# Elastic-plastic Buckling Analysis for A Combined Structure of Cylindrical Shells

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## ABSTRACT

A model of combined shells, that is consisting of three cylindrical shells with different thickness and three strengthen rings, is used to simulate the bottom supported main vessel of FBR. Buckling tests of the model under two sets of loads simultaneously are carried out. Each set of loads includes horizontal force, moment and identical axial force. The elastic and elastic-plastic buckling loads and buckling mode of the model are calculated by using ANSYS or Sup-SAP93 code. The influences of boundary conditions, initial imperfection of the model and of the plasticity of the model material on buckling load and buckling mode are discussed. The calculation results are in reasonably good agreement with those obtained by experiments.

## INTRODUCTION

A bottom supported main vessel of FBR is a complicated combined structure [1]. In order to reduce thermal stress the main vessel of FBR need to be a thin-walled structure. The main vessel can buckle due to strong earthquakes. So it is very important to study the buckling behavior of the main vessel under seismic loads. On account of complexity of the combined structure and stability problem, it is suitable to carry through numerical simulation and experiment investigation. According to specialty of structure shape and loading a experimental and calculation model of combined shells, that is consisting of three cylindrical shells with different thickness and three strengthen rings, is used to simulate the bottom supported main vessel of FBR.

In this paper, the static analogue on buckling behavior under seismic loads for the model is carried out with FEM. The influence of plasticity, boundary conditions and initial imperfections are considered. A series of buckling loads and buckling modes of the model under vertical loads and transverse shearing loads simultaneously are obtained. The calculation results are compared with the experimental results, they are in reasonably good agreement.

## EXPERIMENTAL MODEL AND CALCULATION MODEL

The coordinate system and geometric parameters of the experimental model are shown in Fig 1, where R is inner radius of cylindrical shells, t and H are thickness and height of shell or depth and height of cross section of ring respectively. The sizes of each component are as follows:

The cylindrical shell A: R=160mm, t=1.5mm, H=84mm;

The cylindrical shell B: R=160mm, t=1.5mm, H=41mm;

The cylindrical shell C: R=159mm, t=1mm, H=245mm;

The strengthen ring D (at z=94,145,400 ㎜): t=7.4mm, H=10mm.

Aluminum alloy is adopted for cylindrical shells A, B, C and stainless steel for strengthen rings D. The material properties of aluminum alloy and stainless steel are as follows:

$\rho$ =2700 kg/m$^3$, E=6.7×10$^4$ Mpa, $\nu$ =0.3 (for aluminum alloy)
$\rho$ =7800 kg/m$^3$, E=2.1×10$^5$ Mpa, $\nu$ =0.3 (for stainless steel)
where $\rho$ , E and $\nu$ are mass density, modulus of elasticity and Poisson's ratio respectively.

