTS (MPa)	513				
Flow stress (MPa)	391.5				
True stress (MPa)	300	370	450	520	605
True strain	4.76×10^{-3}	0.0174	0.042	0.079	0.167

DEFINITION OF LIMIT MOMENT

Limit moment at plastic collapse has been evaluated from the moment vs. end rotation curves by twice elastic slope method. In this method, a tangent to the initial linear part of the moment-rotation curve is drawn. The angle (θ_1) that this tangent makes with the vertical axis of moment is measured. Then another straight line is drawn at an angle (θ_2) with respect to the vertical axis such that $\tan(\theta_2) = 2 \tan(\theta_1)$. The intersection of the second line with the moment - end rotation curve is defined as the limit moment. Moment vs. end rotation curves are generated through non-linear finite element analysis. Limit moments are expressed in non-dimensional form $m_L = M_L / 4r_m^2 t \sigma_f$.

RESULTS AND DISCUSSION

Figures 2 & 3 show the moment vs. end rotation curves for elbows with different pipe bend factors under pure closing and opening mode bending moment respectively. Figure 4 shows the comparison of deflection behaviour of thin ($r_m / t = 12.5$) and thick ($r_m / t = 5$) elbow for two different modes of bending moment. It can be seen that closing moment vs end rotation curves becomes almost flat after applied moment exceeds certain values indicating the collapse of the elbow. Whereas, the same elbow when subjected to opening mode bending moment, shows the rising nature of moment vs. end rotation curves. The effect is more prominent in case of thin walled elbow. Figures 5 - 8 show the effect of internal pressure on the moment vs. end rotation curves for both closing and opening mode of bending moment. It can be seen that internal pressure stiffens the elbow compared to when it is subjected to pure bending moment. The stiffening effect is more significant for closing mode of bending moment. From all these moment vs. end rotation curves, limit moments are evaluated by twice elastic slope method. To compare the effect of internal pressure on the limit moments of elbows and the straight pipes, analyses are also carried out on straight pipes having the same ' r_m / t '. Figures 9 - 11 show the effect of internal pressure on the limit moments of straight pipe and elbows. From fig.9, it is clear that limit moments reduce gradually with application of internal pressure for straight pipes. When the large internal pressure is applied, limit moments drop sharply. The stresses generated due to internal pressure are added to the stresses produced by bending moments which expedites the collapse of the straight pipe. In case of elbow, the phenomenon is a bit different. Here, limit moments increase gradually with application of internal pressure, reach a peak and then start falling with further increase in internal pressure. The ovalisation of the elbow cross section plays an important role in its collapse. The application of uniform internal pressure opposes the ovalisation of the elbow cross section, thus delaying the collapse phenomenon. Ovalisation is more prominent in case of thin walled elbow. That is why, internal pressure enhances the limit moments significantly in thin walled elbow. However, this process

continues up to a limit. If the internal pressure is increased further, the hoop stresses due to internal pressure overshadows the beneficial effect on the limit moments and finally the limit moments start reducing with increase in internal pressure.

CONCLUSIONS

The following conclusions can be drawn from the above analysis :

- a) An elbow subjected to opening mode of bending moment is stiffer than when subjected to closing mode of bending moment. In case of closing mode, moment vs. end rotation curves become almost flat after certain deformation. The curves show the rising trend in case of opening mode.
- b) The application of internal pressure enhances the limit moment of an elbow up to a certain limit. Limit moment starts falling beyond this with further increase in internal pressure. The effect of internal pressure is more prominent in thin walled ($r_m / t = 12.5$) elbow compared to thick walled ($r_m / t = 5$) elbow.
- c) The application of internal pressure progressively reduces the limit moments of straight pipes.

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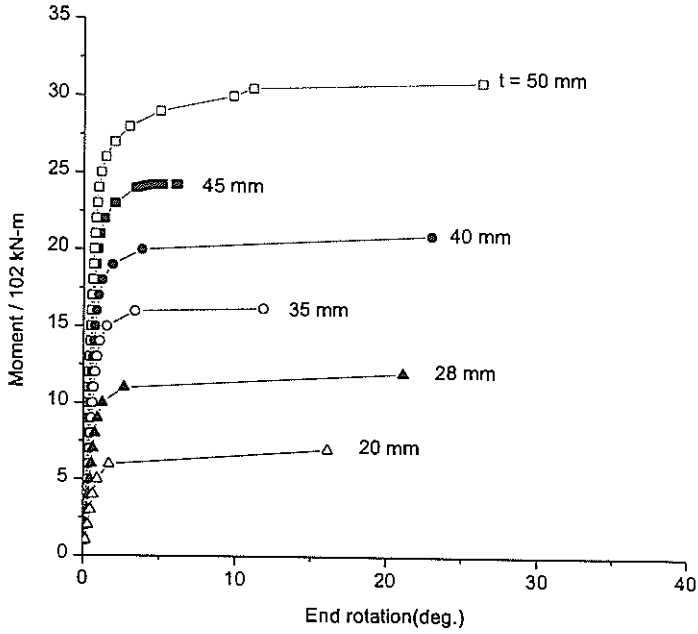


