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Damage Assessment of Structures Using Static and Dynamic Test Data

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ABSTRACT : The purpose of the present study is to propose an improved damage detection and assessment algorithm basis on the method of system identification and to determine the structural parameters from the combined error responses. In this approach, the complete sets of modes or displacements are not needed since the error function involves only the differences between components of those vectors. The present approach allows the use of composite data, which consist of static displacements and eigenmodes. In the dynamic test, the slopes of mode shapes are introduced to formulate the error responses. Using these techniques, a parametric study is conducted and the effectiveness of the method is discussed. Generally, the developed algorithm is also able to detect and assess damages in a structure when the measured data are sparse. To examine the capability of the proposed damage assessment algorithm, a series of tests for a planar truss structure of steel arch bridge are performed.

INTRODUCTION

A variety of system identification methods for evaluating damage in structural systems have been developed and are still actively studied in many areas. The objective of the present paper is to propose an improved method that can identify a finite element model of a structure, capable of providing structural characteristics that are consistent with those measured in static and dynamic tests. In this paper, a modified error function for statistical parameter estimation is presented and a new formula for calculating the standard deviations of estimated parameters is derived. The method maintains the specified finite element character of the model and uses the values of the structural properties originally assigned to the model by the engineer as the starting point. The method then modifies original property values to make the structural characteristics conform to those observed in tests. Accuracy of the values of the test data and the engineer's confidence in the values of the original analytic model properties are incorporated into the procedure. This procedure requires known input to the structure, and the reliability of the algorithm depends on the multiple input test technique.

FORMULATION OF PROBLEM

In a finite element formulation, the characteristics of a structure are defined in terms of the stiffness, damping, and mass matrices. Any variations in these matrices, such as may be introduced by damage, would affect the dynamic response characteristics of the structure. A change in the stiffness matrix alone would influence the static displacement responses. It is very clear that a variation in the system matrices results in a changed response, it is more important from a damage assessment standpoint to relate these differences to changes in specific elements of the system matrices. Since internal structural damage typically does not result in a loss of material, one can assume the mass matrix to be a constant. Typically, the stiffness matrix can be expressed in terms of the sectional properties of various elements such as the cross-sectional area and the bending and polar moments of inertia. There is also a dependence on element dimensions and on the extensional and shear moduli. The mass and stiffness matrices are both functions of the structural parameters of the system. Therefore, if one denotes the structural parameters of interest by $p_j (j=1,\cdots,n_p)$ then the eigenvalues and eigenvectors of the structure and static deflections under prescribed loads will also be implicit functions of these same parameters of the n-degree of freedom system.

The functional relationship between the modal characteristics, static responses, and structural parameters is expressible in terms of a first-order Taylor series expansion. It follows that

$$\begin{Bmatrix} \Lambda \\ \Phi \\ \Gamma \end{Bmatrix} = \begin{Bmatrix} \Lambda \\ \Phi \\ \Gamma \end{Bmatrix}_{P_p} + S\{P - P_p\} \qquad [1]$$

where P is $(n_p \times 1)$ vector of individual structural parameters, p_j, P_p is $(n_p \times 1)$ vector of prior estimates of structural parameters and the subscripts P_p denoting the responses of the system when the parameters are $P = P_p$. Λ is a vector of measured eigenvalues, Φ and Γ are vectors of dynamic and static error responses respectively. Let n_r be the total number of responses for target structure, S is $(n_r \times n_p)$ partial derivative or sensitivity matrix of eigenvalues and dynamic and static error responses. Nelson's method[1] is used for analytical calculation of eigenvector derivatives in this paper.

$$S = \left[\frac{\partial \Lambda}{\partial p_j} \ \frac{\partial \Phi}{\partial p_j} \ \frac{\partial \Gamma}{\partial p_j} \right]^T \qquad [2]$$

