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Self and Coupled Added Mass Evaluation for Indian Pressurised Heavy Water Reactor Calandria Tubes

G. M. Lemuel Raj, R. K. Singh, H. S. Kushwaha and V. Venkat Raj

Bhabha Atomic Research Centre, India

ABSTRACT

This paper highlights the development of an integrated finite element code LAPFEM for carrying out added mass computation and frequency response function evaluation of submerged components with complex geometries. The results from this code for two different submerged tube geometries are compared with analytical and experimental results. The self and coupled added masses along with the earthquake response of the calandria tubes of 500 MWe PHWR are evaluated using this code. The effect of added mass on the dynamic response of the coupled system of tubes is shown to be of small order.

INTRODUCTION

It is well known that the added masses are accounted for dynamic analysis of the submerged structures in absence of two ways coupled fluid-structure interaction. Since literature provides added mass expressions for only a handful of simple geometries [1-3], the ability of finite element method to handle arbitrary configurations is used for more accurate hydrodynamic mass evaluation. An in-house two dimensional finite element code LAPFEM is developed to compute self and coupled added masses of submerged structures along with their frequency response functions. The preparation of the input data is made user-friendly by incorporating interface identifier algorithm in the code LAPFEM to locate the fluid-structure interface. This feature is very useful in evaluating added masses for multiple submerged tubes, especially when their number is large as in the case of 500 MWe PHWR calandria with 392 calandria tubes, concentrically located inside which are the fuel channels. The code is validated by solving a twin concentric tube vibration problem with the ratio of their radii (r_1/r_2) as the variable parameter, and coupled vibration of a 3x3 array of submerged tubes with gap to radius (G/R) ratio as a variable parameter. After validation of the code, a representative 3x3 array of PHWR calandria tube model is analysed for evaluating its added mass and earthquake response. Further, a half model of the calandria with 196 tubes is analysed and the distribution of added mass with respect to the tube location is presented. The global analysis confirms the assumptions made on a local model with a 3x3 array of calandria tubes.

FINITE ELEMENT EQUATION FORMULATION

The governing equations for an inviscid, irrotational and incompressible fluid with small velocity field are given by pressure variable p formulation as

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=0 \qquad \text{inside the fluid domain } \Omega \qquad (1)$$

$\left(\frac{\partial p}{\partial n}\right)_s = -\rho a_n$ at the fluid structure interface Γ with n as outward normal for a fluid of density ρ, subjected to boundary acceleration a_n.

The finite element formulation of these equations is given by

$$[K]\ \{p\} = \{F\} \qquad (2)$$

where K,P and F are given by

$$[K]=\sum\iint_{\Omega}\left(\left\lfloor\frac{\partial N_i}{\partial x}\right\rfloor^T\left\lfloor\frac{\partial N_j}{\partial x}\right\rfloor+\left\lfloor\frac{\partial N_i}{\partial y}\right\rfloor^T\left\lfloor\frac{\partial N_j}{\partial y}\right\rfloor\right)dx.dy \qquad (3)$$

$$\{F\}=\sum\int_{\Gamma}\rho a_n\lfloor N_i\rfloor^T d\Gamma \qquad (4)$$

$\{p\}$ is the column vector of pressure at different node points in the fluid domain.

The finite element code LAPFEM was developed for solving the above equation which uses 4 node isoparametric elements for analysis. This code finds the pressure distribution in the fluid domain due to the acceleration of a structural member in it, and integrates the pressure along the fluid-structure interface to calculate the added masses.

In a generalised manner, the self added mass $M_{i,i}$ and the coupled added mass $M_{i,j}$ for submerged tube geometries are defined [1-3] as

$M_{i,i}$ = Added mass of tube i due to unit acceleration of tube i in x direction.
$M_{i,j}$ = Added mass of tube i due to unit acceleration of tube j in x direction.

Thus a complex configuration of submerged geometries can be analysed to evaluate self and coupled added masses with LAPFEM code. The non-dimensional added mass of a submerged component of any geometry is obtained by dividing its added mass with the mass of fluid it has displaced.

CALCULATION OF ADDED MASS FOR SUBMERGED GEOMETRIES IN WATER

The added masses for various simple geometries were calculated using code LAPFEM and the results were compared with the available results [1].

Benchmark problem No. 1: Evaluation of self and coupled added masses for cylinder in cylinder annular geometry filled with water (fig.1).