Fig. 2 Moment vs end rotation curves for elbows of different thickness under pure closing moment

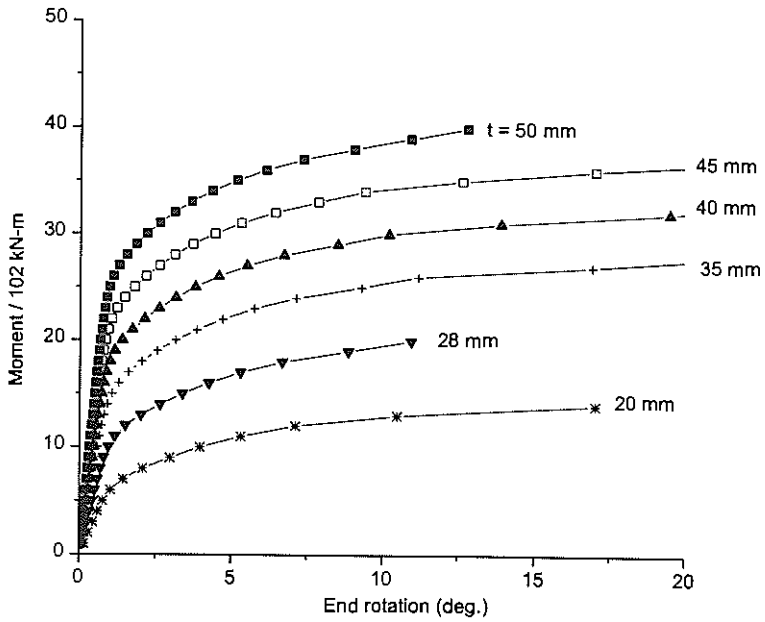


Fig.3 Moment vs. end rotation for elbow with various wall thickness under pure opening moment

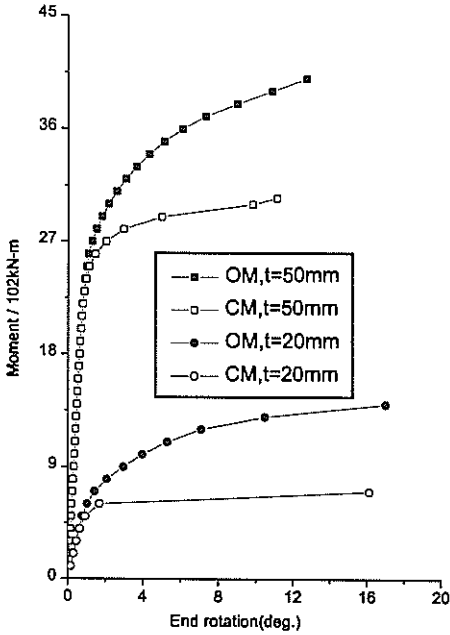


Fig. 4 Different moment - end rotation curves for elbows of thickness 20 and 50 mm under opening and closing mode

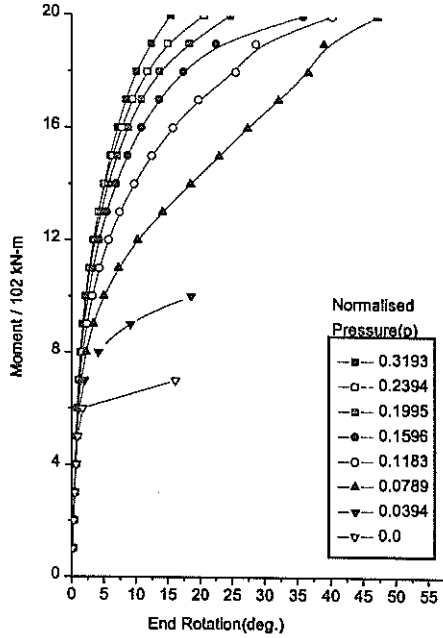


Fig.5 Closing Moment vs end rotation curves for elbow of 20mm thickness under different internal pressure

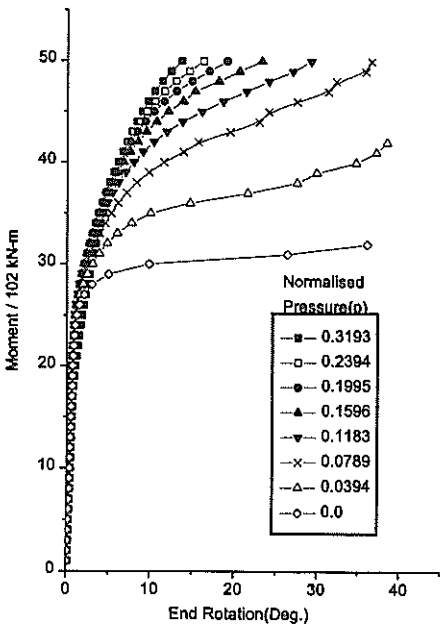


Fig.6 Closing moment vs end rotation curves for elbow of 50 mm thickness under different internal pressure

