A Method for Numerical Solution of Vibrational Random Response of Structures and Its Application

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ABSTRACT

In this paper the mathematical model of the vibrational response of structures to random exiting forces is applied to explore the influence of fluid flow parameters on the dynamic deformations and behaviour of a cylindrical shell. A physical model of the PWR core barrel, considered as a cylindrical shell supported at both ends, is used to demonstrate by numerical expriments the mentioned influence. The mean velocity vector of the fluid flow is the basis for the aero-hydrodynamic excitation expressed by a coherence function of fluctuating surface pressure both in the axial and circumferential directions, stated as the dependence on the values of the correlation lengths. The results of the courses of the generalized spectral loadings, of the ms amplitudes of the displacement and stress distributions are shown at the dependence on the flow parameters.

INTRODUCTION

Vibrations, and consequently, their deformations induced by fluid flow, are sometimes considered to be a secondary parameter in design, but only until a failure has occurred. Failures resulting from the vibrations of fluidodynamic origin have not been avoided in nuclear reactors [1,2], though extraordinary attention has been given to their safe operation. The problem under discussion has an interdisciplinary character and lies on the interface of several scientific fields. This paper applies the method of the numerical solution of vibrational random structure response to explore, by numerical experiments, the influence of fluid flow characteristics on dynamic deformations and the behaviour of a cylindrical shell representing a simple model of a core barrel surrounded by a fluid flow. The magnitudes and the distributions of the stochastic displacements and stresses are studied in relation to the dependence on the values of the mean flow velocity and on the pressure wave correlation length.

THEORETICAL BACKGROUND

The cylindrical shell under consideration is replaced by a r-degree-of-freedom system. The ith element of a cylinrical shell is regarded as a continuum. The frequency modal properties of the system including a cylindrical shell with surroundings are known. To simplify the problem, we assume: (1) there is not coupling between individual modes of vibrations, (2) no coupling between the flow excitation and cylindrical shell response is introduced, (3) the flow random excitation, expressed in terms of the fluctuating pressure $p(t)$ of the fluid where it is in contact with the cylindrical shell is stationary and ergodic.

Using modal analysis and a Fourier transform to solve the equations of motion, we obtain an expression for the vector $s_y(\omega)$ of dimension r of the power spectral density (psd) of the cylindrical shell displacements. In matrix symbolical form we have [3]:

$$s_y(\omega) = \widetilde{W} \,|\, {}^D H(\omega) \,\|\, \widetilde{l}\,(\omega)\,| \,, \qquad (1)$$

where $\widetilde{W}$ is the matrix of dimension ($r \times r^2$) of modal vectors with elements $w_{\alpha}^{(i)} w_{\beta}^{(i)}$ of the associated conservative system; ${}^{D}H(\omega)$ is the diagonal matrix of dimension ($r^2 \times r^2$) of generalized spectral compliances with elements H_{α}, H_{β}^{*} (the asterisk indicates a complexly conjugated function); $\widetilde{l}(\omega)$ is the vector of dimension r^2 of generalized spectral loadings given by:

$$\widetilde{l}\,(\omega) = {}^{T}\overset{\approx}{W}\,\overline{p}(\omega)\,, \tag{2}$$

where $\overset{\approx}{W}$ is the matrix of dimension ($r^2 \times r^2$) of modal vectors with elements $w_{\alpha}^{(i)} w_{\beta}^{(k)}$; $\overline{p}\,(\omega)$ is the vector of dimension r^2 the elements of which can be expressed by the following expression:

$$\overline{p}_{ik}\,(\omega) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T}\int_{-T}^{T}\int_{Si}\int_{Sk} p_i(t_1)\,p_k(t_2)\times\exp(\,\omega(t_1 - t_2))\,\mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}i\,\mathrm{d}k, \tag{3}$$

