Analysis of Axial Crushing of Metallic Round Tubes

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ABSTRACT

The large deformation behavior of thin walled structures under axial loading, both static and dynamic, has been studied extensively for its application to the design for crash energy absorbing devices. In particular round tubes are the most frequently used devices that absorb energy by material deformation and have therefore received considerable attention of researchers.

Models for axial crushing of cylindrical tubes have been developed for both total inside as well as total outside folding. These early models do not satisfy the experimental observations where it has been observed that the folds formed are partly inside and partly outside. In the present paper a simplified straight fold model with partly inside and partly outside folding has been developed and the proportion of inside and outside fold lengths have been determined by minimizing the average crushing load. The size of fold and mean as well as variation of crushing load has been computed. The difference in the value of yield stress of material in compression and tension has been incorporated in the analysis. The variation of circumferential strain during the formation of a fold has been taken into account. The total inside and total outside fold models can also be easily derived from the present model. The results have been compared with experiments and reasonably good agreement has been observed. The results are of help in understanding the phenomenon of actual fold formation.

INTRODUCTION

Aircrafts, ships and space vehicles etc. have to withstand an accidental crash for which efficient energy absorbing devices are required. These energy-absorbing devices may be classified under three different categories on the basis of the principle of (a) extrusion, (b) friction, and (c) material deformation. Cylindrical tubes are the most frequently used devices that absorb energy by material deformation.

Many investigators have carried out analysis of axial crushing of cylindrical tubes in concertina mode by considering the formation of plastic hinges [1-7]. For the calculation of mean crushing load, Alexander [7] had taken mean circumferential strain. But the calculation of variation of crushing load requires the consideration of variation of circumferential strain with change in the rotation of fold (see[1]). Alexander [7] considered total outside folding, whereas Abbas et al. [2,3] considered total inside folding in their formulation. These early models, in general, do not satisfy the experimental observations [8,9] where it has been observed that the folds formed are partly inside and partly outside the initial mean diameter of the tube. In some recent studies [8], models were developed with partly inside/outside folding but the folding parameter, m, which is the ratio of inside to total fold length was either taken from experiments or it was assumed. The authors [5,6] developed some useful models for determining the value of folding parameter mathematically and incorporated the change in the thickness of the tube during fold formation.

In the present study, a simplified straight-fold model with partly inside and partly outside folding has been developed. The models take into account two different types of variation in the thickness over the fold length as well as the parameter r, which is the ratio of the yield stress values of the tube material in compression and tension. The value of the fold parameter has been determined by minimizing the average crushing load. For r = 1, and without consideration of the change in thickness, the fold parameter is found to be more than 0.5, whereas, experiments [8] show that its value should be less than 0.5. Incorporating changes in thickness in this model, the fold parameter comes closer to experiments. Expressions have been developed in terms of the parameter r. The influence of its variation is considered and is found to help in explaining the process of fold formation. The total outside and the total inside fold models can be easily derived from the present model by simply putting m=0 and 1, respectively.

ANALYSIS FOR AXIAL CRUSHING OF TUBES

Considering a cylindrical tube of mean diameter D and initial thickness t_0 for the purpose of its analysis. The tube is undergoing axi-symmetric axial crushing. Let h be the half fold length out of which mh is inside and (1-m)h is outside the mean diameter as shown in Fig. 1(a). Neglecting the change in thickness due to axial forces, there will be decrease in