Formulation of Dynamic Error Response

The changes in the curvatures of mode shapes are noticeable in the region of damage compared to the changes in the displacement of mode shapes.[2] Therefore, the differences in the curvatures of mode shapes can be used to

constitute the error response vector as well as to be instructive in estimating the uncertainty of structural parameters. Due to the structural characteristics of a target structure, there are some cases where the curvatures of mode shapes cannot be applied. In this case, the slopes of mode shapes can be applied. From the writer's experience, the changes in the slopes of mode shapes are noticeable in the vicinity of damaged zone compared to the changes in the displacement of mode shapes. Therefore, a new parameter called the slope of mode shape, which corresponds to each member, is investigated as a possible candidate for formulation of error response in this paper. The measured mode shapes, ϕ_i, are orthonormalized against the mass matrix[3] and then only the translation degree of freedom is considered in the analysis. From the displacement mode shapes and the length of the element L, the curvatures of mode shapes are obtained numerically by using a central difference approximation as

$$\phi''_{ji} = \frac{\phi_{(j+1)i} - 2\phi_{ji} + \phi_{(j-1)i}}{L^2} \qquad [3]$$

and the slope of each member is obtained as

$$\phi'_{li} = \frac{\phi_{mi} - \phi_{ni}}{L} \qquad [4]$$

where, subscript i is the number of mode shapes, j is the number of nodes, l is the number of elements, m is the number of nodes at one end and n is the node at the other end. According to the target structure, the dynamic error response vector Φ may be composed of the curvatures of mode shapes or the slopes of mode shapes.

Formulation of Static Error Response

The objective of this section is to formulate the error response of static test using the parameter identification at the element level. Static forces are applied at one subset of degrees of freedom and displacements are measured at another subset of degrees of freedom. Each set of forces should not be equal to any other set nor a linear combination of the previous sets of applied forces. Therefore, static governing equation is partitioned into x_a and x_b, which are the measured and unmeasured displacements, respectively.

$$\begin{Bmatrix} f_a \\ f_b \end{Bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{Bmatrix} x_a \\ x_b \end{Bmatrix} \qquad [5]$$

The vector of unmeasured displacements x_b can be condensed out. Sanayei and Onipedo[4] suggested a new error function in the condensation approach for the static damage detection algorithm. This is

$$\gamma(p_j) = [K_{aa} - K_{ab}K_{bb}^{-1}K_{ba}]x_a + K_{ab}K_{bb}^{-1}f_b - f_a \qquad [6]$$

Vectors f_a, f_b, and x_a are obtained from the test data. The analytical stiffness matrices K_{aa}, K_{ab}, K_{ba} and K_{bb} are functions of structural parameters p_j. In order to identify the structural parameters in this paper, the static error response of Eq. [6] is used. All elements of static error response corresponding to each structural parameter are placed into a vector $\Gamma(p_j)$.

Derivation of the Statistical System Identification Equations

In Eq. [1], the number of structural parameters is not the same as the number of measured responses generally and the inverse matrix of S is not always in existence. That is, a unique solution of Eq. [1] may not exist, and special mathematical techniques may be needed. Therefore, an estimate matrix T can be defined as

$$\Delta P = T \Delta R \qquad [7]$$

Let us define here C_p and C_r, which are the covariance matrices of the errors on initial structural parameters and measured responses respectively, and they are diagonal matrices. The matrices D_p and D_r are the weighting matrices for the structural parameters and measured responses, which can be assumed as inverse matrices of C_p and C_r respectively. The objective of this procedure is to find a best linear unbiased estimator of ΔP based on measured values of Λ, Φ, Γ and prior estimates of the structural parameters P_p. Define a scalar performance error function in the Bayesian estimation theory[5] for the derivation of the statistical system identification formula as

$$E_f = (R - R_k)^T D_r (R - R_k) + (P_k^* - P_k)^T D_p (P_k^* - P_k) \qquad [8]$$

