Comparative Study of Detailed Dynamic Analysis and Equivalent Static Analysis of Structures

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ABSTRACT

Detailed seismic analysis of nuclear structures is carried out in order to obtain seismic induced stresses, which are used for design of various members of the structure. An alternative method to detailed seismic analysis is the Equivalent Static Method (ESM). This method is simpler, conservative and cost-effective. In many design codes and standards such as IEEE, USNRC and ASCE, it is stated that this equivalent static method is recommended only for those structures, which can be represented by a simple model and has simple dynamic characteristics as that of a cantilever beam.

However for a tall tower supporting a flare stack, though a simple cantilever structure, it is observed that, in the dynamic analysis some of the members exhibit a local mode along with the global mode of the structure. For such members, equivalent static analysis will be conservative as compared to dynamic analysis if these local and global modes appear together and the dominant frequency of this tuned mode does not lie at the peak of the response spectrum. The ESM also can be used in the case the local and global frequencies differ by a factor of 2. But, ESM does not give conservative results if the local and global modes are tuned to each other and the tuned mode lies at the peak of the response spectra. It is observed that for such cases, the stresses in the members are found to be higher in dynamic analysis compared to equivalent static analysis. The reasons for such type of behavior are discussed considering a spring mass system and a box type structure. A method is evolved to judge the applicability of ESM, to a system to be qualified for seismic loads.

INTRODUCTION

Seismic qualification of nuclear structures requires detailed dynamic analysis using either the time history method or the response spectrum method. Time history methods require large computational efforts and are thus time consuming. Hence, response spectrum method is generally adopted for the dynamic analysis of structures. The response spectrum is a representation of the maximum responses of idealized single degree of freedom systems as a function of natural frequencies. The individual modal response of any system is thus obtained from the response spectra. The total response is further obtained by combining all the modal responses using the method such as SRSS or 10% SRSS.

An alternative simpler, cost-effective and conservative method used instead of detailed dynamic analysis is the equivalent static method (ESM). In this method, the system is analyzed by subjecting it to an equivalent static load proportional to its mass distribution in
the direction of the earthquake. The load magnitude is determined by multiplying the mass distribution with a factor, which is obtained as the product of the maximum spectral acceleration of the design response spectrum and a static coefficient, which accounts for the effect of the higher mode. In many design codes and standards such as USNRCS[1, IEEE[2], and ASCE[3], this static coefficient has been mentioned as 1.5. Also it has been stated in these codes and standards that ESM can be applied only to those structures which can be represented by a simple model [4] like that of a cantilever beam. Based on these criteria, a simple cantilever structure such as a tall tower supporting a flare stack is analysed.

A TALL TOWER SUPPORTING A FLARE STACK

A model of tall tower supporting a flare stack is shown in Fig.1(a). All the members of the tower are rolled steel sections. The height of the tower is 116m. The free vibration frequency analysis and also the dynamic analysis is carried out using computer package COSMOS/M. The first two fundamental modes having frequencies 0.88 Hz and 1.7 Hz, are basically global modes representing full structure without any local modes. However, the third mode appears in four modes along Y- direction as shown in Figs. 1b, 1c, 1d and 1e. This happens due to the interaction of global mode along Y- direction with the local modes along Y and Z directions. The total mass participation of these modes along Y- direction is equal to the mass participation in the third mode along X- direction.

The response spectrum analysis of the structure is carried out using the spectrum shown in Fig.2 having ZPA 0.03g [5]. Equivalent static analysis is carried out and the results of the analysis are tabulated in Table 1. It can be observed from Table 1, that the dynamic stresses in elements such as 91, 197, 159, 184, 182, 174 exceed the stresses obtained by equivalent static analysis. These elements are identified as the members participating in the local modes. This occurs due to the following three reasons.

1. The acceleration value at a frequency of this tuned mode is equal to the peak of the response spectrum.
2. The mass participation is equally distributed amongst the four tuned modes.
3. The mode shapes of these modes show large local displacements in Y-direction and Z-direction as shown in Figs. 1(b), 1(c), 1(d) and 1(e).