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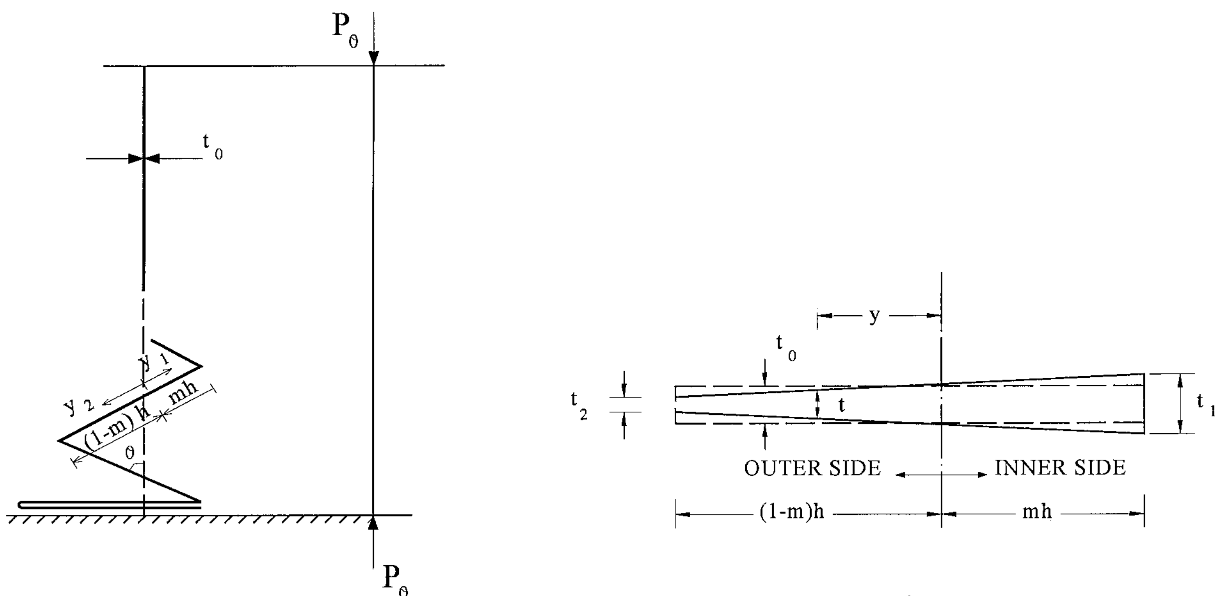
thickness in outside portion of the fold due to circumferential stretching and increase in thickness during circumferential compression in its inside portion.

In total four models have been considered in the present study. Models A and B do not consider change in the thickness of the tube, whereas the other two models (Models C and D) incorporate the change in the thickness of the tube. Stepped variation in thickness has been considered in Model C and linear variation in Model D as shown in Fig. 1(b). For clarity in presentation, stepped variation is not shown in this figure. Mean circumferential strain as considered by Alexander [7] has been incorporated in Model A and in all the other three models circumferential strain varies with the rotation of the fold. The yield strength of the material of the tube in compression and tension has been taken as f_{yc} and f_{yt} respectively.

The analysis of partly inside and partly outside folding of a cylindrical tube is presented in the subsequent subsections. The plastic moment of resistance of the material of the tube has been taken as $M_p = \frac{1}{2\sqrt{3}} f_{yt} t^2$, where, t is the instantaneous thickness of the tube.



(a) (b)

Fig. 1. Axial crushing model of a cylinder. (a) Folding mechanism; and (b) Variation of thickness

Energy Absorbed in Crushing

The energy absorbed in axi-symmetric crushing of the tube has been calculated in the following for all the four models. The Model A has been considered as a special case of Model B, so it has not been considered separate.

Model B –No change in thickness

The absorption of bending energy is assumed to be concentrated at the hinges only. Therefore work done in bending W_b in the rotation at hinges for total crushing of the fold is given by:

$$W_b = 2\pi M_p \left(\pi D + 2h - 4mh\right) = 2\pi k f_{yt}\, t_0^2 \left(\pi D + 2h - 4mh\right) \qquad (1)$$

Considering variation in circumferential strain with the rotation of the fold, the work done in circumferential deformation W_c in rotation of fold up to $\pi/2$ can be calculated from [5,6]:

$$W_c = 2\pi f_{yt} t_0 h^2 \left[r m^2 \left(1 - \frac{2mh}{3D}\right) + (1-m)^2 \left\{1 + \frac{2(1-m)h}{3D}\right\} \right] \qquad (2)$$

Where, $r = f_{yc}/f_{yt}$

Taking mean value of circumferential strain as considered by Alexander [7], approximate value of W_c is given by:

$$W_c \approx 2\pi f_{yt} t_0 h^2 \left\{ rm^2 + (1-m)^2 \right\} \tag{3}$$

For Model B, W_c is given by Eq. (2), whereas for Model A W_c is given by Eq. (3).