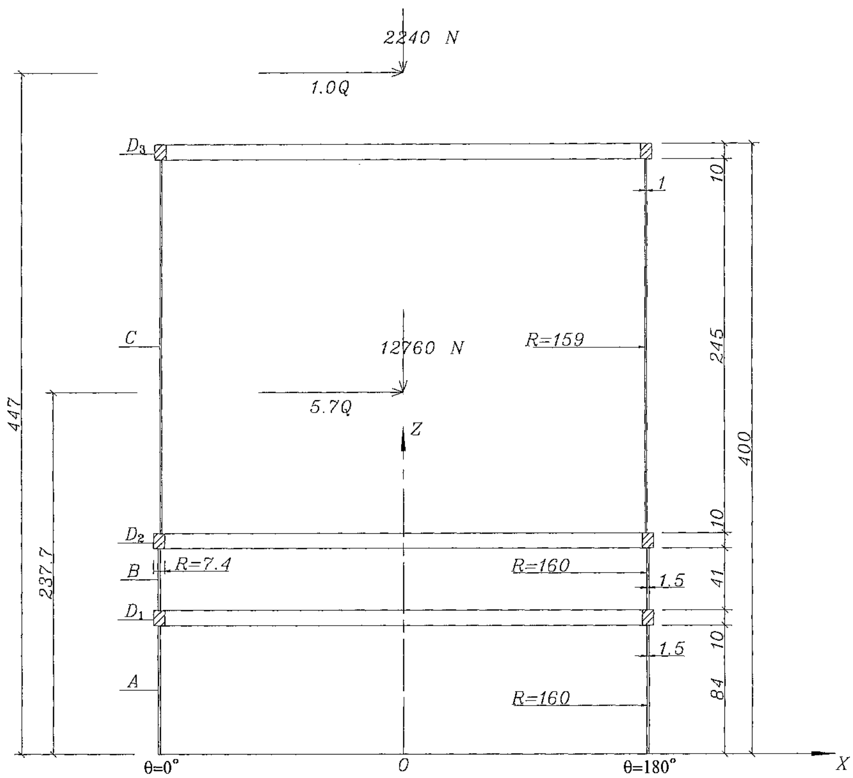



**Fig.1 Geometry of the Experimental Model**

The scale ratio $\alpha_c$ of experimental model to prototype is taken as 1/25. Thus the load ratio should be $\alpha_c^2$=1/625. The experimental model is subjected to two invariable vertical loads, which simulate deadweight of components in reactor, and two transversal loads, which simulate inertial forces at places of gathering mass of components in reactor due to transverse earthquake. Fig 1 shows applied loading situation for the experimental model, that is, invariable vertical loads are $F_{z1}$= -2240N (at $z_1$=447 mm) and $F_{z2}$= -12760 N (at $z_2$=237.7 mm), transversal loads are $F_{x1}$=1.0Q (at $z_1$=447 mm) and $F_{x2}$=5.7Q (at $z_2$=237.7 mm) respectively. These loads are transferred to the top of strengthen rings $D_3$ and $D_2$ (at $z_3$=400 mm and $z_4$=145 mm) respectively by using loading equipment.

For calculation model, the distance from symmetrical axis z to middle plane of cylindrical shells A, B, C and rings D is taken as 160.75 mm. Besides this the geometric sizes and material properties of calculation model are same with experimental model. Three-dimensional shell element with 4-nodes is adopted for all cylindrical shells. While three-dimensional beam element is elected for strengthen rings. Each shell and each ring is divided into 36 elements in circumference direction. The whole combined structure is divided into 1512 elements with 1649 nodes.

The constraint condition of calculation model is that all nodes at the bottom of cylindrical shell A are fixed. On 36 nodes at $z_3$=400 mm transversal loads $F_{x1}$ and invariable vertical loads $F_{z1}$ are exerted. Loading values of each node of these nodes are 1/36 of $F_{x1}$ and $F_{z1}$. While the moment $M_1$= $F_{x1}$×($z_1$- $z_3$) is substituted with resultant moment composed of z-direction force of each node. The magnitude of z-direction force of each node is direct ratio with distance of this node to y-axis. Similarly, on 36 nodes at $z_4$=145 mm $F_{x2}$, $F_{z2}$ and $M_2 = F_{x2} \times (z_2 - z_4)$ are exerted.

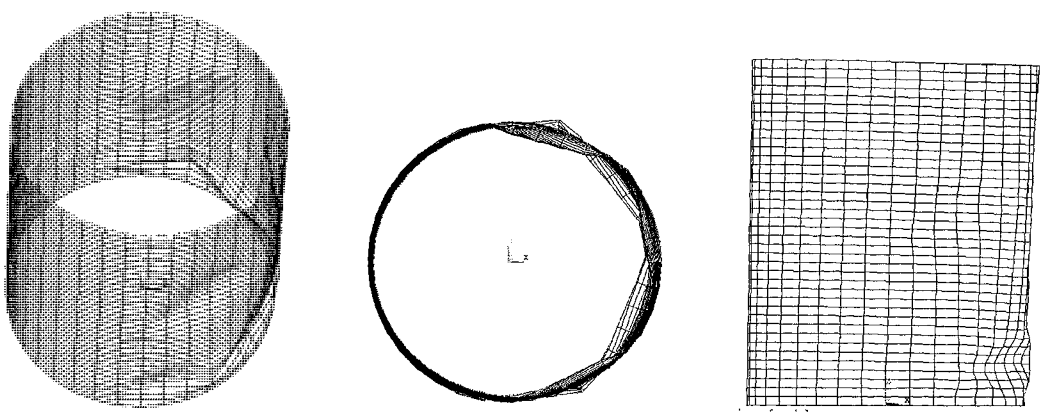
## NUMERICAL RESULTS AND COMPARISON WITH THE EXPERIMENTAL MEASUREMENTS

Both ANSYS5.4 and ALGOR FEAS（SUPER SAP93）codes x are employed to study the static buckling of above-mentioned calculation model under transversal loads and invariable vertical loads.

