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On the Numerical Integration of Multiple Cracks Elastoplastic Models for Concrete

A. Millard

CEA SACLAY-DMT/SEMT, France

ABSTRACT

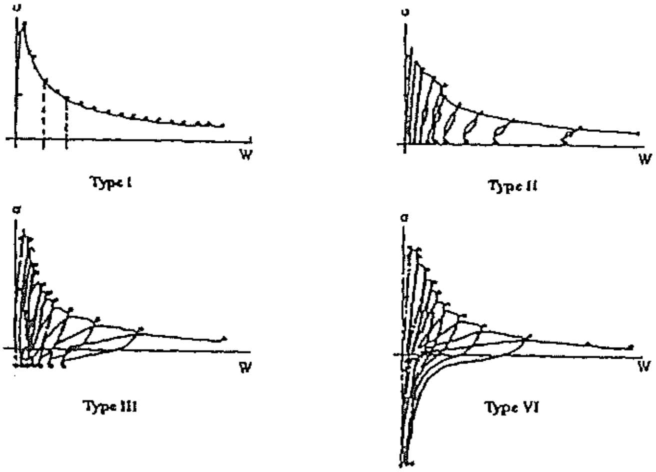
The accurate prediction of cracks pattern in a complex structure subjected to various loadings, is a major issue in safety evaluations of nuclear buildings. In this framework, the smeared crack approach is often preferred to handle distributed cracking. To be of practical use, models must incorporate various features like multiple cracks, induced anisotropy, degradation of elastic stiffnesses, permanent strains, closure and reopening of cracks, etc. These features lead to specific difficulties in the numerical integration of the constitutive equations, in particular because of the mutual influence of cracks at a given point. In this paper, we propose a robust algorithm based on a sub-stepping strategy coupled to the tangent cutting plane algorithm, to handle these difficulties.

1. INTRODUCTION

The importance of cracking of concrete, in the general non linear response of a reinforced concrete or prestressed concrete structure, is now well recognised. The modelling of concrete cracking has led recently to a very large amount of work, in particular in conjunction with some intrinsic difficulties like loss of uniqueness and localization of strains. For some structures, like nuclear reactor containments, the prediction of a realistic crack pattern is a fundamental issue, with respect to the evaluation of the possible leak of radioactive products, for example in case of a severe accident.

In the same way, the evaluation of the ductility of a structure during an earthquake, and the prediction of its non linear dynamic response, require to be able to model the cracks development, as well as their evolution under alternate loadings.

At this stage, the difficulties arise from the variety of aspects which must be properly described in order to reproduce a realistic concrete behaviour. Some of them are illustrated on figure 1, from ref. [1], which shows the uniaxial response of a concrete sample under cyclic loadings.



I : Monotonous uniaxial traction test
II : Cyclic uniaxial traction test
III : Cyclic uniaxial traction + low compression test
IV : Cyclic uniaxial traction + high compression test

Figure 1 : Uniaxial response of concrete under cyclic loads (from [1])

Examination of tests results available in the literature leads to the following list of properties required for a realistic model :

- non linear behaviour under compression, in pre-pic regime,
- strain-softening in compression and in traction (post-pic regime),
- permanent strains in compression and traction,
- cracking induced anisotropy,
- elastic stiffness degradation,
- partial closure of cracks under load reversal.

This list is not exhaustive, since it does not mention neither specific behaviours of concrete under high confinement, nor time dependent features. Of course, some of the above properties may be irrelevant for some specific applications. However, for the safety problems outlined before, they must be accounted for.

In this paper, we shall focus on the behaviour of concrete under traction loads, because of its importance for the problems considered and its consequences for the numerical integration of constitutive laws.