where T is half of the realization time of the random process and $S_{i,}\ S_k$ is the contact surface of the ith and kth elements respectively, of the cylindrical shell. An analytical expression can be gained for $\widetilde{l}(\omega)$. Then, the generalized spectral loadings, $L_{\alpha\beta}(\omega)$, related to a unit surface are done:

$$L_{\alpha\beta}\,(\omega) = \frac{1}{S^2}\int_S\int_S w_\alpha(A_1)\,w_\beta(A_2)\times G_{p1p2}(A_1, A_2, \omega)\,dA_1\ \ dA_2, \tag{4}$$

where A_1 and A_2 are sites on the contact surface S of the cylindrical shell; $w_\alpha(A_1)$ and $w_\beta(A_2)$ are the coeficients of the distribution of displacement amplitudes of the α and β modes, respectively, of vibration at A_1 and A_2 respectively; $G_{p1p2}(A_1\,,\,A_2\,,\,\omega)$ is the coherence function of the pressure field of the fluid flow at the sites A_1, A_2. Equation (4) represents one of the possible forms of the so-called acceptance integral first introduced by Powell [4]. If the function of the fluctuating contact surface pressure is a separable function then, consequently, the functions of the generalized spectral loadings are also separable functions. So, we have spectral loadings in the axial and in the circumferential directions, and a total generalized spectral loading. To express the coherence function of a homogeneous turbulent flow a Bessel function of the first kind of zero order, dependent on fluid flow parameters, was used in the computations. The vector of the amplitude of the root mean square (rms) displacements $\overline{y}$ of dimension r can be obtained from the psd of the displacements through integration over all the angular frequencies ω,

$$\overline{y} = \left(\frac{1}{2\pi}\int_0^\infty s_y(\omega)d\omega\right)^{1/2}. \tag{5}$$

The distribution of this vector is used for the approximate estimations of the vectors of ms stress amplitudes in the axial, $\widetilde{\sigma}_x$, and hoop, $\widetilde{\sigma}_\varphi$, directions:

$$\widetilde{\sigma}_x = \left(K_x\frac{d^2\overline{y}}{dx^2}\right)^2, \qquad \widetilde{\sigma}_\varphi = \left(K_\varphi\frac{d^2\overline{y}}{d\varphi^2}\right)^2 \tag{6}$$

where K_x, K_φ are constans given by the dimensions and material properties of the cylindrical shell.

RESULTS OF NUMERICAL EXPERIMENTS

To study the dynamic deformations in a cylindrical shell due to various changes of the exciting fluid flow parameters, the ODEZ 4 code was developed. For the proper computations, a physical cylindrical shell model of the PWR core barrel of dimensions: t/a=0.046, a/l=0.231 was used (a – middle surface radius, t – thickness, l – length). The shell model was considered to be supported at both ends continuously and replaced by the system of 2400 degrees

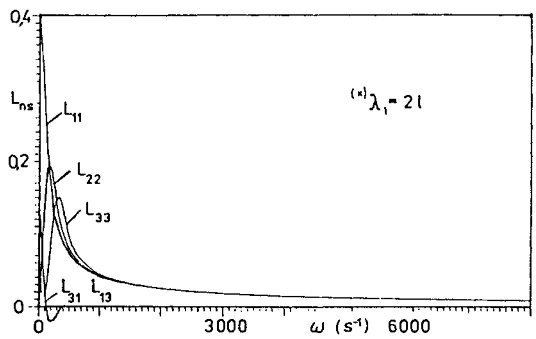


Fig. 1. Axial spectral loadings at parameter $^{(x)}\lambda$.

