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An Analytical Study on the Effects of End Conditions on the Behavior of 45- and 90-Degree Elbows Subjected to Bending

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ABSTRACT

Regarding the high seismic vulnerability of elbows as important parts in all piping systems, in this paper two types of elbows, 45 degrees and 90 degrees, with two common types of connections, namely simply welded, and flange connection, have been considered to investigate analytically the effect of end conditions on the ultimate state behavior of elbows. Several geometrical conditions have been considered in each case, and various lengths have been supposed for the straight part of the pipe connected to the elbow, including *D/2, D, 2D*, and *3D* (*D* is the diameter of the elbow). Loading cases include both in-plane and out-of-plane bending moments applied to one end of the connected pipe. The analyses have been performed by using ANSYS computer program. In each case both yielding moment and failure moment have been obtained. Failure moment here is defined as the moment value in which the relative rotation of the two ending part of the elbow begins to vary rapidly with a little increase in the applied moment value. Results show that in the case of welded elbows by increasing the length of the straight part both yielding and failure moments increase, and ovalization occurs in higher moment values. In each case there is a limit value for the length of the straight part, more than which the increase has no significant effect on the ultimate sustained values of loading moments. This length can be called as the effective length in each case. Generally, the in-plane applied moments have greater effects on the elbow behavior, and the 45 degree elbows are less sensitive to the end conditions comparing with the 90 degree elbows. In the case of flange connections the effect of end conditions is much less and it can be said that in this case the flange makes the pipe to behave almost as a built-in beam.

INTRODUCTION

Elbows are one of the most vulnerable parts in all piping systems, specially when subjected to extreme conditions such as earthquake induced movements. Although elbows can show plastic behavior in extreme conditions and absorb a large amount of the seismic input energy, and therefore, decrease the damage enforced to the whole piping system, they themselves suffer from heavy damages such as cracks. These kinds of damage to elbows can often lead to adverse consequences such as post earthquake fires and environment pollution. Therefore, it is necessary to design these parts as much flexible and ductile as possible.

Since early 1980s the seismic behavior of piping systems elbows, especially those of NPP facilities, have been taken into consideration by several researchers. As one of the first works in this regard the stress indices and stress intensification factors of 45-degree elbows in pressure vessel and piping components have been studied [1]. As another work a methodology have been presented for designing piping systems, including elbows, and their supports and restraints to satisfactorily perform under normal, expected, and postulated dynamic loads [2]. The presented methodology relies on a synthesis process utilizing pre-analyzed equivalent piping elements such as straight spans, spans with concentrated weights, cantilever spans, and elbow spans, and a set of procedural rules to construct the piping system. The application of the methodology has been illustrated for the case of seismic excitations, so chosen because the present methodology could have been directly compared with previously used simplified dynamic analysis methods. Comparison of that methodology with other existing methods have indicated that it more closely predicts the dynamic behavior of piping systems as determined by detailed computer analyses.

A nonlinear analysis of piping have been reported, in which the following cases have been considered [3]. (1) The static equivalent analysis of a large piping system (a few hundred degrees-of-freedom) with nonlinear supports and nonlinear elements; (2) The analysis of a piping system under extreme loading due to incompatible differential displacements of the supports; and (3) The dynamic analysis of a piping system under seismic excitation with nonlinear piping and supports. For each case, the following items have been discussed. (1) First yielding and plasticity propagation, particularly for shell element assemblies; (2) Differences between the analysis discussed and elastic analysis, particularly for the load distribution on the supports; and (3) Points of comparison with simplified methods such as nonlinear floor response spectra, with particular reference to the definition of ductility factors. Computations in that work has used the PAULA code, a 3D nonlinear code for the analysis of piping with a finite element library (straight pipe, elbow, any assembly of shell elements, such as T joint, nozzles, and restraints) which makes possible a complete modeling of the pipe.