where R is the experimentally measured response, R_k and P_k are predicted vectors of responses and structural parameters at step k respectively, and P_k^* is a vector of objective parameters. Generally, the objective parameters are not known but could be estimated. In Eq. [8], each response is combined into one scalar value for the constitution of error function. Collins et al.(1974) used the results of previous iteration for P_k^*, thus the second term on the right-hand side of Eq. [8] is removed, and the final results will be same as the statistical parameter estimation formula given in their paper. According to the study of Torkamani et al.[5], P_k^* was estimated as the result of the fifth iteration in the least squares method, and then the final results were the same as least squares method. In this study, P_k^* is assumed to be initial structural parameter values in the first iteration. From the second iteration, most of the objective parameters are estimated as the revised parameters of previous iteration. However, from the results of previous iteration, the portion of enhanced stiffness is ignored, and if some of the revised parameters are decreased more than the corresponding standard deviations it is assumed that the corresponding objective parameters are the same as these values, which are the prior estimates of structural parameters minus the corresponding standard deviations of previous iteration. Substituting

Eq. [1] into Eq. [8] and differentiating the resulting error function with respect to the structural parameters, using the symmetry condition of D_r and D_p, and setting the resulting equation equal to zero, the following expression is obtained.

$$P_k = \hat{T}_k(R - R_{k-1}) - \hat{T}_k S_k(P_k^* - P_{k-1}) + P_k^* \qquad [9]$$

where,

$$\hat{T}_k = C_p S_k^T (S_k C_p S_k^T + C_r)^{-1} \qquad [10]$$

In order to derive the covariance matrix of vector P_k, the statistical characteristics of the basic random variables in Eq.[7] must be verified. In this paper, the results established by Collins et al.[6] have been used. In the first place, one may assume that the mean of P_k is the objective parameter P_k^*. The covariance for the estimate of P_k is then defined as

$$\begin{aligned} \mathrm{Cov}[P_k, P_k] &= \mathrm{E}[(P_k - P_k^*)(P_k - P_k^*)^T] \\ &= \mathrm{E}[(\Delta P_k - \Delta P_k^*)(\Delta P_k - \Delta P_k^*)^T] \end{aligned} \qquad [11]$$

As the recursive analysis is performed, the estimated objective parameters are approximated as the values of revised structural parameters at previous step. Therefore, the estimate matrix of kth iteration, T_k, can be treated as $\hat{T}_k$. The covariance matrix of vector P_k is derived as the following equation.

$$\begin{aligned} \mathrm{Cov}[P_k, P_k] &= \mathrm{E}[(T_k \Delta R_k - \Delta P_k^*)(T_k \Delta R_k - \Delta P_k^*)^T] \\ &= T_k(S_k C_p S_k^T + C_r) T_k^T - T_k S_k C_p - C_p S_k^T T_k^T + C_p \end{aligned} \qquad [12]$$

EXAMPLE

The structure under consideration for the simulation study is a planar truss structure with known geometry and topology as shown in Fig. 1. This structure consists of 214 elements with 212 degrees of freedom. Since this structural model consists of axially loaded members only, each element has one random structural parameter, cross sectional area, thus element mass and length values are considered as constants.

The capability of the proposed algorithm is tested with one damage state where pertinent members are damaged with different percentage rate of severities. In order to estimate the feasibility of the algorithm, three members in the target structure are simulated to be damaged. The damaged members are simulated as 20% reduction in the cross sectional area for element 50, and 30% reduction for element 92, and 50% reduction for element 10. Thus, the cross sectional areas are reduced from 0.114 m^2 to 0.091 m^2 in element 50, from 0.100 m^2 to 0.071 m^2 in element 92, from 0.092 m^2 to 0.046 m^2 in element 10. In the present study, the dynamic test data as well as static displacements have been