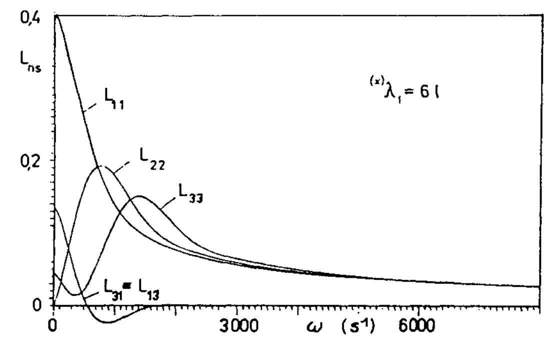


Fig. 2. Axial spectral loadings at parameter $^{(x)}\lambda$.

of freedom. The frequency spectrum of the model was fixed in both air and water. The values of natural frequencies and of generalized damping ratios were considered for 24 selected vibration modes including the influence of liquid and of limited surroundings [5].

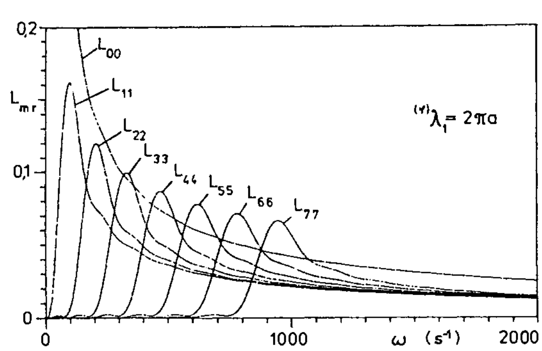


Fig. 3. Hoop spectral loadings at parameter $^{(\varphi)}\lambda$.

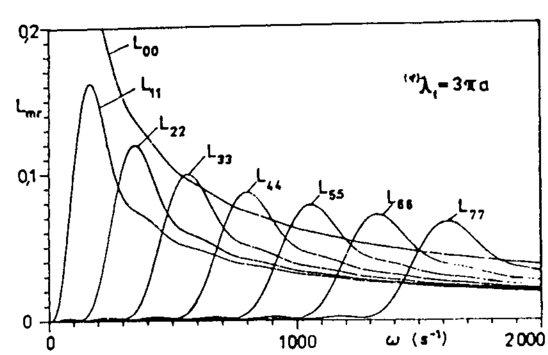
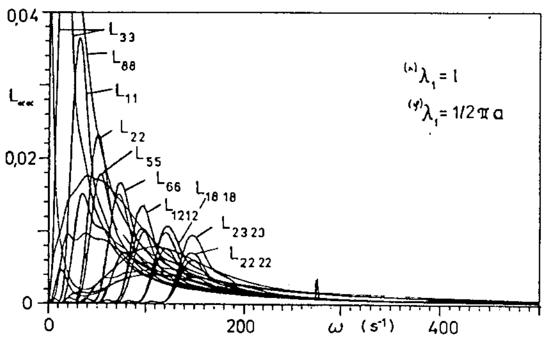


Fig. 4. Hoop spectral loadings at parameter $^{(\varphi)}\lambda$.

Generalized spectral dimensionless loadings were computed for the various pressure wave correlation lengths λ. Some results of computations are plotted in Figs. 1 to 6. The courses of axial generalized spectral loadings for some



Figs. 5. Total spectral loadings at parameters $^{(x)}\lambda$, $^{(\varphi)}\lambda$.

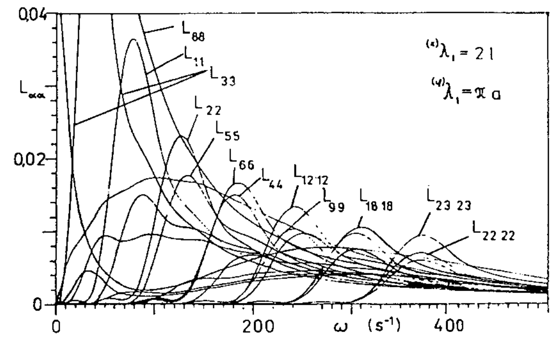


Fig. 6. Total spectral loadings at parameters $^{(x)}\lambda$, $^{(\varphi)}\lambda$.

joined and cross factors at two different correlation lengths are shown in Figs. 1 and 2, while Figs 3 and 4 present the

