Cavity Formation in Interface between Power Low Creep Particle and Elastic Matrix Subjected to a Uniaxial Stress

Yong-Son Lee¹, Sang-Tae Kim¹, Jae-Do Kwon¹ and Kil-Soo Kim²

1) Yeungnam University, Korea
2) Korea Institute of Nuclear Safety, Korea

ABSTRACT

The paper attempts to estimate the incubation time of a cavity in the interface between a power law creep particle and an elastic matrix subjected to a uniaxial stress. Since the power law creep particle is time dependent, the stresses in the interface relax.

Through previous stress analysis related to the present physical model, the relaxation time is defined by $\alpha_2$ which satisfies the equation $\Gamma_0 (1 + \alpha_2 \kappa_1) = 1 - \alpha_2$ [19].

The volume free energy associated with Helmholtz free energy includes strain energies caused by applied stress and dislocations piled up at the interface (DPU). The volume free energies caused by both applied stress and DPU are a function of the cavity size ($r_c$) and elapsed time ($t$) and arise from stress relaxation of the interface.

The critical radius $r_c^*$ and incubation time $r^*$ to maximize Helmholtz free energy is found in present analysis. Also, the kinetics of cavity formation are investigated using the results obtained by Riedel [16]. The incubation time is defined in the analysis as the time required to satisfy both the thermodynamic and kinetic conditions.

1. Introduction

The failure of metals and alloys in high temperature service is a formidable technological problem to which much time and expertise has been devoted [1,2,3]. Since the pioneering work of Greenwood[4] it has become well established that the prime cause of high temperature creep fracture is the formation of intergranular creep cavities, which lead to cracks under tensile loading.

Svensson and Dunlop [5] have reviewed the literature regarding growth of intergranular creep cavities. They conclude that the creep fracture process can be divided into the following stages [5]. The stages are: 1) nucleation of cavities, 2) growth of cavities, 3) interlinkage, 4) crack propagation, and 5) final failure. It should be noted that these stages are not necessarily discrete. They may overlap, even to the point where they all occur simultaneously.

The majority of physical metallurgical research on creep fracture has concentrated on the nucleation and growth stages of cavitation with the implicit assumption that these stages are time dominating in the overall fracture process. The validity of this assumption
may in many cases be questionable [5]. Thus Attempts have been made to formulate satisfactory creep fracture criteria [6,7]. In general it has now been accepted that, once nucleated, cavities can grow either by diffusion, by power law creep, or by a combination of both [5,8]. The mechanism for cavity nucleation, however, is still controversial.

Gifkins [9] and Chen, and Machlin [10] first proposed that slip impingement produces offsets at the grain boundary, which are then opened up as cavities by grain boundary sliding. Harris [11] and Ishida and Mclean [12] have pointed out that even if such a process is feasible during a sliding burst, the cavities will sinter up by diffusion immediately after they are formed. They [11,12] concluded that the step models are impracticable.

Raj and Ashby[13] have shown that the stress concentration set up by grain boundary sliding can drive vacancies to cluster, forming cavities. However, these stress concentrations relax readily at high temperature. Raj [14] has shown that for cavity nucleation to be feasible, the stress concentration must be maintained over a specific period of time, called the incubation time for the cavity nucleation. When stress relaxation prevails, Raj[14] has shown that cavity nucleation may be possible at the second phase particles in the interface, where the critical vacancy cluster is small. Raj [14] indicated that cavities nucleate predominantly during the primary creep stage, when the strain rate is high. Argon et al. [15] have examined the role of sliding in cavity nucleation in materials where relaxation occurs by power law creep. They [15] have concluded that cavity nucleation is unlikely during a steady state creep, unless the boundary sliding is spasmodic. Treating sliding boundaries as shear cracks, while allowing relaxation by both diffusion and power law creep, Riedel [16] has concluded that sliding induced cavitation is not feasible under the usual creep conditions. Hirth and Nix [17] concluded that although cavity nucleation by vacancy clustering involves short range diffusion, vacancy supersaturation at high stress concentration sites requires long range diffusion and a large build-up time. Therefore, sliding is not the cause for cavitation at high temperatures.

