A Method for the Measurement of Flow Rate in Pipe Using Microphone Array

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ABSTRACT

When plane waves propagate through a medium flowing in a pipe, the general relation between the pressures at three adjacent points equally spaced has been derived. A new method is proposed to measure the mean flow velocity in a pipe from multiple measurements of acoustic pressure using a microphone array. It is based on the fact that the variation of mean flow velocity affects the change of wave number. This approach offers advantages in that it does not obstruct the flow field and can provide the time-spatial mean flow velocity. Some numerical simulations and experiments are accomplished to verify the validity of this method.

1. INTRODUCTION

The pitot-tube, turbine meter and orifice meter are the instruments generally used to measure the flow velocity of the medium flowing inside of a pipe. However, these measurement instruments shortcoming is that they fail to take the fine measurement of the original flow velocity, because they must be inserted into the pipe directly, which disturbs the flow of fluid and transforms the flow field. To overcome this problem, a new measurement instrument applying supersonic waves has been devised and in use at present, but its high price has hindered its widespread use. Existing measurement instruments have also the defect of failing to measure the precise mean flow velocity inside of a pipe. They measure the flow velocity at the very point where their measuring probe is located, not the mean velocity.

In this paper a new approach is introduced which can measure the mean flow velocity without affecting the flow of a fluid using acoustic waves propagating through a fluid inside of a pipe. The wave number of a sound wave is used as an information carrier that contains the flow velocity information, since the wave number tends to decrease in the direction of the flow of a fluid and to increase in its opposite direction. Kim and Kim² have presented a method to derive the mean flow velocity inside of a pipe using three sensors (accelerometers), that is, it is to apply the preservation of the propagating acoustic energy achieved by ignoring the sound attenuation caused by viscosity or turbulent flow of a fluid.

In this study, with the flow velocity existing, the general relation formula of the acoustic pressure on the acoustic field created inside of a pipe is derived giving consideration to the attenuation components of a sound wave, and the approach for measuring the inner mean flow velocity is advanced and verified through the simulation and actual measurement.
2. DEDUCTION OF MEAN FLOW VELOCITY INSIDE OF A PIPE

Fig. 1 Plane waves propagate though moving fluid with flow velocity \( U \) in a pipe.

As Fig. 1 shows, in the case of a flowing medium at the mean flow velocity \( U \) inside of a pipe with an even cross-sectional area, simple is the one-dimensional sound wave equation in the direction of the length of the pipe as showed in the following:

\[
(c_s^2 - U^2) \frac{\partial^2 p}{\partial x^2} - 2U \frac{\partial^2 p}{\partial x \partial t} - \frac{\partial^3 p}{\partial t^2} = 0
\]  

(1)

Here, \( c_s \) denotes the propagating velocity of a sound wave, given the stationary medium. Provided that the plane wave propagating under the uniform flow condition is the time harmonic form of each frequency \( \omega \), the equation would be,

\[
p(x,t) = \{ P^+ \exp(-jk^+x) + P^- \exp(jk^-x) \} \exp(j\omega t)
\]  

(2)

where (+) means the sound wave propagating downstream and (-), the sound wave propagating upstream, and according to the mean flow velocity and direction of a medium the value of the wave number is determined by \( k^\pm = k/(1 \pm M) \).\(^{09}\) \( M \) (Mach number) is \( U/c_s \) and in general \( k \) takes a complex number of \( \nu - j\gamma \), where \( \gamma \), the attenuation coefficient, is composed of the visco-thermal attenuation and turbulent attenuation and mainly affected by the wall admittance at the boundary, and \( \omega/c_s \) is \( \nu \) which shows the propagating characteristics of a sound wave. As Fig.1 shows, if applying the Fourier transform to the acoustic pressure at 3 consecutive points which are equally spaced then,

\[
P_n = P^+ \exp(-jk^+x_n) + P^- \exp(jk^-x_n)
\]

\[
P_{n+1} = P^+ \exp(-jk^+x_{n+1}) + P^- \exp(jk^-x_{n+1})
\]

\[
P_{n+2} = P^+ \exp(-jk^+x_{n+2}) + P^- \exp(jk^-x_{n+2})
\]

(3)

Subsequently, by eliminating the common terms of the equation \( P^+ \) and \( P^- \), derive the relation formula between the acoustic pressures \( P_n, P_{n+1}, P_{n+2} \) at each of the three points and wave number \( k^+, k^- \), then,

\[
P_n \exp[j(k^- - k^+)(dx)] + P_{n+2} = -P_{n+1} \{ \exp(jk^-dx) + \exp(-jk^+dx) \}
\]  