Concerning the numerical treatment of concrete cracking, there are basically two kinds of approach : one, which is called discrete crack approach and consists in modelling explicitly the kinematic discontinuity associated with the crack, and the other one which is called smeared crack approach, and consists in accounting for the discontinuity by means of the material law. The former approach is more difficult to implement, since the location and the direction of the crack are not known before hand, and therefore, its use is limited to a very small number of propagating cracks. The latter, originally proposed by Rashid [2], is well adapted to distributed cracking as expected in reinforced concrete structures. The relation between these two approaches was introduced by Hillerborg [3], by means of the fictitious crack concept : If a localized cracking is modelled using the smeared crack approach, then the material model used must dissipate the same amount of energy during the crack formation.

In this paper, we adopt the smeared crack approach. However, the choice still exists between very many different models such as elastic non linear models, plasticity models, damage models, etc. Although a large number of models available in the literature do not offer the desired characteristics as enumerated before, the model proposed by Ottosen and Dahlblom [4] has been retained because it brings simple solutions to the aforementioned problems, together with the use of the fictitious crack concept. In the next paragraphs, we will recall the Ottosen's model and then detail the numerical integration procedures.

2. PRESENTATION OF OTTOSEN'S MODEL

In order to represent the behaviour of cracked concrete in a smeared way, the model assumes a strain partition into an elastic part (which may be related to the behaviour of intact concrete between cracks) and a fracturing part (which must be understood as the smeared contribution of the cracks) :

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{f} \tag{1}$$

The elastic strains are classically related to the stresses through Hooke's law, such as :

$$\varepsilon_{ij}^{e} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\,\delta_{ij} \tag{2}$$

where E stands for Young's modulus and ν for Poisson's ratio.

Following the fictitious crack concept, the fracturing strains are derived from relations between stresses and crack opening and shearing displacements. A simplifying assumption of Ottosen's model is that, in the general tridimensionnal situation, three orthogonal cracks may develop at a given point. Once a crack is initiated, its direction is fixed and induces some anisotropy in the response of the cracked concrete. A crack is initiated when the normal stress in some direction exceeds the concrete tensile strength σ_t. For the clarity of the presentation, we will assume that the local axis oxyz define the three directions normal to the crack planes at a given point. For example, for the crack orientated by the normal vector ox, the crack opening displacement is described by a general expression :

$$W_{xx} = W_{xx}(\sigma_{xx}) \tag{3}$$

which has been taken as linear for simplicity (Note that a more complex and probably more realistic function might be taken without difficulty) :

$$W_{xx} = \frac{1}{N}\left(\sigma_{xx} - \sigma_t\right) \tag{4}$$

The normal response of the crack is illustrated on figure 2.

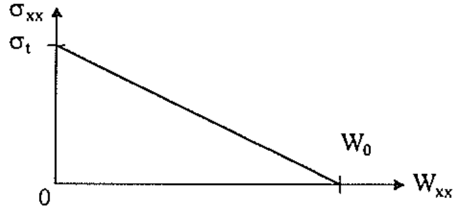


Figure 2 : Normal response of a crack

The smearing of the cracks is achieved by means of equivalent lengths, dependent of the finite element mesh size (see figure 3), and relating displacements and strains such as :

$$\varepsilon_{xx}^{f} = \frac{W_{xx}}{L_x} \tag{5}$$

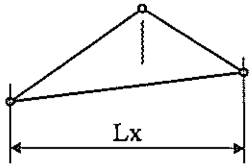


Figure 3 : Illustration of element size characteristic length

If a crack does not yet exist along a direction, then the inverse of the corresponding characteristic length is set to zero. If the opening displacement reaches a rupture value W_0, then the concrete is completely cracked and can no longer sustain any load along the corresponding direction. At this stage, it is very easy to incorporate in the model some basic features to handle closing and reopening of cracks. Indeed, for closure, it is assumed that only partial closure of the crack may happen. Thus, in the closing phase, a linear relation is assumed :

$$W_{xx} = \left[\alpha + (1-\alpha)\frac{\sigma_{xx}}{\sigma_{xx}^{max}}\right] \cdot W_{xx}^{max} \tag{6}$$

where σ_{xx}^{max} and W_{xx}^{max} correspond to the stress and crack opening displacement reached before unloading. The parameter α may be adjusted to simulate total crack opening recovery ($\alpha = 0$) or zero recovery ($\alpha = 1$), as illustrated on figure 4 :