In an attempt to resolve the above conflict, Lim [18] proposed a model of pile ups of grain boundary dislocations(GBD), to explain cavity nucleation at high temperatures. He [18] showed that high strain energy ahead of the pile up is a steady state phenomenon during secondary creep. It helps to compensate for the large capillarity forces in the formation of sub-micron sized cavities, thereby rendering cavity nucleation barrierless. He showed the existence of a threshold stress below which the cavities cannot grow to effect fracture. Through his results [18] he concluded that cavity nucleation is feasible in single phase metals and alloys at the intersection of cell and grain boundaries. The model suggests that during hot tensile tests, a ductility trough should occur at intermediate temperatures. The results [18] were compared with the experimental observations and were found to be in good agreement.

Lee et al. [19] performed a stress analysis of a power law creep particle embedded in an elastic matrix subjected to a uniaxial stress. They found that stress relaxation caused by power law creep occurs in the interface between the particle/matrix. The relaxation time is characterized by \( \alpha_2 \) to satisfy the equation \( \Gamma_0 |1 + \alpha_2 \kappa|^{-n} = 1 - \alpha_2 \), where \( \kappa = 3 - 4 \nu \) and \( \Gamma_0 = 2(1/\sqrt{3})^{1+n}(\sigma_{\infty}/2\mu)^n(\sigma_b/\sigma_{\infty})^m \).

For particle in an elastic matrix, \( m = 1.0 \), \( \Gamma_b = \sigma_b/(3\mu) \), and \( \Gamma_0 \) corresponds to the ratio of a particle's viscosity to matrix shear modulus. If \( m = 1.0 \), then \( \alpha_2 = (1-\Gamma_b)/(1+\Gamma_b \kappa) \) is identical to the expression "A" in Dundurs and Mura's result.
Comparing $A = (1 - \Gamma_0) / (1 + \Gamma_0 \kappa)$ in the result of [20], with $\alpha_2$ in the present analysis, we see that "$A$" is time independent, while $\alpha_2$ is time dependent. This time dependency results from a power law creep particle, while time independency corresponds to an elastic particle and matrix. The pile up stress in the vicinity of the particle is obtained from the interaction force of a dislocation found by Dundurs and Mura [20], replacing the Burger's vector $b$ with $n_b$, where $n$ is number of dislocations piled up in the interface. The quantity $\Gamma_0$ of power law creep material is substituted into the quantities $A$ and $B = (\kappa_2 - \Gamma_0 \kappa_1) / (\kappa_2 + \Gamma_0)$ ($\kappa_2 = 1$, $\kappa_1 = 3 - 4 \nu$) [24], assuming the expression of interaction force given in [24] does not differ from the expression for a power law particle in an elastic matrix.

In the previous analyses [13, 14], this "volume free energy" is considered to be time independent, while in the present analysis it is time dependent, because of $\Gamma_0$ in the power law creeping particle. There are two dependent variables in the free energy $\Delta G$: $r_c$ (cavity radius) and $t$ (elapsed time). The values of $r_c^*$ and $t^*$ to maximize $\Delta G$ are found from the energy equation. The incubation time is found from the kinetics equation proposed by Riedel [16]. Through the analysis it is found that, even though vacancies can condense when the thermodynamic condition is satisfied, they can not grow right after the condensation takes place because of sluggish kinetics.

2. Numerical results and Discussions

Riedel [16] gives the characteristic relaxation time of an elastic stress singularity by a power law creep. It is given by

$$ t_p = \frac{1}{(n+1)EB \sigma_p^{n-1}} $$

(1)

where $\sigma_p$ is the average stress of the particle, $\sigma_p = \tau (\lambda / p)^2$, $\lambda$ is spacing of a cube shaped particle with a diameter $p$, $\tau$ is the shear traction applied on the shear crack connected between two triple grain boundary caused by the remote stress (see Fig. 2 in [16]). Eq. (1) was derived using a different model from that of the present analysis. However, there is a similarity between Riedel’s and the present results. The relaxation time caused by power law creep in the present case is given in eq. (1a) in Appendix A. Rewriting eq. (1a) and eq. (1), those are shown as:

$$ t = 2^n \left( \frac{1}{\sqrt{3}} \right)^{1 + \frac{n}{2}} \left( \frac{\sigma_0}{2\mu} \right)^{n-1} \left( \frac{\kappa_2 \kappa_1}{1 - \kappa_2} \right)^{n} $$