Model C – Stepped variation in thickness

Considering uniform increase in thickness along the inside length of fold and uniform decrease in thickness along the outside length of fold, the changed thickness t_1 and t_2 for inside and outside portions of fold are given by:

$$t_1 = \frac{Dt_0}{D - mh\sin\theta} \quad \text{for inside portion of fold} \tag{4a}$$

$$t_2 = \frac{Dt_0}{D + (1-m)h\sin\theta} \quad \text{for outside portion of fold} \tag{4b}$$

Considering stepped change in thickness, the modified value of W_b is given by [5]:

$$\begin{aligned} W_b &= 4\pi k f_{yt} D^2 t_0^2 \frac{\left(D^2 - 2m^2h^2\right)}{A^{1.5}}\left[\tan^{-1}\left\{\frac{D - mh}{\sqrt{A}}\right\} - \tan^{-1}\left\{\frac{-mh}{\sqrt{A}}\right\}\right] \\ &+ 4\pi k f_{yt} D^2 t_0^2 \frac{\left\{D^2 - 2(1-m)^2h^2\right\}}{B^{1.5}}\left[\tan^{-1}\left\{\frac{D + (1-m)h}{\sqrt{B}}\right\} - \tan^{-1}\left\{\frac{(1-m)h}{\sqrt{B}}\right\}\right] \\ &+ 2\pi k f_{yt} D^2 t_0^2 h\left[-\frac{m}{A} + \frac{(1-m)}{B}\right] \end{aligned} \tag{5}$$

where, $A = D^2 - m^2h^2$ and $B = D^2 - (1-m)^2h^2$. W_c can be calculated by [5]:

$$W_c = \frac{2}{3}\pi f_{yt} t_0 h^2\left[4rm^2 + \frac{rmD}{h}\ln\left(1 - \frac{mh}{D}\right) + 4(1-m)^2 - \frac{(1-m)D}{h}\ln\left\{1 + (1-m)\frac{h}{D}\right\}\right] \tag{6}$$

Model D – Linear variation in thickness

Here we consider the thickness of the tube to be varying linearly from a maximum at the inside most point of the fold to a maximum at the outside most point of the fold. Considering that the volume of half-fold length h remains constant at any stage of deformation, the changed thickness t at any distance y from the initial mean radius of tube is obtained as

$$t = t_0\left[1 - \frac{2y\sin\theta}{D + (1-2m)h\sin\theta}\right] \tag{7}$$

It is to be noted here that y is positive for the outside portion of the fold and negative for the inside portion of the fold. The thickness t_1 at the inside most and t_2 at the outside most points of the fold can be obtained from

$$t_1 = t_0\left[1 + \frac{2mh\sin\theta}{D + (1-2m)h\sin\theta}\right] \tag{8a}$$

$$t_2 = t_0\left[1 - \frac{2(1-m)h\sin\theta}{D + (1-2m)h\sin\theta}\right] \tag{8b}$$

Considering linear change in thickness, the modified value of W_b is given as [6]:

$$W_b = 2\pi k f_{yt} t_0^2 \int_0^{\pi/2} \left[1 + \frac{2mh\sin\theta}{D + (1-2m)h\sin\theta}\right]^2 (D - 2mh\sin\theta)d\theta$$
$$+ 2\pi k f_{yt} t_0^2 \int_0^{\pi/2} \left[1 - \frac{2(1-m)h\sin\theta}{D + (1-2m)h\sin\theta}\right]^2 \{D + 2(1-m)h\sin\theta\}d\theta \tag{9}$$

The work done in circumferential deformation W_c is found as [6]:

$$W_c = \frac{4\pi f_{yt} t_0 hD}{3(1-2m)^3} \ln\left\{1 + (1-2m)\frac{h}{D}\right\}\left\{rm^3(2-m) + (1-m)^2(1-m^2)\right)$$
$$+ \frac{2\pi f_{yt} t_0 h^2}{3(1-2m)^2}\left\{rm^2(3 - 8m + 10m^2) + (1-m)^2(5 - 12m + 10m^2)\right\}$$
$$- \frac{2\pi h f_{yt} t_0 h^2}{3D(1-2m)}\left\{rm^3(2-m) + (1-m)^2(1-m^2)\right\} \tag{10}$$