used. The first elastic mode shapes are calculated by eigenvalue analysis, where vertical modal displacements are available at 35 stations over the structure. In the case of simulated static test, the applied load case is only one. The concentrated force of 50 Ton is applied downward to the 20th nodal point. The measured degrees of freedom for the static test are selected at the 29 degrees of freedom from the bottom nodes of the truss model. The covariance matrix of the errors on prior parameters, C_p, used in this example is a diagonal matrix with each element on the diagonal representing the estimated variance of a sectional property of each element. In this case, the standard deviations of all structural parameters are assumed to be 10% of their original values. The C_r matrix is also diagonal with each element representing the variance of a measurement. The standard deviations of all measured modal displacements are chosen to be 0.2% of the largest mean displacement in each respective mode. In this example, it is assumed that the uncertainty of static error responses is neglected.

From the measured displacement mode shapes, the slopes of mode shapes are obtained by using Eq. [4]. Hence two types of the dynamic error responses are calculated. Fig. 2(a) shows first type of the error response vector ΔR, which is made up of the differences in the displacements of mode shapes between the intact and the damaged structures. The dotted line corresponds to the case of second mode shape. Fig. 2(b) shows the case of slope differences of mode shapes, the maximum value of the error response vector ΔR for each member's slope generally occurs in the damaged region. The changes of the dynamic error responses for the displacements of mode shapes do not seem to localize to the damaged zone, as shown in Fig. 2(a). The differences in the slopes of mode shapes between the intact and the damaged structures can, therefore, be used to constitute the error response vector effectively, and better results of damage assessment can be expected.

In this example, 29 measurements of static test, 35 measurements of dynamic test, and 64 measurements of the combined data are used to revise the 106 parameters, respectively. In the first case, the static responses are used only. As one can see from the results shown in Fig. 3(a), the algorithm cannot detect and assess damages successfully in this case. The next case study is conducted by using the dynamic responses of first mode shape only. The slopes of mode shapes are used to formulate the error response vector. The results are given in Fig. 3(b) and it is seen that the identified values of first damaged member is relatively close to exact value even though the locations of damaged regions are not accurate. However, in Fig. 3(c) of the last case study, the best results are obtained by using the combined data which are composed of the first mode shape and the static deflections. The effect of adding static displacements gives a improvement in assessing the damage. It is again seen that the proposed algorithm of using combined data gives good performance.

CONCLUSION

An improved formulation using the combined modal and static information is proposed for the damage detection and assessment in this paper. The proposed algorithm is based on the iterative method. Both modal data and static displacements are used in the identification procedure, with the static data

providing the advantage of lower computational cost. It has been observed that the proposed algorithm can detect and assess damage effectively when the combined data of eigenmodes and static displacements are used. In the formulation of dynamic error response, the curvatures and slopes of mode shapes are presented. The changes in the curvatures and slopes of mode shapes are noticeable in the region of damage compared to the changes in the displacements of mode shapes. Thus, the differences in the curvatures and slopes of mode shapes between the intact and the damaged structures are used to constitute the error response vector effectively, and better results of damage assessment are produced. The feasibility of using the proposed procedure has been demonstrated by the example problems. From the case studies, it has been proved that the proposed algorithm can detect and assess damage realistically even under sparse measured data.