### Elastic Buckle of the Perfect Structure

The buckling load multiplier f and the buckling load Q for perfect structure subjected to transversal loads and invariable vertical loads under normal atmospheric temperature (20℃) are listed in Table 1, where $Q_1 = f \times 36$ is the resultant transversal buckling loads of 36 nodes at z=400mm, $Q_2 = 5.7 \times f \times 36$ is the resultant transversal buckling loads of 36 nodes at z=145mm, $Q = Q_1 + Q_2$ is the total transversal buckling loads of entire model. In Table 1 "boundary condition 1" means 36 nodes at bottom of calculation model are all fixed, while "boundary condition 2" means 18 nodes at bottom of calculation model are all fixed, the other 18 nodes are free. The fixed node and unengaged node are placed alternately. The latter is the same as boundary condition of experiment model.

The elastic buckling load under boundary condition 1 for perfect structure is Q=235.556 kN (ANSYS5.4 code) and Q=236.456 kN (SUPER SAP93 code). Their relative error just is 0.38%. The calculation shows that the buckling will take place mainly in the middle part of the cylindrical shell A. There is a little buckling in the middle part of the cylindrical shell C. The buckling mode is shown in Fig. 2. It can be seen that the unstable shape is localized and symmetrical about x-axis. There are 2 waves in vertical direction and 5 waves in circumference direction.



(a) (b) (c)

**Fig.2 Elastic Buckling Mode under Boundary Condition 1 for Perfect Structure**

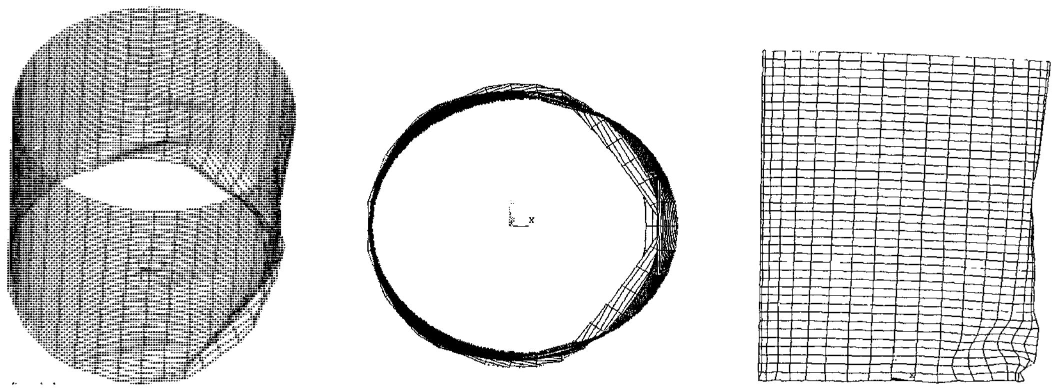
The elastic buckling load under boundary condition 2 for perfect structure is Q=216.571 kN. It reduces about 8.1% than the buckling load under boundary condition 1. The local buckling will take place mainly in the middle part of the cylindrical shell A. There is a little buckling in the middle part of the cylindrical shell C. The buckling mode is shown in Fig. 3. It is similar to that under boundary condition 1. It can be seen that the unstable shape is a symmetrical about x-axis and local buckling. There are 2 waves in vertical direction and 5 waves in circumference direction.

The relative error of the buckling loads under different boundary conditions for perfect structure is listed in Table 1,where $Q_{b1}$ and $Q_{b2}$ denotes the buckling load under boundary condition 1 and boundary condition 2 respectively.