(4)

This is the general relation formula reached under the assumption that inside of a pipe with an even cross-sectional area the plane waves propagate together with the flow of fluid. However, Eq. (4) would not be effective in the following occasion if in the process of deriving Eq. (4) the singular condition concerning the specific frequency and intervals of each sensor would be like the following,

\[
\exp(jk^-dx) = \pm \exp(-jk^+dx)
\]  

(5)

Should the degree of attenuation be small enough to ignore, the wave number is
\[ k^2 = (\omega/c)/(1 \pm M), \text{ therefore the singular condition of Eq. (5) is,} \]
\[ \omega = m\pi(1 - M^2)c/(2dx) \tag{6} \]

or, if the frequency of the sound wave is \( \lambda \),
\[ dx = (1 - M^2) m\lambda/4 \tag{7} \]

where, \( m = 1, 2, K \). We find that this is equal to the condition which makes it impossible to separate sound waves between sensors \( n \) and \( n+2 \). Accordingly, in the scope excluding the singular condition of Eqs. (6) and (7) the acoustic pressure relationship among the three points assumes the recursive form like in Eq. (4) and can be constructed in the form of a matrix determinant.

\[
\begin{bmatrix}
  P_1 & P_3 \\
  P_2 & P_4 \\
  M & M \\
  P_{n-2} & P_n
\end{bmatrix}
\begin{bmatrix}
  G^- \\
  G^+
\end{bmatrix}
= 
\begin{bmatrix}
  P_1 & P_3 \\
  P_2 & P_4 \\
  M & M \\
  P_{n-2} & P_n
\end{bmatrix}
\begin{bmatrix}
  G^+ = \left\{ \exp(jk^+dx) + \exp(-jk^+dx) \right\} \\
  G^- = G^+ \exp\{j(k^- - k^+)dx\}
\end{bmatrix}
\tag{8}
\]

Here, the propagating characteristics of a sound wave is implied in \( G^- \) and \( G^+ \) and the unknown quantity \((G^-, G^+)\) shall be induced through Eq. (8) using the measured acoustic pressure at \( N \). In this case, Eq. (8) can be rewritten as the following after applying another formula \( k^\pm = k/(1 \pm M) \),
\[ G^-/G^+ = \exp\left\{ j2Mkdx/(1 - M^2) \right\} \tag{9} \]

Therefore, the information on the mean flow velocity and attenuation coefficient inside of a pipe can be sought in the following way.

\[ \frac{M}{1 - M^2} = \frac{c}{2\omega dx} \arg \left( \frac{G^-}{G^+} \right) \tag{10} \]

\[ \gamma = \frac{-\omega}{c} \log \left| \frac{G^-}{G^+} \right| \tag{11} \]

* The mean flow velocity information:

\[ \gamma = \frac{M}{1 - M^2} \arg \left( \frac{G^-}{G^+} \right) \]

* The attenuation coefficient information:

Hence, if the exact value of \( G^- \) and \( G^+ \) in Eq. (8) can be inferred from the measured acoustic pressure, the precise mean flow velocity and attenuation coefficient can be found. If \( N = 4 \), where the form of the matrix is a regular square, we can get a unique solution, but if \( N > 4 \), where the number of columns is more than that of rows, the Least Square Error method which enables to minimize an error can be employed to find the appropriate value of \( G^- \) and \( G^+ \). This has a thread with connection with the occasion of using the pseudo inverse. For instance, in case of the general matrix \( A = B \), the pseudo inverse is \( A^+ = (A^H A)^{-1} A^H \) and \( x \) can be sought through \( (A^H A)^{-1} A^H B \). Hence Eq. (8) can be rearranged from a point of the Least Square Error. That is,
\[
\begin{bmatrix}
\sum_{n=1}^{N-2} P_n P_n' & \sum_{n=1}^{N-2} P_n P_{n+2} \\
\sum_{n=1}^{N-2} P_{n+2} P_n' & \sum_{n=1}^{N-2} P_{n+2} P_{n+4}
\end{bmatrix} \begin{bmatrix}
G^-

\end{bmatrix} = \begin{bmatrix}
\sum_{n=1}^{N-2} P_n P_{n+1}'

\end{bmatrix} \begin{bmatrix}
G^+

\end{bmatrix}
\]

(12)