All these factors lead to the response of these elements obtained from dynamic analysis is more than that from equivalent static analysis.

In order to understand the above phenomenon in details, two case studies such as simple spring mass system and a cantilever box structure are considered. These systems are defined such that their local and global modes appear simultaneously as in the case of tower structure. Analysis using ESM and response spectrum method is carried out and the response ratio is plotted as a function of frequency ratio. Finally, a factor by which the local and the global frequencies must differ for ESM to be conservative is suggested.

A SPRING MASS SYSTEM

A simple, two degrees of freedom spring mass system as shown in Fig. 3(a) is considered. The top and bottom springs have stiffness as 58000 N/m and 40000 N/m respectively. The top and bottom masses are 22 Kg and 36 Kg respectively. Free vibration analysis for this system is carried out and the frequencies and the mode shapes obtained are shown in Fig. 3(a). Next, two springs are added to the system as shown in Fig. 3(b) such that the individual frequency of the added springs is equal to the fundamental frequency of the initial system. The total mass on the added springs is assumed to be approximately 1% of the total mass of
the initial system. Free vibration analysis is carried out for this system and the frequencies and mode shapes as shown in Fig. 3(b) are obtained.

It is observed from Fig. 3(b) that the mass participation of first mode, which was initially 57 kg (Fig. 3(a)) is reduced to 33.74 Kg and the third mode is similar to the first mode having frequency close to the first mode. It has mass participation of 23.95 Kg. The second mode is the local mode of the added spring masses. The fourth mode of the system is similar to the 2nd global mode of the system shown in Fig. 3(a).

Thus, it is observed that the presence of the 2-spring mass systems whose local frequencies is near the global frequency of the system makes that particular global mode appear in two different modes. Also, the mass participation is distributed amongst the two modes, such that the total mass participation is almost equal to that of the global mode if the local spring mass systems were absent.

A detailed study is made in which the frequencies of the local springs are increased and decreased by changing the mass of the local springs. The mass of the global springs are changed correspondingly so that the total mass of the system remains same and the global frequency remains the same. Analysis using ESM and response spectrum method is carried out. In Fig. 5, response ratio of spring forces obtained in response spectrum method and ESM are plotted as a function of frequency ratio of local to global modes. In Fig. 6, percentage modal mass as a function of frequency ratio is plotted for first and second global modes.

A CANTILEVER BOX STRUCTURE:

As a second case study a box structure as shown in Fig. 4(a) is considered for the analysis. The walls of the structure has a thickness of 0.3 m, width of 0.5 m, and a height of 8m. There is a slab on the top of the walls of thickness 0.15m. The material considered has properties such as Density = 2500 Kg/m$^3$, Modulus of elasticity = 0.07 x 10$^{10}$ N/m$^2$. Poisson’s ratio =0.2. The modulus of elasticity is an arbitrary value taken such that the frequency of the box structure lies at the peak of the response spectrum. A spring mass system is attached to the middle of the two walls of the box structure along Y- direction as shown in Fig. 4(a). The mass of the springs is such that it is about 1% of the total mass of the box structure. The frequency of each of the spring mass system is equal to the first fundamental frequency of the box along Y- direction. Free vibration analysis of the finite element model is carried out using computer package, COSMOS/M. It is found that the springs attached to the walls of the box structure exhibit local modes which appear along with the first fundamental global mode. This global mode appears twice with 37% and 35% mass participation respectively in both the global modes as shown in Fig. 4(b). Similar to above case study, analysis is carried out and various dynamic parameters obtained are plotted in Fig. 5 and Fig. 6.