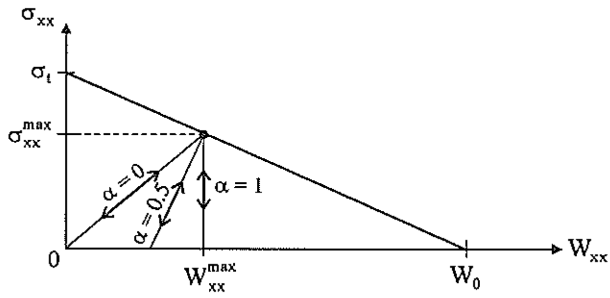


Figure 4 : Closing and reopening of a crack

When the concrete is completely cracked, i.e. for $W_{xx} > W_0$, closing will occur for $W_{xx} = \alpha W_0$.

3. NUMERICAL INTEGRATION OF MULTIPLE CRACKS MODEL

When integrating a model such as the above presented one, a major difficulty arises from the conjunction of the strain-softening behaviour together with the couplings between different directions through Poisson's effect. As a consequence, in a single step, a cracked direction may undergo non monotonous regime, like first an elastic loading, then a plastic loading and finally an elastic unloading, due to the non linear behaviour along the other directions. Such a situation is illustrated on figure 5.

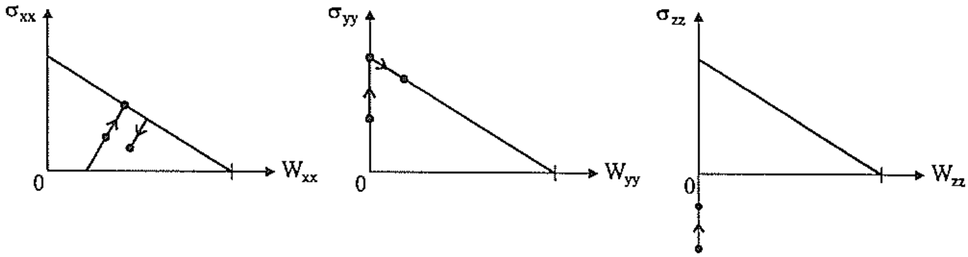


Figure 5 : Non monotonous response due to Poisson's effect

One solution, which is currently proposed in the literature, consists in setting Poisson's ratio to zero as soon as cracking occurs. However, this is not satisfactory since it has no physical basis, and, moreover, it does neglect some couplings which indeed exist between the various cracking directions and it may then lead to an erroneous response of the material (for example, in the above configuration, direction x may go on with plastic loading instead of elastic unloading). Another difficulty lies in the treatment of the totally cracked material, associated with a zero stress. Again, a trick often used consists in assuming that the softening branch is described by an exponential law, such that the zero stress be never reached. Here also, it is desirable to represent the fully cracked material for a given finite crack opening displacement W_0.

For the purpose of numerical integration, we consider the material as an elasto-plastic damaged material. The yield domain is described by a multi-criterion : Each potential crack plane is associated with a maximum principal stress criterion. Moreover, it is easy in such an approach to account for other criteria such as a compression criterion, or even a cap criterion for high confinement pressure loadings.

Various methods have been proposed for the integration of constitutive equations in case of non-smooth multisurface plasticity. We refer here to the work by Simo and colleagues [5]. However, we shall use a modified version of the algorithm they have proposed, according to the following points :

- because of the coupling effects between cracked directions, we must introduce a sub-stepping strategy, where the sub-steps size will be limited by the physical changes occurring during the time step,
- instead of a "closest - point - projection" algorithm, which is an implicit algorithm, we propose to use the “tangent cutting plane” algorithm, introduced by Ortiz and Simo [6], which will facilitate the determination of the sub-steps size, due to its explicit nature.

In fact, the choice of an implicit projection algorithm within each sub-step could be made, leading to a non linear equation to be solved for the sub-step size. Note that the idea of the sub-stepping for concrete cracking can be found in a paper by R. de Borst in 1987 [7]. However, to our knowledge, it does not seem that the method has been much implemented in computer codes.