Besides, the inverse matrix must find the unknown \( G^- \) and \( G^+ \) in the matrix determinant (12). In general a solution error for simultaneous equation's matrix can be minimized when small is the condition number defined as the ratio of the maximum to the minimum eigenvalue. For convenience, on the basis of having no flow velocity, the following are the conditions that make the condition number have the maximum and minimum values:

* The condition for the minimum value: \( dx = (2m-1)\lambda/8 \)  

(13)

* The condition for the maximum value: \( dx = m\lambda/4 \)  

(14)

This means that when the intervals between sound waves and sensors satisfy Eq. (13), the optimal condition of the inverse matrix shall be fulfilled, and conversely when they satisfy Eq. (14), the inverse matrix shall not be sought. On one hand the condition of Eq. (14), we can find, corresponds to the singular condition of Eq. (7), and on the other hand it becomes the condition to stop separating the sound wave. Consequently, on the occasion of having a moderate flow velocity, when the intervals between microphones is \( dx \) based on Eq. (13), the optimal wavelength of a sound wave can be said as \( \lambda = \frac{8}{(2m-1)}dx \).

3. SIMULATION

Through simulation we have examined the effect caused by the signal to noise ratio(SNR) and the difference of characteristics between sensors, which are factors that can possibly create errors during the actual measurement. Simulation conditions were selected at random and applied equally to every case.

* air with temperature of 20 degrees

\[ |P^+| = 10 \text{ [Pa]}, \quad \angle P^+ = 45 \text{ [degree]} \], \quad |P^-| = 5 \text{ [Pa]}, \quad \angle P^- = 77 \text{ [degree]}

First, we examined the variation of the condition number in accordance with the frequency and intervals between sensors. We assumed 10 sensors and set the frequency band as 10Hz–1kHz where the intervals between sensors are 8cm and 16cm respectively.

![Fig. 2 Condition number vs. frequency; sensor space (a) dx=0.08m (b) dx=0.16m](image)

Fig. 2 shows the condition numbers according to the frequency variation when having the mean flow velocity inside of a duct such as 0m/s, 6m/s, 12m/s respectively, which, we can notice, are too similar to classify and in good agreement with the results of Eq.s (13) and (14). Therefore, as in the event of the small Mach number, little difference occurs according to the flow velocity variation, we can reaffirm that it does not matter to calculate under the criterion that the flow velocity is 0m/s. To conduct the simulation under the same conditions to those of the actual system of measuring the mean flow velocity, we used 8 sensors and set the
intervals between sensors at 8cm. In this case, since the optimal frequency condition is about 536Hz according to Eq. (13), in simulation we carried out the calculation under the criterion of 500Hz. Fig. 3 shows the results of the simulation which was performed when the flow velocity inside of a duct was 12m/s. (a) and (b) represent the acoustic pressure signals over time, from the sensors to measure the maximum and minimum acoustic pressure, while (c) indicates the magnitude of the standing wave.

![Fig. 3 Time signal and standing wave pattern in ideal case](image)

The case of sensors having different characteristics: We can give consideration to the occasion that there is no measurement noises but the specificity difference between sensors. First, we examined the occasion of \( \hat{H}_n = (1 + a \cdot \varepsilon_n)H \), that is an error \( \varepsilon \) occurs merely to the magnitude of the transfer function \( H_n \).

<table>
<thead>
<tr>
<th>Table 1. Random distribution of ( \varepsilon ) on each channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch. 1</td>
</tr>
<tr>
<td>( \varepsilon )</td>
</tr>
<tr>
<td>Ch. 5</td>
</tr>
<tr>
<td>( \varepsilon )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Estimation of mean flow velocity and error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (\text{m/s}) )</td>
</tr>
<tr>
<td>( U = 0 ) (error)</td>
</tr>
<tr>
<td>( U = 6 ) (error)</td>
</tr>
<tr>
<td>( U = 12 ) (error)</td>
</tr>
</tbody>
</table>