The non-dimensional self added mass per unit length of tube 1 is equal to $\left(r_2^2 + r_1^2\right)/\left(r_2^2 - r_1^2\right)$, and the non dimensional coupled added mass is $2/\left(r_2^2 - r_1^2\right)$. The calculated results are compared with the finite element results for different r_1/r_2 values in fig.2. It can be seen that the present numerical results are in good agreement with the theoretical results.

Benchmark problem No.2: Non dimensional Added mass (C_m) of the central tube of a 3x3 array of tubes in a large domain of water for different gap to radius ratios (G/R) (fig. 3).

This model was analysed for the case when the central member is only vibrating and all other tubes are stationary. The analysis was carried out for models with different G/R ratios and in each case the non-dimensional added mass was calculated. The FEM and experimental results [1] showing C_m values with different G/R ratios are shown in fig.4. The FEM results are in good agreement with the reported experimental results [1].

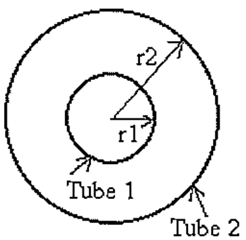

Fig.1: Fluid filled annular geometry

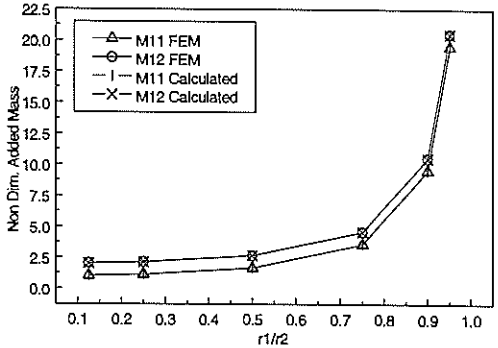

Fig.2: Non-dimensional Self and coupled added masses of inside cylinder.

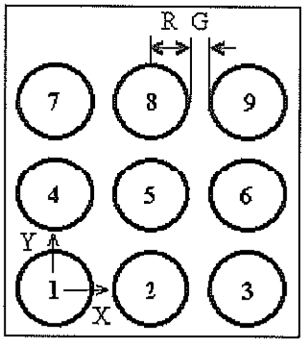

Fig.3: Submerged 3x3 tube assembly

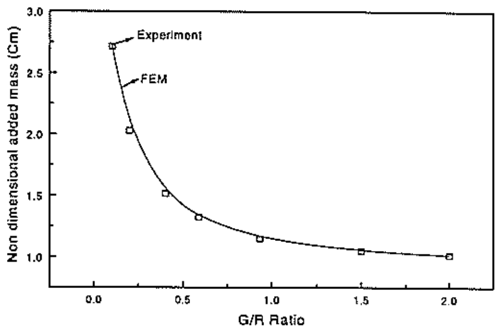

Fig.4: Comparison of FEM and experimental results

DYNAMIC ANALYSIS OF 3X3 CALANDRIA TUBE ARRAY

The self and coupled added masses of a representative 3x3 array of calandria tubes are evaluated using the code LAPFEM. This assumption of considering a 3x3 tube array to represent the features of whole array of 392 calandria tubes can be justified by the fact that the effect of neighbouring tubes on the self added mass of a particular calandria tube is negligible. It can be seen from the results presented in fig. 4 that the value of C_m for the array of calandria tubes (G/R = 2.317) is close to1.0. Since the x-y cross coupling terms are observed to be negligible, only 9x9 added mass matrix is considered. The non dimensional self added mass of the central tube is computed as 1.0204. Maximum non dimensional coupled added mass is found to be 0.12 which confirms that the coupling is weak. Every element $M_{i,j}$ in this matrix is the x-added mass of ith tube due to the motion of jth tube in x-direction. The actual mass matrix of this system is obtained by adding the mass of a single CT-PT assembly to all the principal diagonal terms of the added mass matrix. The dry natural frequency of the calandria tube pressure tube assembly (CT-PT) with garter springs (fig.5) is found using a finite element model with beam and spring elements. The fundamental frequency of this system without added mass obtained by analysis is 6.9 Hz. This frequency value is used to calculate the stiffness of a simple beam with fixed boundaries at both ends. The stiffness matrix of this system is a 9x9 identity matrix multiplied with the stiffness value of this equivalent beam.