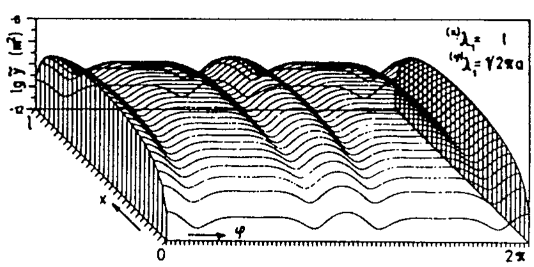


Fig. 7. Distribution of ms displacement amplitudes at parameters $^{(x)}\lambda$, $^{(\varphi)}\lambda$.

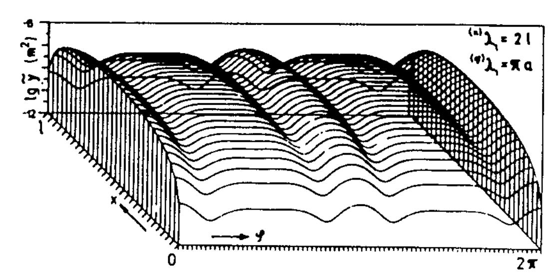


Fig. 8. Distribution of ms displacement amplitudes at parameters $^{(x)}\lambda$, $^{(\varphi)}\lambda$.

courses of the circumferential spectral loadings. An illustration of obtained values of the total spectral loadings is displayed in Figs. 5 and 6. Comparing the plotted cross loading with the joined one (Fig. 1) it is obvions that the cross terms are smaller and become very quickly damped with increasing frequencies. As for correlation length, we can see

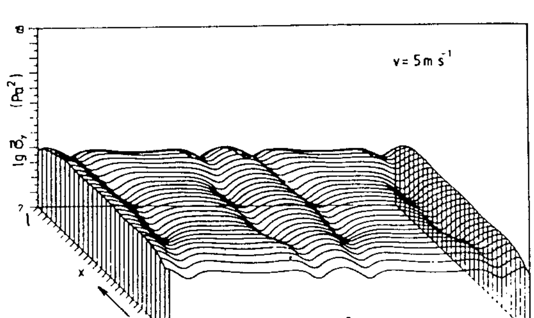


Fig. 9. Distribution of ms axial stress amplitudes at parameter v.

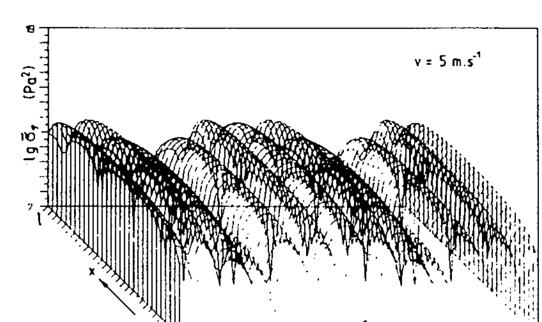


Fig. 10. Distribution of ms hoop stress amplitudes at parameter v.

that the spectral loading values are rising and their maxima shift to regions of higher exciting frequencies, with its growth.

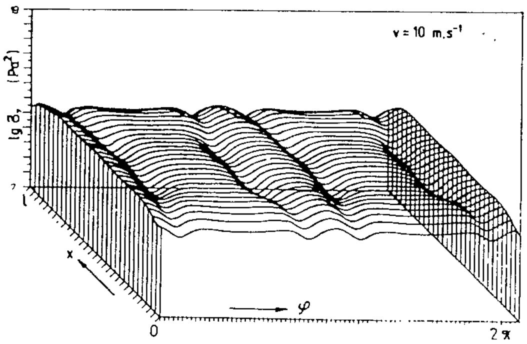


Fig. 11. Distribution of ms axial stress amplitudes at parameter v.