Here, \( 8\varepsilon_n, s \), as shown in Table 1, were optionally abstracted from the set of the random numbers with a normal distribution of mean 0 and standard deviation 1, and \( a \) is the constant for adjusting the error magnitude. Table 2 reveals the consequent estimation reached when the magnitude difference exists between the transfer functions of the sensors for the following mean velocity cases is 0m/s, 6m/s, 12m/s respectively. It says that the error magnitude is not sensitive to the variation of \( a \) and in spite of the large difference in the magnitude of the transfer functions of each sensor, the approximate mean flow velocity inside of a pipe can be estimated. What is worthy of notice here is the fact that the estimated error of the mean flow-velocity remains almost unchanged according to the flow velocity. In other words, the actual mean flow velocity can be revised using the value of the flow velocity which is estimated under the condition of no flow velocity. Now, let’s take into account the occasion of having the same magnitude of the transfer functions of each sensor while only the phase difference exists. \( \hat{H}_n = \exp(ja \cdot \varepsilon_n \cdot \pi/180)H \) can be quoted as the instance where there is a phase gap between the transfer functions of each sensor. Likewise above, \( 8\varepsilon_n, s \), as shown

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in Table 3, were optionally abstracted from the set of the random numbers with a normal distribution of mean 0 and standard deviation 1, (the unit is [degree]) and a is the constant for adjusting the error magnitude. Table 4 displays the estimated consequence when the mean velocity is 0m/s, 6m/s, 12m/s respectively. This says that when there is a phase difference the error is much bigger than in the case of having a magnitude difference of the transfer functions of each sensor.

Table 3. Random distribution of $\epsilon$ on each channel

<table>
<thead>
<tr>
<th></th>
<th>Ch. 1</th>
<th>Ch. 2</th>
<th>Ch. 3</th>
<th>Ch. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>-0.5975</td>
<td>-1.2106</td>
<td>-0.7027</td>
<td>0.3564</td>
</tr>
<tr>
<td></td>
<td>Ch. 5</td>
<td>Ch. 6</td>
<td>Ch. 7</td>
<td>Ch. 8</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.6526</td>
<td>0.2157</td>
<td>-0.2639</td>
<td>1.804</td>
</tr>
</tbody>
</table>

Table 4. Estimation of mean flow velocity and error

<table>
<thead>
<tr>
<th>(m/s)</th>
<th>a = 0.5</th>
<th>a = 1</th>
<th>a = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>U = 0</td>
<td>1.3502</td>
<td>2.6732</td>
<td>7.6762</td>
</tr>
<tr>
<td>(error)</td>
<td>(+1.3502)</td>
<td>(+2.6732)</td>
<td>(+7.6762)</td>
</tr>
<tr>
<td>U = 6</td>
<td>7.3543</td>
<td>8.6795</td>
<td>13.6912</td>
</tr>
<tr>
<td>(error)</td>
<td>(+7.3543)</td>
<td>(+8.6795)</td>
<td>(+13.6912)</td>
</tr>
<tr>
<td>U = 12</td>
<td>13.3738</td>
<td>14.7055</td>
<td>19.7429</td>
</tr>
<tr>
<td>(error)</td>
<td>(+13.3738)</td>
<td>(+14.7055)</td>
<td>(+19.7429)</td>
</tr>
</tbody>
</table>

The reason is that the information on the mean flow velocity is included in the phase of the measured acoustic pressure. However, the estimated error in this case, also, shows a constant value, almost regardless of the variation in the mean flow velocity. Consequently, the indwelling difference of characteristics between sensors shall bring about a constant error in estimation irrespective of a flow velocity variation, so that the actual mean flow velocity can be revised using the estimated value under the condition of no flow velocity.

The case of having measurement noise: In general, actual measurement noise different from object signal has always existed. Should there be a small SNR, the object signal shall be contaminated by noise and in the end the distorted results will be estimated. To only examine the effect of noise, we used the same sensors with no changes to the characteristics between the transfer functions of each sensor.

Table 5. Estimation of mean flow velocity and error

<table>
<thead>
<tr>
<th>(m/s)</th>
<th>SNR = 5dB</th>
<th>SNR = 10dB</th>
<th>SNR = 20dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>U = 0</td>
<td>-0.1625</td>
<td>-0.0801</td>
<td>-0.0220</td>
</tr>
<tr>
<td>(error)</td>
<td>(-0.1625)</td>
<td>(-0.0801)</td>
<td>(-0.0220)</td>
</tr>
<tr>
<td>U = 6</td>
<td>5.8344</td>
<td>5.9188</td>
<td>5.9789</td>
</tr>
<tr>
<td>(error)</td>
<td>(-0.1656)</td>
<td>(-0.0812)</td>
<td>(-0.0211)</td>
</tr>
<tr>
<td>U = 12</td>
<td>11.8405</td>
<td>11.9277</td>
<td>11.5904</td>
</tr>
<tr>
<td>(error)</td>
<td>(-0.1595)</td>
<td>(-0.0723)</td>
<td>(-0.0096)</td>
</tr>
</tbody>
</table>