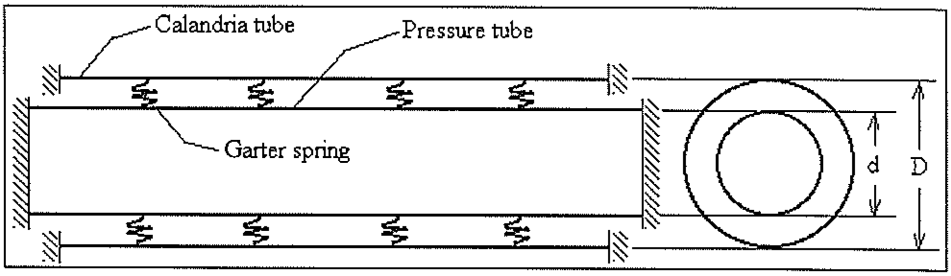

Fig.5: Calandria tube - Pressure tube assembly with Garter springs

The natural frequencies of this coupled system of calandria tubes are obtained by solving the eigen value problem. The mode shapes and the natural frequencies are indicated in fig. 6. Further the dynamic analysis of this system is performed using two methods:

First, the non dimensional frequency response functions of the coupled system are obtained for a ground motion excitation, $\ddot{u}_g(t) = A_g \sin(\omega t)$, at different damping ratios, in the range 6.0 Hz to 6.5 Hz. The equation of motion of this system is given as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{1\}\{\ddot{u}_g(t)\}.$$

It can be shown that the maximum displacement response of the i th tube in this multi degree of freedom system is given by,

$$(u_i)_{\max} = \sum_{n=1}^{9} \left| \phi_{in} \frac{F_n^*}{\sqrt{(k_n^* - m_n^* \omega^2)^2 + (c_n^* \omega)^2}} \right|$$

Where ϕ_{in} is the i th element of the n th mode shape vector $\{\phi_n\}$, $F_n^* = \{\phi_n\}^T [M]\{1\} A_g$, $m_n^* = \{\phi_n\}^T [M]\{\phi_n\}$, $c_n^* = \{\phi_n\}^T [C]\{\phi_n\}$ and $k_n^* = \{\phi_n\}^T [K]\{\phi_n\}$.

The non-dimensional response of tubes $\left|(u_i)_{max}/\omega^2 A_g\right|$ as a function of excitation frequency ω, for tubes 1, 2, 4 and 5 at different damping ratio values, is shown in the fig. 7. The response of other tubes can be found from symmetry.

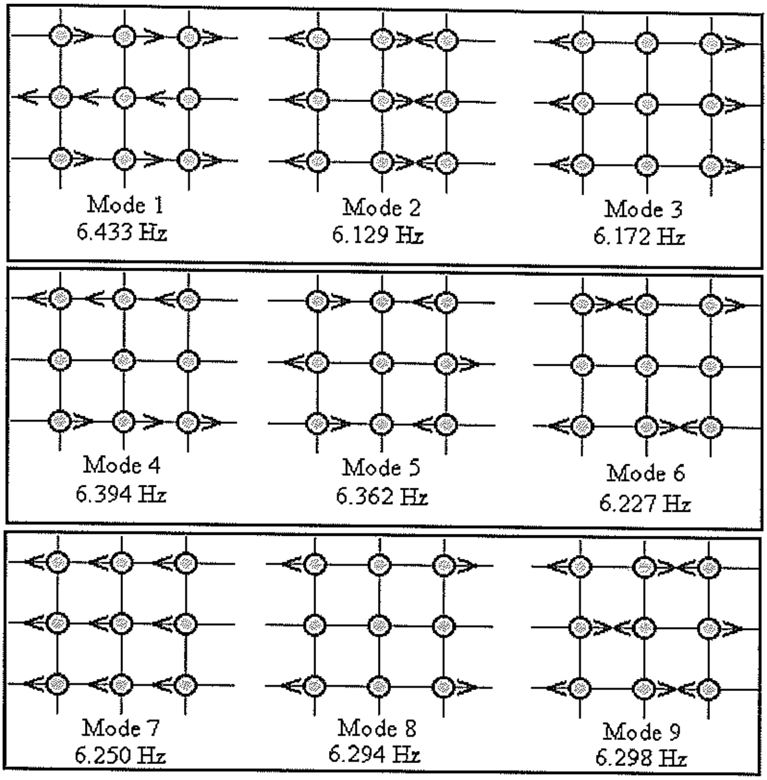

Fig. 6: Natural frequencies and mode shapes of 3x3 calandria tube assembly

Secondly, the maximum displacements and forces for every tube are evaluated using response spectrum method. A response spectrum, with a unit value of acceleration response for all the frequencies is used to evaluate the maximum displacements and forces for all the 9 tubes for every eigen mode separately. These values are tabulated in the tables 2a and 2b. These values can be used to calculate the actual response for any given response spectra $S_a(\omega_n)$ in m/s^2, for damping ratio 0.01.