**Elastic-plastic Buckle of the Perfect Structure**

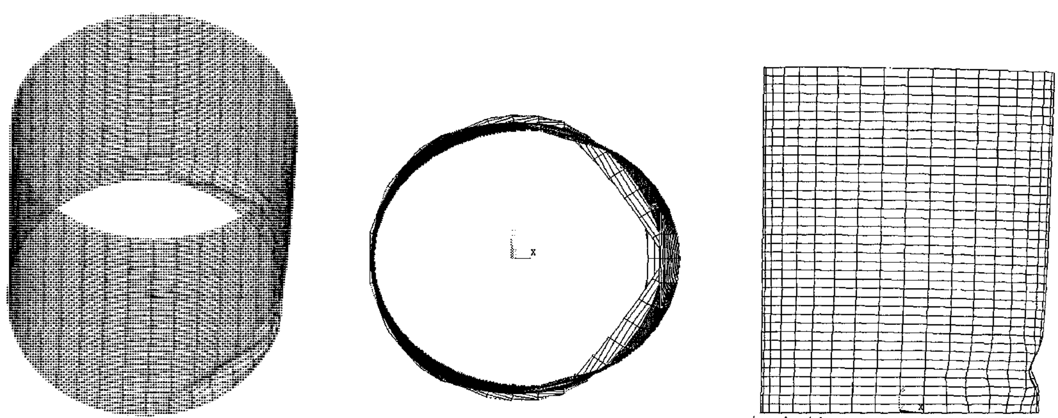
In above-mentioned elastic buckling calculation it is found that the entire model except a majority of the cylindrical shell C is on plastic state when buckling is happened. So the influence of plasticity on buckling load must be considered. The curve of stress-strain of material in elastic and plastic state is measured by using experiment. And the curve of stress-strain is simplified into bilinear, where the yield stress $\sigma_Y$=114Mpa and the slope of the curve (here is simplified to line) in plastic state $E_t=0.12715866\times10^{11}$ N/m$^2$.

The elastic-plastic buckling load under boundary condition 1 for perfect structure is Q=98.434 kN. The buckling will take place in the middle part of the cylindrical shell A, and no buckling in the rest parts of the combined shell. The buckling mode is shown in Fig.4. It can be seen that the unstable shape is also localized and symmetrical about x-axis. There are only 1 waves in vertical direction and 5 waves in circumference direction.



(a) (b) (c)

**Fig.3 Elastic Buckling Mode under Boundary Condition 2 for Perfect Structure**



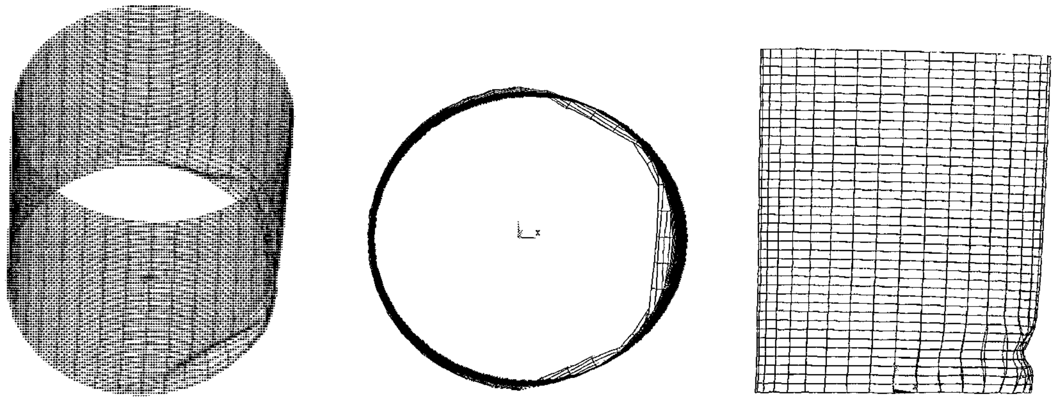
(a) (b) (c)

**Fig.4 Elastic-plastic Buckling Mode under Boundary Condition 1 for Perfect Structure**

The elastic-plastic buckling load under boundary condition 2 for perfect structure is Q=87.242 kN. Compared to that under boundary condition 1 it reduces by 11.4%. The buckling will take place only in the middle part of the cylindrical shell

A. The buckling mode is shown in Fig.5, and similar to that for boundary condition 1.

The relative error of the buckling loads for elastic buckling and elastic-plastic buckling is also listed in Table 1,where $Q_e$ and $Q_{ep}$ denotes the buckling load of elastic and elastic-plastic buckling respectively.