Equation is not valid for m = 0.5 for which the expression gets transformed to

$$W_c = \frac{\pi}{6D} f_{yt} t_0 h^2 (r+1)\left(3D - \frac{h^2}{2D}\right) \tag{11}$$

Average Crushing Load

Assuming that the energy is absorbed in plastic deformations in bending and circumferential deformations only and thus neglecting the energy absorption in axial and shear deformations, we have the energy balance by equating the external work done to the energy absorbed in bending and circumferential deformation, and we get

$$P_m 2h = W_b + W_c \tag{12}$$

where, P_m is the mean crushing load. The value of P_m can thus be obtained from this equation for different models by putting the corresponding values of W_b and W_c.

Size of Fold, h and Folding Parameter, m

The values of the size of the fold, h, and the folding parameter, m can be obtained by minimizing the average crushing load. The expression for average crushing load being very complex especially for Models C and D, closed-form solution for the h and m will not be convenient and therefore these have been determined numerically.

Load Variation

The crushing load P_θ at any instant of crushing when the rotation of the fold is θ can be calculated by

$$P_\theta = \frac{dW_\theta}{dz} = \frac{d(W_{b\theta} + W_{c\theta})}{d\{2h(1-\cos\theta)\}} = \frac{1}{2h\sin\theta}\frac{d}{d\theta}(W_{b\theta} + W_{c\theta}) \tag{13}$$

where, $W_{b\theta}$ and $W_{c\theta}$ are the work done in bending and circumferential deformation respectively in the rotation of fold by angle θ.

COMPARISON WITH EXPERIMENTAL OBSERVATIONS

Some experimental results involving crushing of cylindrical tubes reported in an earlier paper [8] have been used in the validation of the analysis presented in earlier sections. The experimental results are reported in Table 1. The folding parameter determined from the analysis has been compared for all test results given in the table, but the variation of crushing load has been compared with a typical data no. 7. A parametric study has also been carried out for studying the influence of the difference in the compressive and tensile strength of both the materials by taking the parameter r as 1.0, 1.5 and 2.0.

Table 1 Experimental results involving crushing of cylindrical tubes [1]

S. No.	Diameter, D (mm)	Thickness, t_0 (mm)	D/t_0	$D t_0$	Size of fold, h (mm)	Parameter, m	Mean crushing load, P_m / P_0
			Aluminium tubes (f_y = 160 MPa)				
1.	49.10	1.70	28.88	83.50	15.30	0.320	-
2.	37.10	1.50	24.73	55.70	13.70	0.321	-
3.	43.10	1.75	24.63	75.40	16.70	0.290	-
4.	37.10	1.60	23.19	59.40	11.30	0.319	-
5.	49.60	1.60	31.00	79.36	06.74	0.257	0.455
6.	36.90	1.40	26.36	51.70	11.92	0.284	0.450
			Steel tubes (f_y = 400 MPa)				
7.	43.00	1.80	23.89	77.40	17.50	0.274	-
8.	26.90	1.20	22.42	32.30	08.36	0.282	-
9.	53.10	2.30	23.13	122.10	17.40	0.316	-
10.	25.00	2.00	12.50	50.00	10.10	0.188	-
11.	30.35	1.85	16.40	56.10	12.70	0.276	-
12.	37.40	1.60	23.38	59.80	15.10	0.318	-
13.	39.30	1.80	21.82	70.70	17.73	0.305	-

Folding Parameter m

The size of fold and folding parameter have been calculated for all the four models by taking different values of r numerically by minimizing the average crushing load and the folding parameter has been reported in Table 2.