In another study a six-inch line from a nuclear power plant has been analyzed into the plastic regime to evaluate the response characteristics of a typical piping system subjected to a seismic overload, primarily in order to ascertain the effects of the plasticity on the seismic response of the pipe and supports [4]. The pipe has been modeled to include elastoplastic material behavior. The non-linearity introduced by distortion of the pipe elbow cross section, has been found to be insignificant for the load levels considered, has not been modeled into the nonlinear elbow element. The supports have been modeled as linearly elastic restraints. A linearly elastic model of the piping system has been also developed for use in comparative studies with the nonlinear model. The piping models have been analyzed using three different procedures: two time history seismic analyses, one of the linear and one of the nonlinear model, both producing Fourier transforms which have then been used to develop a frequency-dependent ductility factor, and two response spectra analyses have been performed using the linear and the nonlinear response spectra. Those spectra have been used with the linear piping model in two response spectrum analyses to obtain the linear and equivalent nonlinear seismic responses. The results of these analyses have then been compared with the elastic and inelastic time history responses to evaluate the ductility factor.

Some experiments have been performed on a three-inch-diameter pressurized pipe whipping against a rigid restraint or a concrete slab [5]. Eleven experiments have had pipes with various geometries impacting a stiff structure either at an elbow or on a straight segment. Pipe motion, impact force, impact time, and pipe crushes have been measured. Two experiments have had pipe elbows impacting a six-inch-thick and a three-inch-thick reinforced concrete slab. The thicker slab experiences little impact damage, whereas the thinner slab is perforated by the elbow. Pipe displacement, re-bar strains, and forces on slab supports have been measured. Some laboratory and field tests and vibration measurements have been also conducted, in which pipe samples 30 in. and 18 in. in diameter have been subjected to three-edge-bearing, cyclic loading, torsion, drop, impact, and bending tests [6]. A 90-degree elbow has been also subjected to bending tests. Results of the tests are presented. Based on the results a conventional, commercially produced cement-mortar lined pipe has been evaluated in its intended function in a low-to-moderate seismic area.

The methods of toroidal elasticity theory have been used to determine the fields of stress induced by seismic components acting in two directions, X and Y, on a 90 degree elbow or pipe bend of hollow circular cross section [7]. These methods have been also extended to the problem of determining the stress fields in a hollow circular elbow or pipe bend subjected to seismic accelerations, which are represented by equivalent body forces, X, Y, and Z, acting in arbitrary directions [8]. These forces may also be viewed as upper bounds obtained from seismic response curves. The critical stresses are circumferential and meridional at the inner and outer walls of the elbow. Curves exhibiting these stresses have been presented and depend upon the representative values for a nuclear elbow given by s subscript b = 0.35 and s subscript a = 0.30. These values correspond to a typical design for a liquid metal fast breeder reactor. Stress fields produced by a vertical seismic force acting on a 90 degree elbow or pipe bend have been studied as well [9]. A vertical component of seismic force acting upon a 90 degree elbow or curved pipe bend lying in an X-Y plane have been considered. The stress field due to the magnitude force has been established; the field consists of an initial stress field and a corrective stress field. The solution has satisfied the equilibrium and compatibility equations of toroidal elasticity.

Correlation analyses of selected Canadian Electrical Assn. pipe whip tests have been performed using the ABAQUS-EPGEN finite element code [10]. The analysis includes the effects of large deformation and strain rate on resisting moment and energy absorption at the impact region. Three modeling approaches have been compared, two of which have modeled the entire pipe section consisting of beam elements with a nonlinear spring derived from static crushing of a circular ring simulating dynamic pipe crushing at the impact region. The third one has modeled the straight section of pipe as a beam with full shell representation at the elbow impact region. The time-impact relationships, and impact velocities and forces have been compared to test data. The measured impact time and forces have been compared with those obtained by the approach described in ANSI Standard 58.2.

Dynamic analysis of fluid-filled piping systems has been studied by using finite element techniques [11]. Two finite element procedures have been described for predicting the dynamic response of general three-dimensional elastic piping systems. The first approach, a low-frequency procedure, models each straight pipe or elbow as a sequence of beams. The contained fluid is modeled as a separate coincident sequence of axial members (rods) which are tied to the pipe in the lateral direction. The model includes the pipe hoop strain correction to the fluid sound speed and the flexibility factor correction to the elbow flexibility. The second modeling approach, an intermediate frequency procedure, follows generally the original Zienkiewicz-Newton scheme for coupled fluid-structure problems except that the velocity potential is used as the fundamental fluid unknown to symmetrize the coefficient matrices. Beam model predictions have been compared to experimental data and results calculated using the three-dimensional model.