DISCUSSIONS AND CONCLUSIONS:

It can be observed that as the frequency ratio increases from 0.5 to 1.25, the modal participation increases and further it remains constant. On the other hand in the second mode it decreases and remains almost zero after the frequency ratio of 1.25. This shows that there is no interaction between global and local modes when the frequency ratio goes beyond 1.25. In the case of response forces, the forces in the bottom spring (main spring) of case study 1 and bottom element of box structure (case study 2), obtained using response spectrum method remains lower than that of equivalent static analysis. Whereas, the forces obtained in response spectrum method in the local members (local springs) are higher (maximum by a factor 4) in the frequency ratio range approximately 0.75 to 2.75. Therefore, conservatively it
can be said that one has to take into account the interaction effects when the local to global frequency ratios are in range of 0.5 to 3.0. In the case of tower supporting flare stack, by changing the local frequency (i.e. 3 times more than global frequency) it was observed, that the stresses obtained in response spectrum method in the local members are less than those obtained using ESM [5].

The following steps can be followed while checking the adequacy of ESM:

For a simple cantilever type structure:

a) Obtain the global frequency using classical formulae or FE model.
b) Obtain the local frequencies using classical formulae or FE model.
c) Check the ratio of local to global frequencies.
d) If the ratio of frequencies is in the range of 0.5 to 3.0 perform the analysis using response spectrum method otherwise ESM is adequate.

REFERENCES

(1) IEEE Recommended practice for Seismic Qualification of class 1E Equipment for Nuclear Power Generating Stations, IEEE Std 344-1987, Institute of Electrical and Electronic Engineers, pp. 6-16.


(3) American Society of Civil Engineers (ASCE) Standard, Seismic Analysis of safety related Nuclear structures and Commentary on Seismic analysis of Safety Related Nuclear Structures, ASCE 4-86, September 1986.


TABLE 1 STRESSES IN THE MEMBERS (2% damping)

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>$\sigma_{dl}$ (1-150 modes) N/m²</th>
<th>$\sigma_{dz}$ N/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>174 (Level-2)</td>
<td>$6.49 \times 10^7$</td>
<td>$1.56 \times 10^7$</td>
</tr>
<tr>
<td>159 (Level-2)</td>
<td>$7.35 \times 10^7$</td>
<td>$1.37 \times 10^7$</td>
</tr>
<tr>
<td>184 (Level-2)</td>
<td>$7.77 \times 10^7$</td>
<td>$1.99 \times 10^7$</td>
</tr>
<tr>
<td>182 (Level-2)</td>
<td>$5.13 \times 10^7$</td>
<td>$1.52 \times 10^7$</td>
</tr>
<tr>
<td>91 (Level-2)</td>
<td>$1.02 \times 10^7$</td>
<td>$0.89 \times 10^7$</td>
</tr>
<tr>
<td>97 (Level-2)</td>
<td>$1.65 \times 10^7$</td>
<td>$1.20 \times 10^7$</td>
</tr>
</tbody>
</table>

Note:

$\sigma_{dl}$ = stress computed using response spectrum method
$\sigma_{dz}$ = stress computed using ESM
Fig. 1(a) FEM Model of a Flare stack

Fig. 1(b) Mode shape of the tower (frequency=3.81 Hz, mass participation=5.5 %)

Fig. 1(c) Mode shape of the tower (frequency=3.86 Hz, mass participation=4.64 %)

Fig. 1(d) Mode shape of the tower (frequency=3.94 Hz, mass participation=6.5 %)

Fig. 1(e) Mode shape of the tower (frequency=3.95 Hz, mass participation=1.2 %)
Fig. 2 Response spectra for 2% damping with 0.03g ZPA.

Fig. 3(a) Frequency and mode shapes of 2 DOF spring mass system

Fig. 3(b) Frequency and mode shapes of 4 DOF spring mass system
Fig. 4(a). FEM Model of a Box structure

Mode 1 Frequency = 4.14 Hz.  
Mass participation = 37%

Mode 2 Frequency = 4.79 Hz.  
Mass participation = 35%

Fig. 4(b) Mode shapes of first bending mode of a cantilever box
Fig. 5 Response ratio v/s Frequency ratio for two cases

Fig. 6 Mass participation v/s Frequency ratio for the two cases