Table 2 Folding parameter, m

S. No.	Experimental	Model A	Model B			Model C	Model D
		r = 1.0	r = 1.0	r = 1.5	r = 2.0	r = 1.0	r = 1.0
			Aluminium tubes				
1.	0.320	0.540	0.606	0.383	0.252	0.403	0.466
2.	0.321	0.543	0.616	0.389	0.256	0.401	0.464
3.	0.290	0.543	0.616	0.389	0.256	0.401	0.464
4.	0.319	0.545	0.620	0.392	0.258	0.401	0.463
5.	0.257	0.539	0.602	0.380	0.250	0.404	0.466
6.	0.284	0.542	0.612	0.386	0.254	0.402	0.464
			Steel tubes				
7.	0.274	0.544	0.618	0.390	0.257	0.401	0.463
8.	0.282	0.545	0.622	0.393	0.259	0.400	0.463
9.	0.316	0.545	0.620	0.392	0.258	0.401	0.463
10.	0.188	0.561	0.673	0.424	0.279	0.391	0.458
11.	0.276	0.553	0.646	0.408	0.269	0.396	0.460
12.	0.318	0.545	0.619	0.391	0.258	0.401	0.463
13.	0.305	0.546	0.624	0.394	0.260	0.400	0.462

As seen from Table 2, the value of folding parameter predicted by Models A and B for r=1.0 is more than 0.5, whereas, Models C and D with r=1.0 predict m less than 0.5. The reason for m being greater than 0.5 for Models A and B is that the energy absorption in the inside portion of the fold in bending as well as circumferential compression is less due to reduction in the diameter of this portion. Consideration of the increase in the thickness of the tube in the inside portion of the fold causes the value of m to be reduced in Models C and D as compared to Models A and B. Though the values of m predicted by Models C and D are close to 0.40 and 0.46 respectively, a close examination (Fig. 2) reveals that m decreases almost linearly as $\sqrt{t_0 / D}$ increases. The equations obtained for the two models are:

$$m = -0.1233 \sqrt{t_0 / D} + 0.4262 \quad \text{for Model C} \tag{14a}$$

$$m = -0.0785 \sqrt{t_0 / D} + 0.4796 \quad \text{for Model D} \tag{14b}$$

Thus, the value of m depends upon $\sqrt{t_0 / D}$ whose value for the data set taken for analysis (Table 1) varies from 0.180 to 0.283. The variation in $\sqrt{t_0 / D}$ in the experiments considered being small, the variation in m is also small, varying between 0.391 and 0.404 for Model C and varying between 0.458 and 0.466 for Model D. Though the values predicted by Models C and D are higher than those observed in experiments but the consideration of change in thickness has brought the values closer to experiments as compared to its non-consideration. The effect of the parameter r in Model B on the folding parameter can be seen in Table 2, where it is observed that the increase in the value of r reduces the value of folding parameter. It is due to this reason that the inside movement of fold length gets reduced because of the material yield stress in compression is higher than in tension when the parameter r is taken as greater than unity.