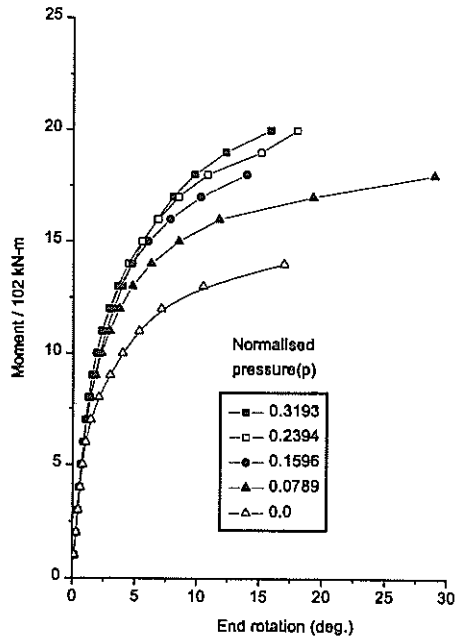


Fig. 7 Opening moment vs end rotation curves for elbow of 20 mm thickness under different internal pressure

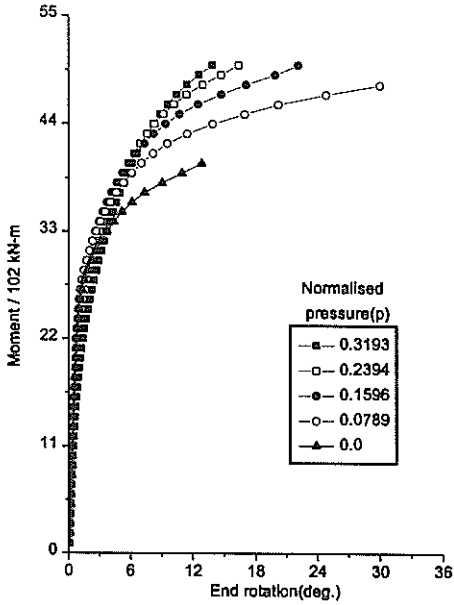


Fig. 8 Opening moment vs end rotation curves for elbow of 50 mm thickness under different internal pressure

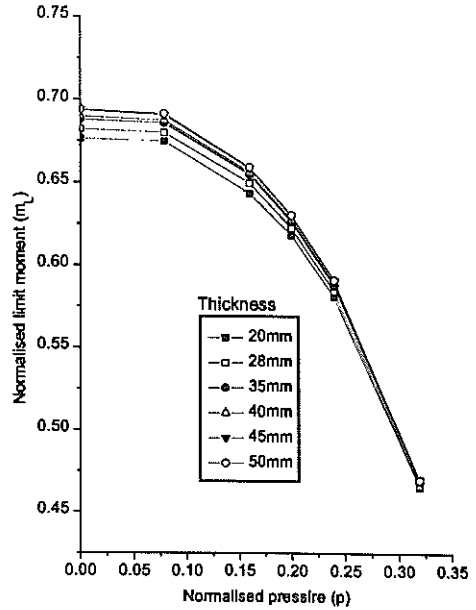


Fig. 9 Variation of limit moments of different straight pipes with internal pressure

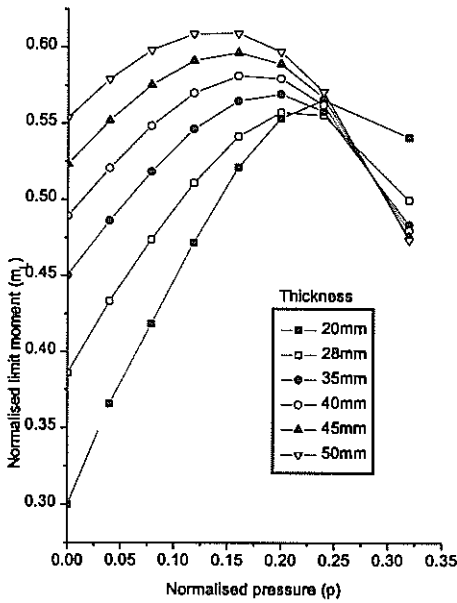


Fig.10 Variation of closing limit moments of different elbows with normalised internal pressure

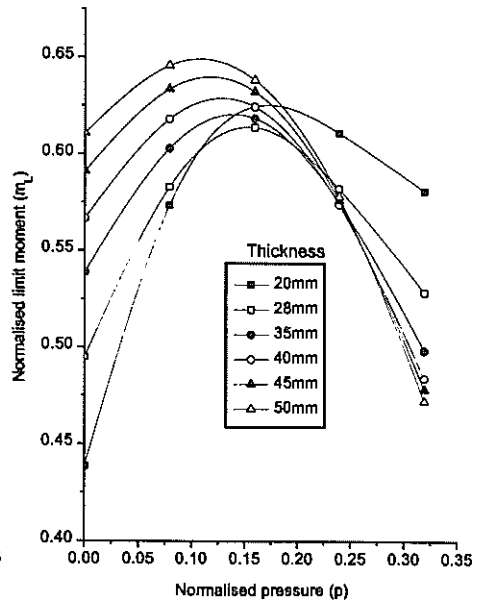


Fig. 11 Variation of opening limit moments of different elbows with normalised internal pressure