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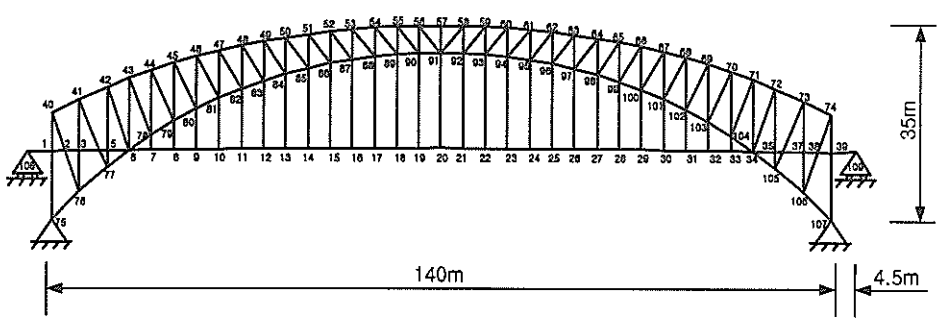
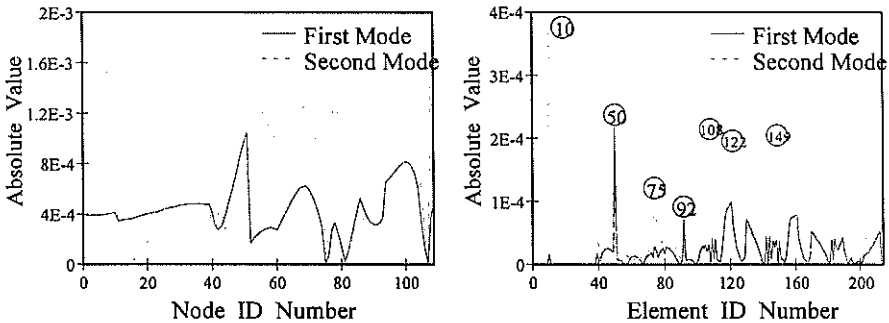


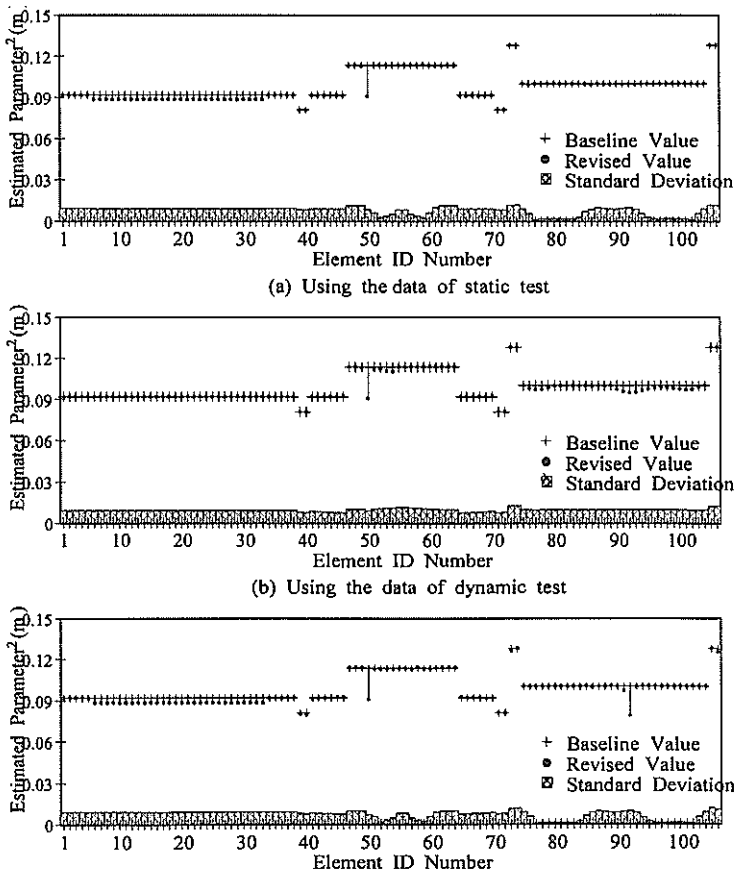
Figure 1. Geometry and physical properties for truss structure (1~109 : node numbers)



(a) Dynamic error response for displacement of mode shape

(b) Dynamic error response for slope of mode shape

Figure 2. Changes of dynamic error responses for planar truss structure



(a) Using the data of static test

(b) Using the data of dynamic test

(c) Using the combined data of static and dynamic tests

Figure 3. Result of parameter estimation for planar truss