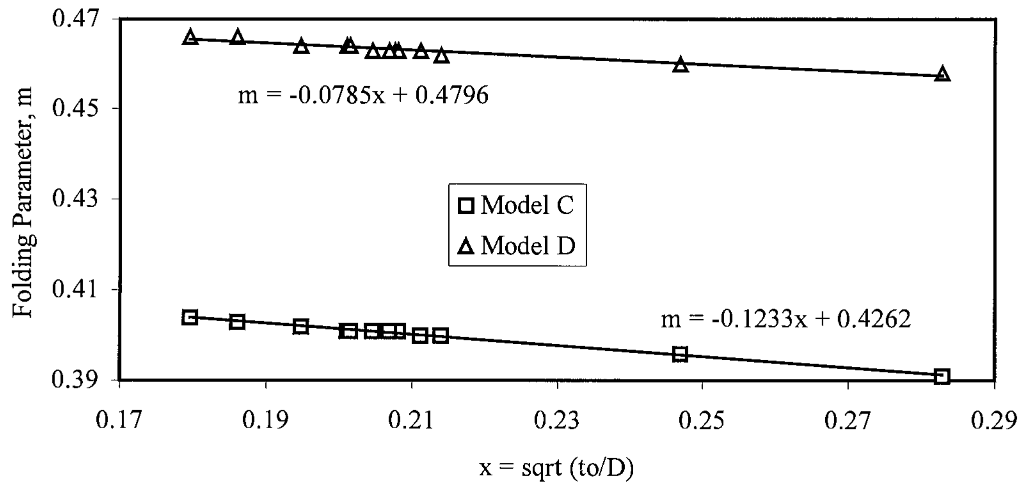


Fig. 2. Variation of folding parameter

Crushing Load variation

The values of h and m were first determined numerically and these values were used for finding out the variation of crushing load. The variation of non-dimensional crushing load P_θ / P_0 along with the experimental curve for aluminium tube no. 7 has been plotted in Fig. 3, where, $P_0 = \pi D f_{yt} t_0$. Results of Model D with r =1.0 are shown in Fig. 3. The curves of Models B and C is close to Model D, so they have not been plotted in the figure.

The analytical load-deformation curves do not start from zero load level due to the neglect of the elastic deformation in the beginning. The consideration of linear change in thickness in Model D reduces the value of parameter m due to the increase in the thickness of the tube for inside portion of the fold and decrease in the thickness of the tube for outside portion of the fold. There will be increase in mean crushing load due to increase in the thickness of inside portion of the fold, whereas, there will be decrease in mean crushing load due to decrease in the thickness of outside portion of the fold. Also, the parameter m gets reduced due to the consideration of the change in thickness, therefore, net effect of the mean crushing load due to the consideration of linear change in thickness is almost negligible as is shown by a comparison of models B and D.

The mean crushing load calculated by all the four models for different values of r has been reported in Table 3. A comparison of calculated mean crushing load by all the models for different values of r with that observed in experiment shows that the predicted values are lower. It is due to this reason that the next fold starts even before complete crushing of previous fold and therefore, the crushing load observed in experiments starts rising even before the vertical crushing reaches 2h (i.e. two times the size of the fold). The mean crushing load is found to increase with the increase in the value of r and comes closer to the experimental values.

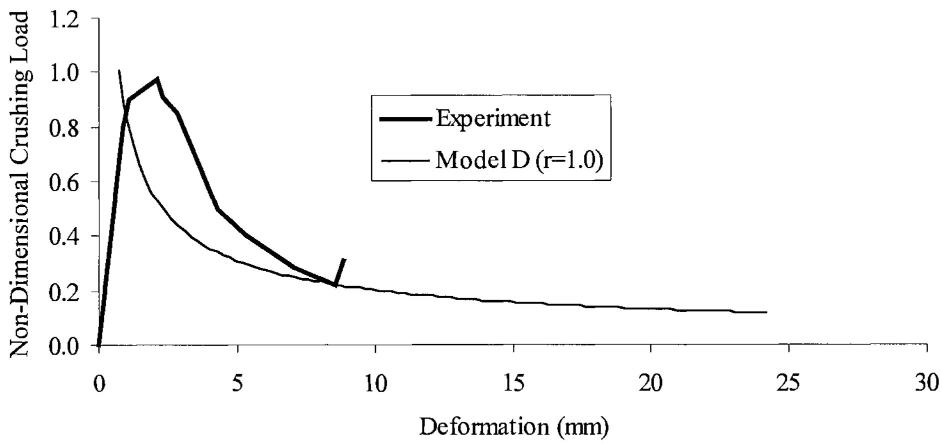


Fig. 3. Load deformation curve of a steel tube of D = 43.0 mm, t = 1.8 mm

The variation of non-dimensional mean crushing load with the folding parameter has been plotted in Fig. 4 for Models B and D. Though the variation by the two models differ significantly, the minimum mean load for both the models with r=1.0 is close to each other. It can also be seen from Fig. 4 that for Model B energy absorption in total inside folding is less than total outside folding model, see [2,3], whereas, the consideration of linear change in thickness in Model D leads to more energy absorption in total inside fold model as compared to the total outside fold model. The increase in the value of parameter r results in increase in the mean crushing load and reduction in the optimal value of m.