(2a)

$$ t_p = \frac{1}{(n+1)(1 + \nu)} \left( \frac{\sigma_0}{2\mu} \right)^{\nu} \left( \frac{\kappa_2 \kappa_1}{1 - \kappa_2} \right)^{\nu} \left( \frac{R}{\lambda} \right)^{2\nu - 2} $$

(2b)

When Riedel [16] compared eq. (2b) with the incubation time obtained by Raj [14], he concluded that the elastic stress concentration has no role in cavity formation.

The required time for cavity formation computed from thermodynamics, (but not kinetics) was found to be approximately $\alpha = 0.2$.

The value of $\Gamma_0$ corresponding to $\alpha_2 = 0.2$ in Fig. 2 is approximated $\Gamma_0 = 0.65$ ($m = 1.0$) and $\Gamma_0 = 0.8$ ($m = 0$). The thermo-dynamically favorable incubation time is approximately $t = \varphi n / 0.73$, where $\varphi$ is given in eq. (3) and the value 0.73 is the mean value of 0.65 and 0.8 for $\Gamma_0$. However, this value does not consider the kinetics of cavity formation.

Even though the strain energy caused by dislocations piled-up is included in $\Delta G$, the
critical radius $r^*$ and $t^*$ do not differ from the results, including energy due to remote
stress. Equation (4') shows $r^*$ and $t^*$. $r^*$ is identical to the results which neglect the
stored energy term from remote stress. Critical time $t^*$, satisfying the thermodynamic law
of the dislocation piled-up model, would not differ from the result of
$t^* = t_i = \phi \cdot \frac{\sigma^*}{0.73^n}$. $t^*$ in eq. (4') is rewritten as:

$$t^* = \phi \cdot \left(1 + \frac{\kappa^2}{2}\right)^n \cdot \left(1.334\right)^n \quad (\kappa = 1.6, \ \nu = 0.35, \ n = 1/m)$$

which is not significantly different from $\phi \cdot \frac{\sigma^*}{0.75^n}$. If eq.(5') is integrated to
$t^* = \frac{\phi}{\nu} \cdot \left(1 + \frac{\kappa^2}{2}\right)^n$, the integrated value is much smaller than 1.0. Therefore the kinetics of
cavity formation do not increase with $t^*$.

The numerical integration of eq. (5') is performed using the properties of copper
given in [18]. The $t^*$ to satisfy eq.(5') is found. The value of $\phi_0 = 40$ and 70 are
considered. Quantity $\phi$ is given in eq. (3') and $c$ is given in eq. (6'), expressed as a
function of the time $r_0 = \phi / \sigma^*$.

The time to satisfy eq. (5') can be found for various $\phi_0$ and $\sigma_0 / \sigma_\infty$. The
incubation time is shown in Fig. 4 for various temperatures and $\sigma_0 / \sigma_\infty$ as $\phi_0 = 40, \ \sigma_\infty = 50$MPa and strain rate exponent $m = 0.1$. Fig. 4 includes the result for $m = 1.0$. The
incubation times for various values of $\sigma_0 / \sigma_\infty$ and temperatures are shown in Fig. 5 for
$\phi_0 = 40, 70, \ \sigma_\infty = 50$MPa and $m = 0.1$. The incubation time variation with respect to strain
exponents (m), under fixed conditions of $\phi_0 = 70$ and $\sigma_0 / \sigma_\infty = 120$ and temperature, is
shown in Fig. 6.

Decreasing $\Delta G^*$ can be seen in eq. (8'), and enveloped $\Delta G^*$ decreases with an
increase of the elapsed time. Fig. 7 shows the schematic variation of several quantities
with elapsed time: critical radius($r^*$), $\Delta G^*$, the integrand(F) of eq. (5'), and integrated
value of eq. (5').