The conditions of the simulation are like the above-mentioned. For comparison under the same conditions, we worked out the same noises and amplified them according to the SNR (5dB, 10dB, 20dB). Fig. 4 shows the time signals and standing wave patterns with a flow velocity of 12m/s. Fig. 4 (a) and (b) are SNR = 5dB and 20dB respectively. When SNR = 5dB, the original signals are so contaminated by noise that the components of the harmonic signal can hardly be noticed. For this case, the results of estimating the standing wave formed inside of a pipe appear in Fig. 4 (c). The solid line represents the standing line reconstructed from the signals of the sensors, while each dot, the standing wave of the signals

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of each sensor. The results of estimating the mean flow velocity are arranged in Table 5. When the SNR is greater than 10dB, the results around the actual mean were obtained and as the SNR increases, error decreases. Consequently, for actual application it is better to raise the SNR to achieve more accurate and precise measurement of the mean flow velocity.

4. MEASUREMENT RESULTS

![Diagram of measurement setup](image)

Fig. 5 Experiment set-up to measure the mean flow velocity using 8 microphones

To directly measure the mean flow velocity inside of a pipe through actual experiment, we prepared the experiments as showed in Fig. 5. The duct was 15cm and 5m in internal diameter and length respectively, and 8 microphones were flush mounted with the internal surface of the duct. And through a ventilator we formed a flow field. To compare measurement results, we situated the common flow velocity measurement system in the center of the interior of the duct to directly measure the flow velocity and in this case the mean flow velocity registered 12.1m/s. To achieve optimal conditions for the experiment, similar to Eq. (13), we set the measurement frequency at 500Hz and 550Hz and carried out experiments in both the flow velocity and no flow velocity cases with each frequency.

![Graphs of measurement results](image)

Fig. 6 Time signal and standing wave pattern of measured pressure data of 500Hz: (a) time signal for U=0m/s case (b) time signal for U=12m/s case (c) standing wave pattern for U=12m/s case

Fig. 6 shows the experiment result with the frequency set at 500Hz. Fig. 6 (a) displays the time signal with no flow and (b), the time signal with the flow velocity of about 12m/s. When comparing (a) with (b), we can notice that the original signals were distorted substantially by a variety of noises. Yet, the pure standing wave can be obtained as shown in Fig. 6 (c) by seeking the acoustic field after separating off just the elements of 500Hz. Fig. 7 reveals the experiment results reached in the case of the frequency at 550Hz, which shows a similar result as the case at 500Hz. Table 6 reveals the estimation of the mean flow taken through the measured data. At this point, the revised value was obtained by subtracting the numerical value taken in the no flow case and as a result, we can find that the mean flow velocity shows the almost same value with both frequencies. However, there is some difference between this result and what is measured through the actual flow meter (12.1m/s).
Because the common flow meter was installed in the middle of the interior of a duct, where the maximum flow velocity is measured, accordingly the actual mean flow velocity should register a smaller value than that. Up to now results have presented a possibility to actually apply the above-suggested method of measuring mean flow velocity.

**Table 6.** Estimation of mean flow velocity from the measured pressure data

<table>
<thead>
<tr>
<th>(m/s)</th>
<th>500 Hz w/o flow</th>
<th>550 Hz w/o flow</th>
<th>500 Hz w/ flow</th>
<th>550 Hz w/ flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0.7471</td>
<td>12.2367</td>
<td>0.2610</td>
<td>11.8837</td>
</tr>
<tr>
<td>U_{nm}</td>
<td>0</td>
<td>11.4890</td>
<td>0</td>
<td>11.6227</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this study, the general relation formula of the acoustic pressure inside of a pipe was derived, from which a new approach was proposed to measure the mean flow velocity. According to the direction of the flow, the propagating velocity of a sound wave varies, hence so does the wave number. And by applying this principle the mean flow velocity inside of a pipe can be measured. We, in the process of the measurement and calculation, sought theoretically the singularity and optimal frequency band in accordance with the intervals between sensors. We also verified the utility of this facts through simulation and examined the effects which the difference of characteristics between sensors and measurement noises take on the estimated result of the mean flow velocity, and lastly contrived methods to revise them. Next, by performing a real experiment in a state of having flow inside of a pipe, we estimated the inner mean flow velocity and as a result we almost obtained the same results as the actual value. From above, we reaffirmed the potential of this approach to estimate the mean flow velocity inside of a pipe.

References