The maximum displacement (metres) response of tube *m* is given by

$$u_m = \sum_{n=1}^{9} \left| U_{m,n} S_a(\xi_n, \omega_n) \right|$$

and the maximum nodal force (Newtons) for tube m is given by

$$f_m = \sum_{n=1}^{9} \left| F_{m,n} S_a(\xi_n, \omega_n) \right|$$

$U_{m,n}$ - displacement coefficients; numerically equal to the maximum nodal displacement for tube m in mode n, when subjected to a unit response spectrum.

$F_{m,n}$ - force coefficients; numerically equal to the maximum nodal displacement for tube m in mode n, when subjected to a unit response spectrum.

The values of $U_{m,n}$ and $F_{m,n}$ are given in the tables below.

Table 2a: Displacement coefficients (s^2)

	Mode 1 (6.433 Hz)	Mode 2 (6.13 Hz)	Mode 7 (6.25 Hz)	Mode 9 (6.295 Hz)
$U_{1,n}$	.301956E-04	.522172E-04	.573446E-03	-.705314E-05
$U_{2,n}$	.407229E-04	-.826198E-04	.672981E-03	.115677E-04
$U_{4,n}$	-.524596E-04	.733992E-04	.622430E-03	.108801E-04
$U_{5,n}$	-.691189E-04	-.117527E-03	.850220E-03	-.152117E-04

Note: $U_{3,n}=U_{7,n}=U_{9,n}=U_{1,n}$; $U_{8,n}= U_{2,n}$ and $U_{6,n}= U_{4,n}$; $U_{m,n}=0$ for n=3,4,5,6 and 8.
The displacement response for modes 3, 4, 5, 6 and 8 is equal to 0. The displacement response for tubes 3, 6, 7, 8 and 9 can be deduced from symmetry.

Table 2b: Forces coefficients (kg.)

	Mode 1 (6.433 Hz)	Mode 2 (6.13 Hz)	Mode 7 (6.25 Hz)	Mode 9 (6.295 Hz)
$F_{1,n}$	.229400E+02	.396701E+02	.435655E+03	-.535837E+01
$F_{2,n}$	.309378E+02	-.627673E+02	.511273E+03	.878815E+01
$F_{4,n}$	-.398542E+02	.557624E+02	.472868E+03	.826572E+01
$F_{5,n}$	-.525106E+02	-.892866E+02	.645924E+03	-.115566E+02

Note: $F_{3,n}=F_{7,n}=F_{9,n}=F_{1,n}$; $F_{8,n}= F_{2,n}$ and $F_{6,n}= F_{4,n}$; $F_{m,n}=0$ for n=3,4,5,6 and 8.
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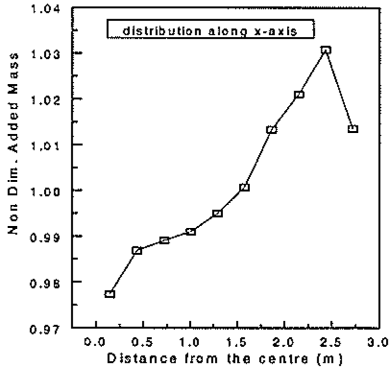