Several experimental studies have been also performed on piping elbows. As an example if the early works, in-plane bending test of a piping elbow can be mentioned [12]. Experimental results have been presented of cyclic tests at room temperature in the elastic/inelastic regimes of an AISI 304 stainless steel elbow, which had been subjected to in-plane closing and opening bending. The purpose of the work has been the analysis of fittings subjected to a specific history of load/deflection in order to verify the correct prediction of the strain/stress state by means of stress intensification factors with the help of standards or elaboration by computer codes. The reported experimental activities have been performed in the framework of a research program supported by the Italian Electricity Board (ENEL-CRTN). The importance of modeling elbow deformation and plasticity has been also demonstrated by studying static and dynamic analysis of flaw stability in piping systems [13], in which the purpose has been providing system design guidance for a large-diameter piping test and gaining insight in modeling dynamic crack behavior in austenitic and carbon steel base metal and weldments.

Another relatively recent experimental study has been done on the hysteretic 3D-behaviour of pipe elbows [14], in which the results of quasi-static cyclic displacement controlled tests with pipe elbows subjected to different 3-D combinations of internal forces have been presented. The results depict the influence of the ratios between different components of internal forces as well as of the internal pressure on the hysteretic 3-D behavior of a pipe elbow. The ability of the computer code ANSYS to capture the recorded behavior has been also discussed.

Experimental data from the EPRI Piping and Fitting Dynamic Reliability Program have been evaluated for high amplitude seismic time history loads, which have been applied to 32 piping components in a cantilevered configuration [15]. Components have included elbows, tees, reducers, a reinforced fabricated tee, a nozzle, and lugs. The test levels have been about 2 to 8 times Level D. The seismic capability of elbows has been shown to be remarkable. The seismic performance of lugs has been judged to be poor. The tests have demonstrated that collapse is a potential failure mode, particularly in low frequency systems. The tests also have demonstrated that a fatigue failure in a single, high level, seismic event is possible. It has been stated that design and fabrication details are crucial to seismic performance, and trends in the data need to be assessed to determine appropriate Section III stress limits for seismic loads. In particular, the impact of component frequency on seismic capability has to be quantified.

An analytical study of the response of piping with mechanical ratcheting under dynamic loads has been performed by finite element analysis, in which a 4-inch-diameter, 4.5-mm-thick piping system with two elbows has been considered [16]. The system has been modeled using both shell and beam elements to examine the response of the piping when mechanical ratcheting at an elbow subjected to internal pressure is induced by dynamic excitation. The piping model has been pressurized internally and one of the elbows has been subjected to an in-plane bending moment by dynamic loads that simulated seismic excitation. Parametric analyses have been performed to study the safety of flexible and rigid piping systems during seismic events.

Finally, in a recent work a simplified method has been presented for calculating the plastic ratchet of elbow-shaped pipes submitted to seismic loading and an internal pressure [17]. The presented method is simplified in the sense that the value of the ratchet has been obtained without the use of finite element method (FEM) calculations. A formula has been derived and used to evaluate the fatigue-ratcheting damage of an elbow. It has been stated the presented approach is applicable to complex plastic response appropriately described by nonlinear kinematics hardening, which is more realistic for stainless steel such as 316-L.

It is seen that in spite of several studies, both analytical and experimental, which have been performed on the behavior of elbows, still little attention has been paid to the effect of end conditions on the seismic response of these crucial components of piping systems. In this paper, regarding the high seismic vulnerability of elbows, two types of elbows, 45 degrees and 90 degrees, with two common types of connections, namely simply welded, and flange connection, have been considered to investigate analytically the effect of end conditions on the ultimate state behavior of elbows. Several geometrical conditions have been considered in each case, and various lengths have been supposed for the straight part of the pipe connected to the elbow. Loading cases include both in-plane and out-of-plane bending moments applied to one end of the connected pipe. The analyses have been performed by using ANSYS computer program. In each case both yielding moment and failure moment have been obtained. Failure moment here is defined as the moment value in which the relative rotation of the two ending part of the elbow begins to vary rapidly with a little increase in the applied moment value. Numerical results include yielding and failure moments, relative rotations, cross-section shape deformation, and stress intensification in elbows for various studied cases.