Table 3 Non-Dimensional Mean Crushing, P_m / P_0

S. No.	Experimental	Model A	Model B			Model C	Model D
		r = 1.0	r = 1.0	r = 1.5	r = 2.0	r = 1.0	r = 1.0
			Aluminium tubes				
1.	-	0.250	0.245	0.307	0.338	0.253	0.244
2.	-	0.270	0.264	0.332	0.367	0.273	0.263
3.	-	0.270	0.264	0.333	0.368	0.274	0.263
4.	-	0.279	0.272	0.343	0.380	0.282	0.271
5.	0.455	0.241	0.237	0.296	0.326	0.244	0.236
6.	0.450	0.261	0.256	0.321	0.355	0.265	0.255
			Steel tubes				
7.	-	0.274	0.618	0.338	0.374	0.278	0.267
8.	-	0.283	0.622	0.349	0.387	0.287	0.276
9.	-	0.279	0.620	0.344	0.381	0.282	0.272
10.	-	0.378	0.673	0.471	0.527	0.383	0.361
11.	-	0.331	0.646	0.410	0.456	0.335	0.319
12.	-	0.277	0.619	0.342	0.378	0.281	0.270
13.	-	0.287	0.624	0.354	0.392	0.291	0.279

CONCLUSIONS

Different aspects of mathematical modeling for the axial crushing of cylindrical tubes with straight fold have been discussed. The models presented incorporate the changes in thickness along the fold length, different yield stresses in compression and tension and the variation in circumferential strain during folding. The model predicts partly inside and partly outside fold formation as is observed in the experiment. The variation in crushing load, proportions of inside and outside fold length, and mean collapse load have been computed.

The consideration of changes in the thickness of tube during folding in the analysis enables us to justify as to why the inside folding is less than 50% of the total size of the fold as observed in the experiments. There do not seem to be significant effect of change in thickness on the crushing load. With the increase in the value of parameter r, mean crushing load increases and the value of folding parameter reduces.

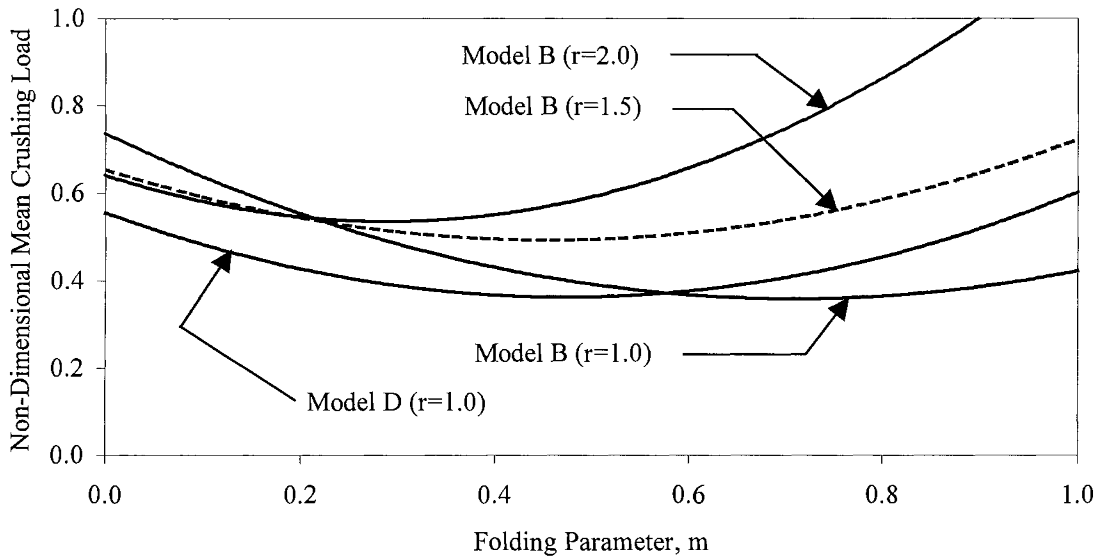


Fig. 4. Variation of mean load with folding parameter of a steel tube of D=25 mm, t=2 mm

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