In order to account for closing and reopening of cracks, we distinguish between inelastic strains associated with closing and reopening cracks and inelastic strains associated with propagating cracks :

$$\Delta\varepsilon = \Delta\varepsilon^{e} + \Delta\varepsilon_1^{f} + \Delta\varepsilon_2^{f} \tag{7}$$

Since the integration method belongs to the category of elastic predictor - plastic corrector algorithm, we start by the damaged elastic prediction of a trial state, such as :

$$\sigma_T = \sigma_0 + \Delta\sigma \tag{8}$$

where

$$\begin{aligned} \Delta\sigma &= D_0 \cdot \left(\Delta\varepsilon^{e}\right) = D_0\left(\Delta\varepsilon - \Delta\varepsilon_1^{f}\right) \\ &= D_I \cdot \Delta\varepsilon \end{aligned} \tag{9}$$

In these equations, σ_0 denotes the initial state of stresses, D_0 the uncracked concrete Hooke's matrix, D_I the damaged concrete elasticity matrix and $\Delta\varepsilon$ the given total strain increment.

The closing - reopening crack strain is thus given by :

$$\Delta\varepsilon_1^{f} = \Delta\varepsilon - D_0^{-1}\, D_I\, \Delta\varepsilon \tag{10}$$

From this trial state, σ_T, it is possible to determine if the stress state is admissible or if some criteria are violated. More generally, during a given sub-step, we will have to distinguish between the criteria which are active, that is on which projection must be carried out, and criteria which are not yet active but which can be reached during the projection. The former will be noted f_i while the latter will be noted F_ℓ. Therefore, only a proportion of the strain increment will be dealt with during the sub-step : $x.\Delta\varepsilon$, where x will be calculated simultaneously with the plastic projection, as described here after.

Suppose that we have, at the beginning of the considered sub-step, n active criteria such that :

$$\forall_i = 1, n \qquad f_i(\sigma_0, Y_0) \cong 0 \tag{11}$$

where Y_0 denotes the initial state of internal variables.

Then, the classical "tangent cutting plane" algorithm [6] can be written as follows :

- <u>Step 1</u> : initialisation by elastic predictor

$$\begin{aligned} k &= 0 \\ \sigma^{(0)} &= \sigma_0 + D_I \cdot \Delta\varepsilon \\ Y^{(0)} &= Y_0 \\ \Delta\varepsilon_2^{f(0)} &= 0 \end{aligned} \tag{12}$$

- <u>Step 2</u> : iterative "plastic corrector" $\quad k = k + 1$

Calculation of "plastic strain" increments :

$$\begin{aligned} \left[\delta\lambda_i^{(k)}\right] &= \left[A^{(k)}\right]^{-1} \cdot \left[f_i\left(\sigma^{(k-1)}, Y^{(k-1)}\right)\right] \\ \delta\varepsilon_2^{f(k)} &= \Delta\varepsilon_2^{f(k)} - \Delta\varepsilon_2^{f(k-1)} = \sum_{i=1}^{n} \frac{\partial g_i}{\partial\sigma}\left(\sigma^{(k-1)}, Y^{(k-1)}\right) \cdot \delta\lambda_i^{(k)} \end{aligned} \tag{13}$$

Updating of stresses and internal variables :

$$\sigma^{(k)} = \sigma^{(k-1)} - D_0 \cdot \delta\varepsilon_2^{f(k)}$$
$$Y^{(k)} = Y^{(k-1)} + \sum_i h_i\left(\sigma^{(k-1)}, Y^{(k-1)}\right) \cdot \delta\lambda_i^{(k)} \tag{14}$$

- *Convergence tests on :*

$$f_i\left(\sigma^{(k)}, Y^{(k)}\right) \quad \forall_i = 1, n$$

In the above equations, g_i denote the non associated flow potentials corresponding to the f_i criteria, h_i are the hardening evolution-functions, and the matrix $A^{(k)}$ has the following expression :

$$A_{ij}^{(k)} = \frac{\partial f_i^t}{\partial \sigma} D_0 \frac{\partial g_j}{\partial \sigma} - \frac{\partial f_i^t}{\partial \sigma} \cdot h_i \cdot \delta_{ij} \tag{15}$$

During the iterations, the positivity of the plastic multipliers :

$$\forall_i = 1, n \qquad \Delta\lambda_i = \sum_k \delta\lambda_i^{(k)} \tag{16}$$

is checked, and we keep as active criteria only those for which $\Delta\lambda_i > 0$.

