Elastic Plastic Finite Element Analysis of a BWR Feed Water Distributor Exposed to an Extreme Pressure Load

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ABSTRACT: During a hypothetical break of a BWR feed water line, the feed water distributor (FWD) inside the RPV is subjected to a high pressure load for a short time (10 ms). Because of the sudden coolant release from the inner volume of the FWD there is a pressure difference between the inner and outer surface. It is conservatively assumed that the pressure difference nearly can reach the operating pressure for a few milliseconds. The distributor box and the ring line of the feed water distributor are modelled with shell and volume elements capable of being used for large strain analyses with elastic-plastic material behaviour. It is demonstrated by non-linear static calculations (without consideration of the inertia of the material) that a buckling instability occurs at about 60\% and 80\% of the maximum pressure load. The arc-length method is used for the numerical solution to overcome these points of instability. To evaluate the influence of the dynamics of the process a non-linear transient analysis is done showing that the maximum strain occurs with a time delay to the pressure peak. The maximum plastic strain differs only insignificantly between static and transient solution. Inspite of the large strains the mechanical integrity is maintained during the hypothetical event.

1 INTRODUCTION

The break of a feed water line outside of the containment is a hypothetical accident scenario for German boiling water reactors (BWR). Fig. 1 shows a simplified scheme of the this scenario. It is assumed that the pressure in the feed water system drops to ambient pressure within a few milliseconds. The feed water distributor (FWD) consists of four hydraulically separated ring lines supported at the feed water nozzles of the RPV. The cross section of the FWD ring lines is rectangular (inner height 210 mm, inner width 155 mm, wall thickness 10 mm). Figure 2 shows a 3D-view of the FWD in the region of RPV.
inlet nozzle. The feed water is normally sprayed into the reactor pressure vessel (RPV) through a number of small nozzles with a rather high flow resistance, located at the top wall of the FWD. Therefore in the case of a feed water line break the pressure within the RPV drops not as fast as the pressure of the feed water system. Thus for a short time the FWD is exposed to high difference pressure between its outer and its inner surface. The pressure maximum pressure peak is 62 bar. The linear stress analysis showed that large deflections and large strains are to be expected at the vertical walls of the ring line and at the back wall of the distributor box.

2 MECHANICAL PHENOMENA AND SOLUTION METHOD

Due to the high pressure load the elasticity limit of the FWD (austenitic steel) is clearly exceeded. Hence the mechanical stress analysis has to be based on non-linear material behaviour. An elastic-plastic material law with an isotropic hardening rule is adopted (section 3). Furthermore the problem is geometrically non-linear because large strain and large deflection have to be expected. Due to the fact that the load is imposed by external pressure the deflection process is connected with instabilities (buckling). Instabilities may occur at the back wall of the distributor box (snap through problem) and at the ring line respectively. The buckling process of the FWD ring pipe is illustrated in Figure 3. The height of the cross section of the ring line is larger than the width, so the vertical walls are bended to the inside and the horizontal walls to the outside. The pressure upon the horizontal walls leads to additional negative membrane stress in the vertical walls and thus causes a stress weakening. As a consequence, the ring line tends to buckling. This process is unstable until the opposite horizontal walls get in contact. This phenomenology can be characterized by the load-displacement curve shown in Fig. 4.

Fig. 2: Feed water distributor (region of RPV nozzle)

Fig. 3: Buckling of the FWD ring line

Fig. 4: Load-deflection curve for buckling
Due to these instabilities special mathematical tools are required to get a converging solution. Like for all non-linear problems the load must be applied in small steps even in a static analysis. Equilibrium iterations are performed at each load step to solve the equations. For stable load-deflection behaviour the classical Newton-Raphson procedure is an adequate algorithm (Zienkiewicz, 1977). The Newton-Raphson procedure iterates the equilibrium at each load step. The solution is converged if the vector of residual forces \( f_r \) is sufficiently small (equilibrium between applied loads and internal forces):

\[
\mathbf{f}_r^{(i)} = \mathbf{K}^{(i)} \mathbf{u}^{(i)} \cdot \mathbf{u}^{(i)} - \mathbf{f}_a
\]

(1)

\( \mathbf{K} \) is the stiffness matrix of the FE-model, \( \mathbf{u} \) the vector of the nodal degrees of freedom (deflection), \( \mathbf{f}_a \) is the vector of the applied loads at the current load step and \( i \) the iteration index.