Figure 8 shows incubation times of $B_2O_3, Pb$ and $CoO$ particles in a copper matrix
for various temperatures. In order to obtain the material constants expressed by a power
law, $\sigma_0 (\sigma = \sigma_0 e^{n \sigma})$ is obtained using Dorn's equation shown in [21] and
material data given in [22, 23] and [24] respectively. The incubation time is obtained
using $t_i = \phi \cdot \left(1 + \frac{\kappa^2}{2}\right)^n$ as seen in the second expression of eq. (4'). The quantity
$(1 / (1 + \kappa^2))^{1/2}$ for copper is given as 0.7495 ($\nu = 0.35$).

3. Conclusions

Through the analysis the following conclusions are derived:

1) The most thermodynamically favorable condition for cavity nucleation is when
$\sigma_0 ~ 0.2$, which is when $\Delta G$ includes the strain energy caused by the remote stress in
the volume free energy, without including the energy due to dislocations piled up. The
kinetics of nucleation should be accounted for by the integration of eq. (5') to the time
corresponding to $a_2 \sim 0.2$. The incubation time corresponding to $a_2 \sim 0.2$ can be found at $\Gamma_0 \sim 0.65 \sim 0.75$ (depending on $m$ value) from Fig. 2 and the time corresponding to $\Gamma_0 \sim 0.65 \sim 0.75$ can be found from eq. (1a'). The integration of eq. (5') is performed to the time corresponding to $\Gamma_0 \sim 0.65 \sim 0.75$ and it was found that eq. (5') is not satisfied. Therefore it can be concluded that strain energy caused by the remote stress can not contribute to the kinetics of cavity nucleation. Also, the strain energy associated with the remote stress in eq. (9') is very small in magnitude and can be neglected.

2) When $\Delta G$ includes the energy change dislocations piled up, the thermodynamically favorable condition is at $c = 0$ or $\Gamma_0 = (2/(1 + \kappa^2))^{1/2}$ and the kinetics of the nucleation are evaluated by the integration of eq. (5') to time $\Gamma_0 = 2/(1 + \kappa^2)^{1/2}$. Through the integration it is found that eq. (5') is not satisfied. Therefore the thermodynamically favorable condition shown in eq. (4') does not satisfy the kinetic condition of eq. (5'). Cavity nucleation can not be expected within a year.

3) Items 1) and 2) satisfy the thermodynamic conditions, but not the kinetic conditions given in (5'). In order to satisfy both the thermodynamics condition to decrease $\Delta G$, and the kinetics condition of eq. (5'), the critical radius $r^*$ should decrease or holds constant with increase of elapsed time until eq. (5') is satisfied. It is noted that variation of $c$ with time or $\Gamma_0$ as seen in eq. (1a') and Fig. 3 does not violate the law of thermodynamics. Therefore, a cavity may not grow right after it is formed, which agrees with the conclusions pointed by [11] and [12].

4) There is a threshold temperature for failure due to cavity formation (Fig. 4, 5 and 6). Longer incubation time promotes smaller particle size ($R$), larger volume fraction of the particle ($f$), smaller applied stress, and larger shear modulus of the matrix ($\mu$). The incubation time increases with the ratio of $\sigma_0/\sigma_\infty$. However, there is instability of $t_i$ for both $\sigma_0/\sigma_\infty$ and temperature (Fig. 4 and 5). The lower strain rate hardening exponent ($m$), increases the incubation time, while $t_i$ does not change for large $m$ ($m > 0.4$). The incubation time is affected significantly by local stress, such as dislocations piled up, rather than strain energy ($\text{eq. (9')}$ and (10')), which is negligible. In addition to the local stress, the incubation time is affected by particle size ($R$), $m$ and volume fraction ($f$).