(a) (b) (c)

**Fig.5 Elastic-plastic Buckling Mode under Boundary Condition 2 for Perfect Structure**

**Elastic Buckle of the Imperfect Structure**

It is supposed that there is initial geometrical imperfection in cylindrical shell A. The initial imperfection is determined according to measured initial surface of cylindrical shell A in experimental model. The maximum value of initial imperfections is 1.09mm. Table 1 shows the calculation results of elastic buckling under boundary condition 1 and 2 for imperfect combined model. Under boundary condition 1 the buckling load for imperfect structure 8.2% is less than that for perfect structure. Under boundary condition 2 the buckling load for imperfect structure 8.9% is less than that for perfect structure. For imperfect structure the buckling load under boundary condition 2, 8.9% is less than that under boundary condition 1. It is obvious that the sensitivity of the combined cylindrical shells considered to initial imperfection is not as significant as cylindrical shell subjected to vertical load [2]. Because of the vertical loads are more less than the transverse loads.

The relative error of the buckling loads under different boundary conditions for imperfect structure is also listed in Table 1.

**Experimental Results**

The measured buckling loads for three experimental models are $Q_{(1)}$=64.856 kN, $Q_{(2)}$=73.912 kN, $Q_{(3)}$=72.696 kN respectively. Their average value is Q=70.488 kN. Nonlinear characteristics can be identified obviously in the test. The buckling in vertical direction were take place in the middle part of the cylindrical shell A. Buckling mode is nonsymmetrical. In circumference direction the bend buckle with concave-protruding type appeared at $\theta$ =180° near the bottom of the cylindrical shell A, and the shearing buckle with drape type appeared at $\theta$ =145° in the same height of the cylindrical shell A.

Compared to experimental value, the calculated result for elastic-plastic buckling of perfect structure under boundary condition 2 is 23.8% greater than the experimental one. If the initial geometrical imperfection is considered in elastic-plastic buckling calculation, the buckling load will be still close to experimental value. However, The wave number in circumference direction obtained by calculation is different from that observed in the test.

**Table 1 Buckling Loads under Different Condition and The Relative Error**

| | perfect structure | | | | imperfect structure | | |
|---|---|---|---|---|---|---|---|
| | buckling loads | boundary condition 1 | boundary condition 2 | relative error | boundary condition 1 | boundary condition 2 | relative error |
| elasticity | f | 976.6 | 897.9 | $(Q_{b2}-Q_{b1})$ | 896.4 | 816.194 | $(Q_{b2}-Q_{b1})$ |
| | $Q_1$(kN) | 35.158 | 32.324 | $\div Q_{b1}$ | 32.270 | 29.383 | $\div Q_{b1}$ |
| | $Q_2$(kN) | 200.398 | 184.247 | = -8.1% | 183.941 | 167.483 | = -8.9% |
| | Q(kN) | 235.556 | 216.571 | | 216.211 | 196.866 | |
| elasticity-plasticity | f | 408.1 | 361.7 | $(Q_{b2}-Q_{b1})$ | | | |
| | $Q_1$(kN) | 14.692 | 13.021 | $\div Q_{b1}$ | | | |
| | $Q_2$(kN) | 83.742 | 74.221 | ≈ -11.4% | | | |
| | Q(kN) | 98.434 | 87.242 | | | | |
| relative error | $(Q_{ep}-Q_e)$ $\div Q_e$ | -58.2% | -59.7% | | | | |

## CONCLUSION

(1) For perfect combined cylindrical shells the calculated buckling load obtained according to linear elasticity theory is 3.1 times larger than the experimental result.

(2) Both experiment and numerical calculation show that influence of plasticity on buckling load is very significant and is the most important factor among others.

(3) The buckling load under boundary condition 2 is about 10% less than that under boundary condition 1. Because the buckling were take place in the middle part of the cylindrical shell A near boundary, the effect of border must be considered for short shell.

(4) The influence of initial geometrical imperfection on buckling load is also significant. For elastic buckling it may reduce buckling load about 9%. It is found that though the influence of initial imperfections for structure mainly subjected to transversal loads is not as evidence as for structure subjected to vertical loads it's influence cannot be neglected yet.

(5) Compared to experimental value the calculation result for elastic-plastic buckling of perfect structure under boundary condition 2 is 23.8% greater than the experimental value. The wave numbers in circumference direction obtained by calculation is different from that observed in the test. It is reasonable yet that the calculated results are greater than the test one. Because the certain factors such as the warp of applied load, actual boundary condition in test are not simulated accurately in calculation or simply unknown.

## REFERENCES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