Fig 8: Distribution of C_m along x-axis

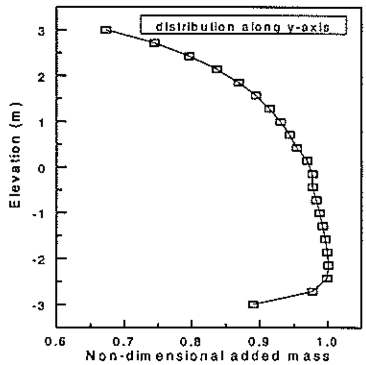

Fig. 9: Distribution of C_m along y-axis

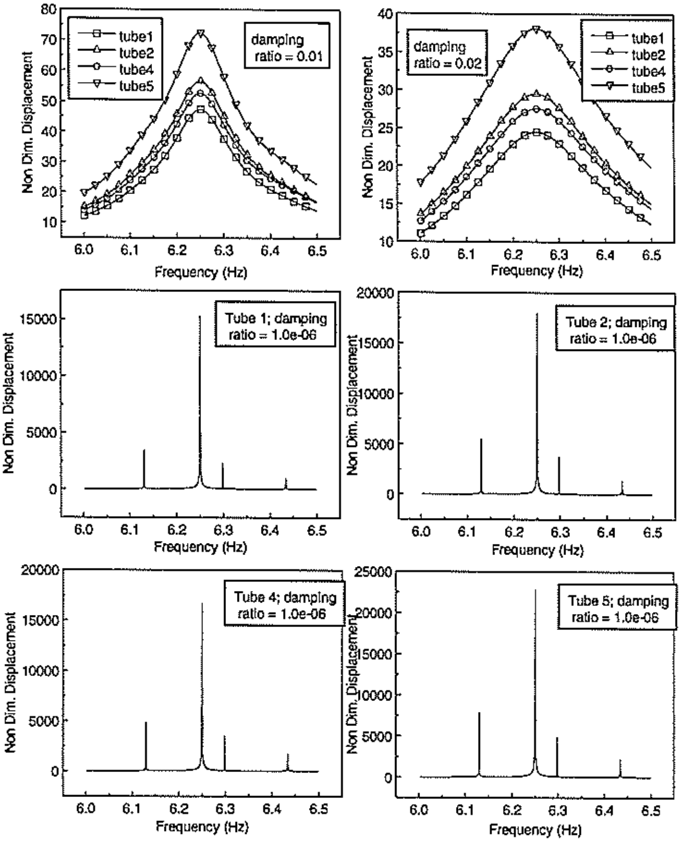

Fig. 7: Frequency response of tubes 1, 2, 4 and 5, for different damping ratios.

The non-dimensional added masses of the calandria tubes due to in-phase motion of all tubes in x-direction are evaluated by using the half model of calandria fluid domain with 196 tubes (fig. 10). The maximum value of non dimensional added mass is 1.0317 (compared to 1.0204 obtained on local model) for the tube that is located at mid height and near the calandria wall. The distributions of the non dimensional added mass values along the x=0 and y=0 axes with the origin at the centre of the calandria are given in figs 8 and 9. The lower values of added masses at higher elevations are due to the presence of free surface where the pressure is zero when sloshing is neglected.

CONCLUSIONS

The non dimensional self and coupled added masses, and the frequency response functions have been evaluated for 500 MWe PHWR calandria tubes on a local 3x3 array of calandria tube model. The effect of coupled added mass is shown to be weak through frequency response analysis. It is shown that the dominant mode is in-phase motion of all the tubes at 6.25 Hz frequency. The response as a result of other coupled modes is shown to be small. This is again confirmed on a global model analysis of submerged calandria tubes array.

The global model accounts for the variation of added mass near the free surface of moderator and near the calandria shell wall. The shift in the natural frequency of the calandria tube assembly due to self and coupled added mass is small. Hence, global analysis with assumption of in-phase motion of all the tubes is justified [5]. The structural mass due to fuel bundles contributes significantly for the net effective tube mass compared to the self and coupled added masses of the moderator.

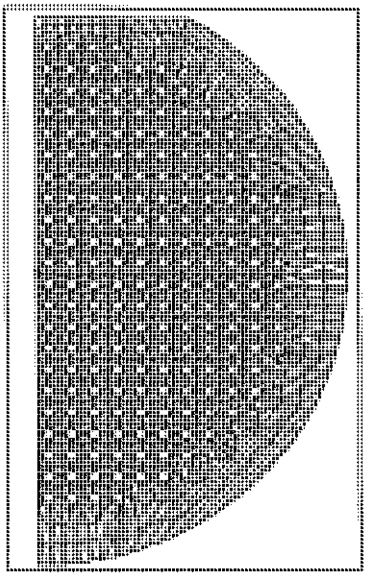

Fig.10: FE mesh of the fluid domain of calandria with 196 tubes (half model).

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