4. References

8. A.C.F. Cooks and M.F. Ashby, "creep in structures" (edited by A.R.S. Pointer and

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Appendix A

\[ K(m) = \left( \frac{\sqrt{3}}{4} \right) \left( 2\sqrt{3} / \sigma_0 \right) \frac{1}{m} \]

\[ I_0 = 2(1/\sqrt{3}) (1 + m) \left[ (\sigma_m/2\mu)^m (\sigma_m \tau^m \sigma_m) \right] \]

\[ \phi = \frac{1}{2\sqrt{3}} (\sigma_m/2\mu) \frac{(\sigma_0 \tau)}{(\sigma_m \tau)} \left[ \frac{1 + x^2}{2} \right] \]

\[ t^* = \left( \frac{2}{\sqrt{3}} \right)^{m} \left( \frac{\sigma_m}{2\mu} \right) \left( \frac{1 + x^2}{2} \right) \]

\[ \rho = \rho_w - 1 - \int_0^c \left( \frac{x}{4} + \phi c^2 \right) \frac{d\sigma}{\sigma^2 K T \left( \frac{x}{4} + \phi c^2 \right)} \]

\[ \frac{4\gamma^2 F_c}{\left( \pi x \right)^{1/2}} \]  

\[ c = A + B = \frac{2 - I_0^2 (1 + x^2)}{1 + I_0 (1 + x) + J_0^2 x} \]

\[ \Delta G = -\gamma^2 F_c \sigma_m \left[ \frac{x}{4} + \phi c^2 \right] + 3 \gamma^2 F_c (\beta) \]

\[ \Delta G^* = 4\sigma^2 F_c / \sigma_m^2 \left[ \frac{x}{4} + \phi c^2 \right] \]

\[ \Delta G = -\gamma^2 F_c \left[ \sigma_r + \frac{3}{2} \omega \right] + 3 \gamma^2 F_c \]

\[ w = -\frac{\nu}{2E} (\sigma_r + \sigma_0) + \frac{1 + \nu}{2E} (\sigma_r^2 + \sigma_0^2) + \frac{1}{2\mu} \sigma_r \theta \]

where \( \sigma_0 \) are given in [19].
Fig. 1 Schematic Diagram of Cavity Nucleation Model

\[ \Gamma_0 = 2(1/\sqrt{3})^{\epsilon} \left[ \left( \sigma_0 / \sigma_{\infty} \right) \right]^m \left( \sigma_0 / \sigma_{\infty} \right) \]
\[ \Gamma_0 = 1 + \alpha \phi \]  
\[ \alpha = 1 - \alpha_2 \]

Fig. 2 Roots of \( \alpha_2 \) Variation of \( \Gamma_0 \) Various and Strain Rate Exponent(m)

\[ C = -2 + P_x(1 + x^2) \]
\[ 1 + \rho_0(1 + x) + P_x \]
\[ \rho_0 = 2(1/\sqrt{3})^{\epsilon} \left[ \left( \sigma_0 / \sigma_{\infty} \right) \right]^m \left( \sigma_0 / \sigma_{\infty} \right) \]

Fig. 3 C Variation of Various \( \Gamma_0 \)

Fig. 4 Incubation Time for various Temperature and Creeping Particle Property

\[ \sigma_0(\bar{\sigma} = \sigma_0 \bar{\varepsilon}^m, m = 0.1) \text{ at } \sigma_{\infty} = 50 \text{ MPa, } \phi_0 = 40 \]

Fig. 5 Incubation Time for various \( \sigma_0 \) and Temperature at \( \phi_0 = 40, 70 \) and \( \sigma_{\infty} = 50 \text{ MPa and } m = 0.1 \)
and Elapsed Time

b : Critical Nucleation Radius Variation versus Elapsed Time

c : Critical Helmholtz Tree Energy Variation versus Elapsed Time

d : Intergrand Function(F) variation versus Elapsed Time

e : Kinetics variation versus Elapsed Time ($\int_0^t F \, dt = 1$)

Fig. 6 Incubation Time for various Strain rate Exponents($m$) and Temperature at $\sigma = 120$, $\psi = 70$ and $\sigma_{\text{no}} = 50$ MPa

(a)

Fig. 8 Incubation Time for various Temperature and various Creeping Particles ($B_2O_3$, Pb and CoO)

Fig. 7 Schematic Diagram

a : $\Delta G$ versus Cavity Nucleation Radius