However, if the load-deflection curve exhibits instability points (negative slope) the classical Newton-Raphson algorithm does not work. A numerical algorithm to solve problems with instabilities is the arc-length method (Kolar and Kamel, 1986). The basic idea of this method is to control the gradual application of the load by a relative load factor \( \lambda \) in such a way that the distance between two iteration points in the load-deflection space is limited by an arc length radius. Thus the load can be increased and decreased during the progress of the equilibrium iterations. Figure 5 illustrates this strategy. The equation for the residual forces (eq. 1) is modified as follows:

\[
\mathbf{f}_r^{(i)} = \mathbf{K}^{(i)} \mathbf{u}^{(i)} \cdot \mathbf{u}^{(i)} - \lambda \mathbf{f}_a
\]

(2)

The load factor \( \lambda \) is determined from the additional condition:

\[
I_i^2 = \Delta \lambda_i^2 + \beta^2 (\Delta u_n^i)^T (\Delta u_n^i)
\]

(3)

where \( I_i \) is the arc-length radius and \( \beta \) a scaling factor to consider the different units of load and deflection, \( n \) is the load step index and \( i \) the iteration index. The arc-length procedure is available with the FE-code ANSYS ®.

The arc-length method is only applicable to static analyses. If the load is time dependent in such a way that the dynamic forces of the structure cannot be neglected, a non-linear transient analysis is to be performed. In this case the inertia and damping forces have to be included into the equilibrium iterations of the classical Newton-Raphson procedure. The residual force vector then reads:

\[
\mathbf{f}_{r,n}^{(i)} = \mathbf{M} \ddot{\mathbf{u}}_n + \mathbf{B} \mathbf{u}_n + \mathbf{K}^{(i)} \mathbf{u}_n^{(i)} \cdot \mathbf{u}_n^{(i)} - \mathbf{f}_{a,n}
\]

(4)

with \( \mathbf{M} \) being the mass matrix, \( \mathbf{B} \) the damping matrix and \( n \) the time index. Equilibrium iterations are performed for each time step of the analysis. For the time integration the Newmark
method is adopted (Bathe, 1982).

3 FINITE ELEMENT MODEL AND LOADING

The FWD is modelled using the FE code ANSYS®. Various models are available. The first analyses were done using a shell model of the complete 90° segment of the FWD (distributor box, redirection box and ring line, Fig. 2). This model is only useful for linear static and dynamic calculations. In the non-linear large strain analyses no convergent solutions could be obtained. That's why the model has to be separated, one model for the distributor box and one model for the ring line. For these parts shell models or volume models can be used. The volume models were developed because of the missing convergence with shell elements (see section 4). All used element types have plasticity, large strain, large deflection and stress stiffening capabilities. To cope with the dependence of the stiffness of the structure on the deflection an updated Lagrangian formulation is used (Mattiasson, et. al. 1986). Contact elements are used to model the touching of the two opposite vertical walls of the ring line.

The material behaves elastic-plasticity with multi-linear isotropic hardening. Figure 6 shows the stress-strain curve up to the yield point at the temperature of 200 °C.

To describe the plastic material behaviour within the 3-dimensional stress state the flow rule of Huber, von Mises and Hencky is used (Zienkiewicz, 1977):

\[
\Phi = \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] - \sigma_p^2 (\varepsilon_p^{pl}) = 0
\]  

(5)

with \( \sigma_p \) being the yield stress and \( \varepsilon_p^{pl} \) the equivalent plastic strain. If equation (5) is fulfilled the material is subjected to a plastic strain which can be calculated from the von-Mises flow law:

\[
d\varepsilon_p^{pl} = d\lambda \cdot \frac{\partial \Phi}{\partial \sigma_{kl}} ; \quad d\lambda = \frac{d\varepsilon_p^{pl}}{2\sigma_p}
\]  

(6)

The load for the FWD is a pressure on its outer surface (pressure difference between RPV and
feed water system). The pressure is time dependent and exhibits a spatial distribution. In Fig. 7 the pressure load at the distributor box is shown over the time. For static analyses the pressure distribution at t=0.045 s (maximum pressure) is used as load. The maximum pressure onto the distributor box is about 62 bars, whereas the pressure onto the ring line is 52 bars at its entry and 31 bars at the azimuthal position of 45°.

For transient analyses the time dependent distribution between t=0 s and t=0.056 s is used.

\[
p(\bar{x}, t) = p(\bar{x}) \cdot \lambda(t) \quad ; \quad 0 \leq \lambda(t) \leq 1
\]

(7)

where \( p(\bar{x}) \) is the spatial distribution of the pressure and \( \lambda(t) \) a relative load factor which corresponds to the pressure curve of Fig. 7 between t = 0.04 s and t = 0.056 s.