In order to determine the size of the sub-step, the above algorithm is modified as follows :

- Step 1 : initialisation by elastic predictor

$$\begin{aligned} k &= 0 \\ Y^{(0)} &= Y_0 \\ \Delta\varepsilon_2^{f(0)} &= 0 \\ \sigma^{(0)} &= \sigma_0 + x^{(0)} \cdot D_1 \Delta\varepsilon \end{aligned} \tag{16}$$

with $x^{(0)} = \underset{\ell}{\mathrm{Min}}\left[F_\ell\left(\sigma_0 + x_\ell \cdot D_1\Delta\varepsilon, Y_0\right)\right] = 0$ and $x^{(0)} \leq 1$

- Step 2 : iterative "plastic corrector" $\qquad k = k + 1$

Calculation of "plastic strain" increments :

$$\left[\delta\lambda_i^{(k)}\right] = \left[A^{(k)}\right]^{-1} \cdot \left\{\left[b^{(k)}\right]\delta x^{(k)} + f_i\left(\sigma^{(k-1)}, Y^{(k-1)}\right)\right\} \tag{17}$$

where $\left[b^{(k)}\right] = \left[\frac{\partial f_i^t}{\partial \sigma} \cdot D_1 \cdot \Delta\varepsilon\right]$

and

$$\delta\varepsilon_2^{f(k)} = \sum_{i=1}^{n} \frac{\partial g_i}{\partial \sigma}\left(\sigma^{(k-1)}, Y^{(k-1)}\right) \cdot \delta\lambda_i^{(k)} \tag{18}$$

Calculation of $\delta x^{(k)}$ and updating of stresses and internal variables :

$$\delta x^{(k)} = \underset{\ell}{\mathrm{Min}}\left[F_\ell\left(\sigma^{(k)}, Y^{(k)}\right) = 0\right] \tag{19}$$

with $x^{(k+1)} = x^{(k)} + \delta x^{(k)} \leq 1$

and

$$\sigma^{(k)} = \sigma^{(k-1)} + \delta x^{(k)} D_I \Delta\varepsilon - D_0 \delta \varepsilon_2^{f(k)}$$
$$Y^{(k)} = Y^{(k-1)} + \sum_i h_i\left(\sigma^{(k-1)}, Y^{(k-1)}\right) \cdot \delta \lambda_i^{(k)} \tag{20}$$

Convergence tests on :

$$f_i\left(\sigma^{(k)}, Y^{(k)}\right) \quad \forall_i = 1, n$$

Note that in Eq. (19), quantities in the right hand member can be expressed explicitly in terms of $\delta x^{(k)}$, thanks to our choice of an explicit integration scheme. However, the final Eq. (19) may be non linear and its resolution may require some iterative process.

The above integration scheme has been implemented in the general purpose finite element code CASTEM 2000 [8] for bidimensionnal (plane strains, plane stresses, axisymmetric) as well as tridimensionnal cases.

4. CONCLUSION

In this paper, we have considered a smeared crack model, capable of representing some basic features of concrete behaviour, which are of prime importance for realistic predictions in case of safety evaluations of nuclear buildings, under various circumstances. The occurrence of multiple cracks at a given material point leads to numerical difficulties in the integration of the constitutive equations, due to the mutual interaction of the cracks and the treatment of closing and reopening of cracks. A robust algorithm based on a sub-stepping strategy, coupled with a "tangent cutting plane" algorithm projection has been proposed and implemented. It has been successfully tested on some critical configurations.

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