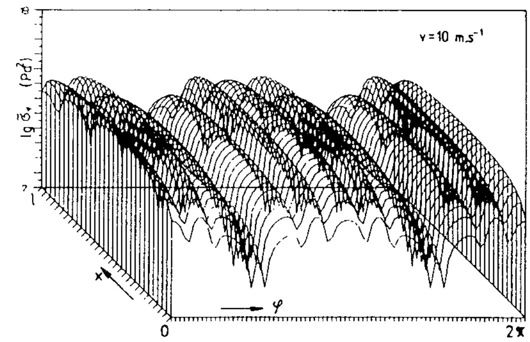


Fig. 12. Distribution of ms hoop stress amplitudes at parameter v.

The graphical expresion of the results of computations of the ms displacements at two different, both axial and circumferential, correlation lengths are presented in Figs. 7 and 8. It can be seen that the amplitudes of ms displacements are significantly dependent on the flow parameter changes. A similar statement can be made for the distributions of the ms stress amplitudes according to Figs. 9 to 12 where the mean flow velocity was changed from 5 m s^{-1} (Figs. 9, 10) to 10 m s^{-1} (Figs. 11, 12).

CONCLUSIONS

From the results presented above and the stochastic deformations and dynamic behaviour of a cylindrical shell due to the random excitation of fluid flow at various turbulent parameter levels, we can conclude:

(a) The pressure wave correlation length and the mean flow velocity significantly affect the magnitudes of the dynamic deformations of the cylindrical shell.
(b) The distributions of the ms hoop stress amplitudes are non harmonic with a number of waves around the circumference.
(c) The maximum values of the ms axial stress amplitude are lower by more than one order in comparison with the hoop ones.
(d) The values of the maxima of generalized spectral loadings in the axial and in the circumferential directions are independent of the flow parameters. With groving pressure wave correlation lengths, however, they shift into the region of higher exciting frequencies.
(e) The distributions of the ms radial displacement amplitudes are symmetrical to the plane dividing the axis of cylindrical shell in half, non-harmonic, with several waves around the circumference which have sufficiently steep tops. Similar symmetries can be observed in the distributions of the ms stresses.
(f) Symmetries in the ms deformation distributions can be employed to reduce the computing time.

NOMENCLATURE

a	radius of the cylindrical shell,
A	place on the contact surface of the cylindrical shell,
G_{p1p2}	coherence function of fluid flow fluctuating pressure,
$H(\omega)$	element of the matrix of general spectral compliances of the cylindrical shell,

l	length of the cylindrical shell,
$\tilde{l}$ (ω)	vector of generalized spectral loadings,
L (ω)	dimensionless generalized spectral loandings,
ms	mean square,
psd	power spectral density,
p (t)	fluctuating surface pressure of fluid flow,
$\overline{p}$ (ω)	vector of cross (spatial) power spectral density of flow fluctuating surface forces,
r	number of degrees of freedom,
rms	root mean square,
s_y (ω)	vector of power spectral density of cylindrical shell displacements,
S	contact surface of the cylindrical shell,
t	time,
T	half of the realization time of the random process,
v	mean fluid flow velocity,
w_α	coefficient of displacement amplitudes of the αth vibration mode of the cylindrical shell,
W	matrix of modal vectors of the cylindrical shell,
x	axial coordinate of the cylindrical shell,
y	radial coordinate of the cylindrical shell,
$\bar{y}$	vector of the rms displacements of the cylindrical shell,
λ	pressure wave correlation length,
$\tilde{\sigma}$	vector of ms stress amplitudes,
φ	circumferential angle,
ω	angular frequency.

Subscripts / superscripts

D	diagonal matrix,
i, k	applies to considered elements of the cylindrical shell,
m, r	applies to circumferential vibration mode,
n, s	applies to axial vibration mode,
p	applies to fluctuating pressure of fluid flow,
T	transposition,
x	applies to axial direction,
y	applies to radial direction,
α, β	applies to natural vibration mode,
φ	applies to circumferential direction.

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