4 RESULTS

![Figure 8: Static analysis of distributor box. Load deflection curve for an outside node on the symmetry axis at the back wall (vertical position 40 mm above thermosleeve axis).](image)

As stated in section 3 the non-linear mechanical analysis for the FWD has to be carried out with separate models for the distributor box and for the ring line, because a convergent solution could not be obtained with the shell model for the complete structure. For both parts the best results were achieved with volume elements. First the static analyses are discussed. Figure 8 shows a load deflection curve for the distributor box. The relative load factor (eq. 7) is drawn versus the x-displacement of a node at the back wall on the symmetry axis (vertical position 40 mm above the thermosleeve axis). Up to about 60% load the curves are linear with a rather steep slope. This is the range of elastic deformation (high stiffness). If the relative load factor gets higher than 60% the plastic deformation starts to develop. The point of instability (snap through point) is about 81% load. At about 25 mm deflection the structure becomes stable again. This is due to the hardening of the material (increasing yield stress due to plastic strain) as well as to the
stress re-stiffening (change of the curvator sign). Figure 10 shows the distribution of the total displacement of the distributor box at 100% load (62 bar).

![Fig. 10: Total displacement of the distributor box at 100% load. Static calculation, volume model.](image)

Figure 9 shows two load deflection curves for the ring line. The green curve is for the node at the inner vertical wall of the FWD ring line, half height at an azimuthal position 15° from the nozzle. The blue curve is for the node displacement of the outer vertical wall at the same axial and azimuthal position. The point of instability is around 68% load. The instability is a result of the stress weakening of the vertical walls (Fig. 4). At about 50 mm deflection of the outer node (blue curve Fig. 9) and 20 mm deflection of the inner node (green) the structure slowly becomes stable again as a consequence of the material hardening. At about 95 mm deflection of the outer node and 54 mm of the inner node the first contact between the opposite wall takes place. This leads to a high stiffness of the structure again (extremely steep slope). The slope of the blue curve is even higher than 90°, because the outer wall is pushed back by the inner wall. Fig. 11 shows the deflection of the structure at 100% load.

![Fig. 11: Deflection (mm) of the ring line at 100% load. Static calculation, volume model. View to the inner wall (upper figure) and to the outer wall (lower figure).](image)

Figure 12 shows a detail of the strain distribution to demonstrate that there is contact between the opposite walls. Inspite of the large deflection the maximum strain (0.14) is clearly below the fracture strain (0.44, Fig.6). The maximum deflection of the outer wall is larger than that of the inner wall. The maximum equivalent stress is about 320 MPa. This agrees to the σ-ε curve of the FWD steel (Fig. 6). The static calculation with the shell model exhibits a worse convergence in comparison with the volume model. At a relative load factor greater than 58% the solution is not convergent.
For the transient calculations the deflection and the relative load factor are displayed versus time (Fig. 13, distributor box). The relative load factor corresponds to the part of the pressure curve in Fig. 7 between 0.04 s and 0.056 s. It is to be seen that there is a remarkable time delay between the maximum pressure peak and the response of the structure. At 100% load there is only a deflection of 5 mm at the distributor box back wall. This is a consequence of the inertia of the material which acts as an additional resistance in the beginning of the deformation. But at the time of the maximum pressure peak there is a high velocity of deformation, and the kinetic energy promotes a further deflection even when the load is already decreasing again. However, the maximum deflection is limited to a value of 50 mm, whereas in the static calculation the distributor box deflection reaches 100 mm (see Figures 8 and 13).

In the case of the ring line there is also a time delay between maximum load and deflection but the motion is only stopped when contact between the walls takes place. The maximum deflection and the maximum strain are very similar to those of the static calculation. The maximum strain is 0.13. The maximum deflection of the outer wall is 85 mm and that of the inner wall is 70 mm. The corresponding values from the static calculation are 97 mm and 58 mm (see Figures 9 and 14).

5 CONCLUSIONS

Within the considered scenario - break of a BWR feed water line - the elasticity limit of the feed water distributor is exceeded. Hence the stress analysis has to be done with consideration of material and geometrical non-linearity. Within a static analysis the arc-length procedure is an
Fig. 14: Transient analysis of the ring line. Deflection curves vs. time for nodes at the inner wall (ABS_UX4) and the outer wall (ABS_UX5) at azimuthal position 15°. Relative load vs. time (Rel_Load).

Inspite of the large deformation of the FWD ring line the maximum strain remains clearly below the break limit.

REFERENCES