CONSIDERED ELBOWS FOR THE STUDY

Two groups of elbows, including 45 degree and 90 degree elbows have been considered for this study, whose geometric specifications are given in Table 1. The parameters D, t, and R are as shown in Figure 1, and the parameter h is the bend factor defined as $h=Rt/r^2$. The values given in Table 1 are real values used in the investigations of other researchers. The structural properties of the elbow material are as follows:

Modulus of elasticity, E = 193 *Gpa*
Yielding stress, F_y = 272 *Mpa*
Poisson ratio, í = 0.2462

These values are also based on the previous studies of other investigators. It has been assumed that the weld materials are of the same properties of the elbows' materials. Some cases of elbows with flange connections have been also studied.

Table 1 . Geometric specifications of the elbows under study

No.	*D/2 = r (mm)*	*t (mm)*	*R (mm)*	*H*
1	193.5	10.4	580.4	0.1615
2	186.9	16.7	560.6	0.2675
3	177.8	26.19	533.4	0.4417

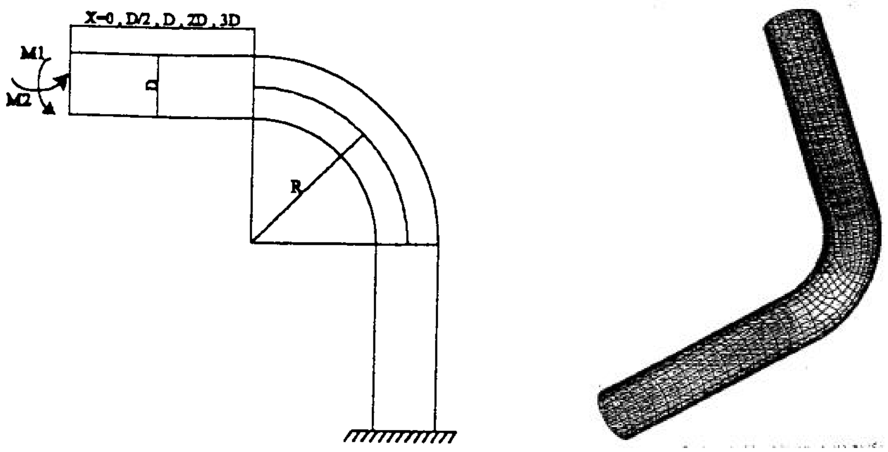


Figure 1. The schematic features of the modeled elbow and its finite element representation

MODELING AND ANALYSIS

The stress strain behavior of the elbow material has been assumed to be elastic-perfectly plastic with yielding strain of 0.00135 as reported in other researches [15]. For the analyses it has been assumed that one end of the elbow is fixed and the loadings consist of moments acting once in plane of the elbow center line and once out of the plane of the elbow center line as shown in Figure 1. A finite element model has been considered for analyses by using the ANSYS computer program. A sample of the finite element representation of elbows can be seen in Figure 1. To study the effect of end conditions on the ultimate behavior of elbows five cases have been assumed for the length of the pipe segment connected to the elbow as shown in Figure 1. These cases include the pipe segment lengths of *0, D/2, D, 2D,* and *3D* respectively. In the cases of elbows with flange connection only two lengths of *0* and *3D* have been considered.

Both geometric and material nonlinearities have been considered in analyses. For finite element analyses the pipe and elbow bodies were modeled by four-node shell elements in which stresses and strains, stress intensification, average plastic strains, and hydrostatic pressures all can be dealt with. To apply the moments to the model of the elbow a rigid zone has been defined at its free end and then a force couple has been introduced equivalent to the desired moment value. In each analysis case the applied moment value has been increased gradually from zero to its extreme value for which the structural model of the elbow in the finite element computer program reaches an unstable state.

NUMERICAL RESULTS

Two groups of elbows, 45 degree and 90 degree, each considered with three various geometry with two types of connections, namely weld and flange, have been analyzed subjected to in-plane and out of plane moments. The numerical results of these analyses include:

- Moments and corresponding rotations:
- Yielding and rupture moment values versus the length of the straight segment of the pipe connected to the elbow, shown hereinafter as X;
- Variation of the angle of the maximum stress plane, ö, versus the X value;
- Variation of elbow flexibility versus the X value;
- Variation of stress intensification factor versus the X value; and finally,
- Variation of pipe section ovalization percent versus the X value.

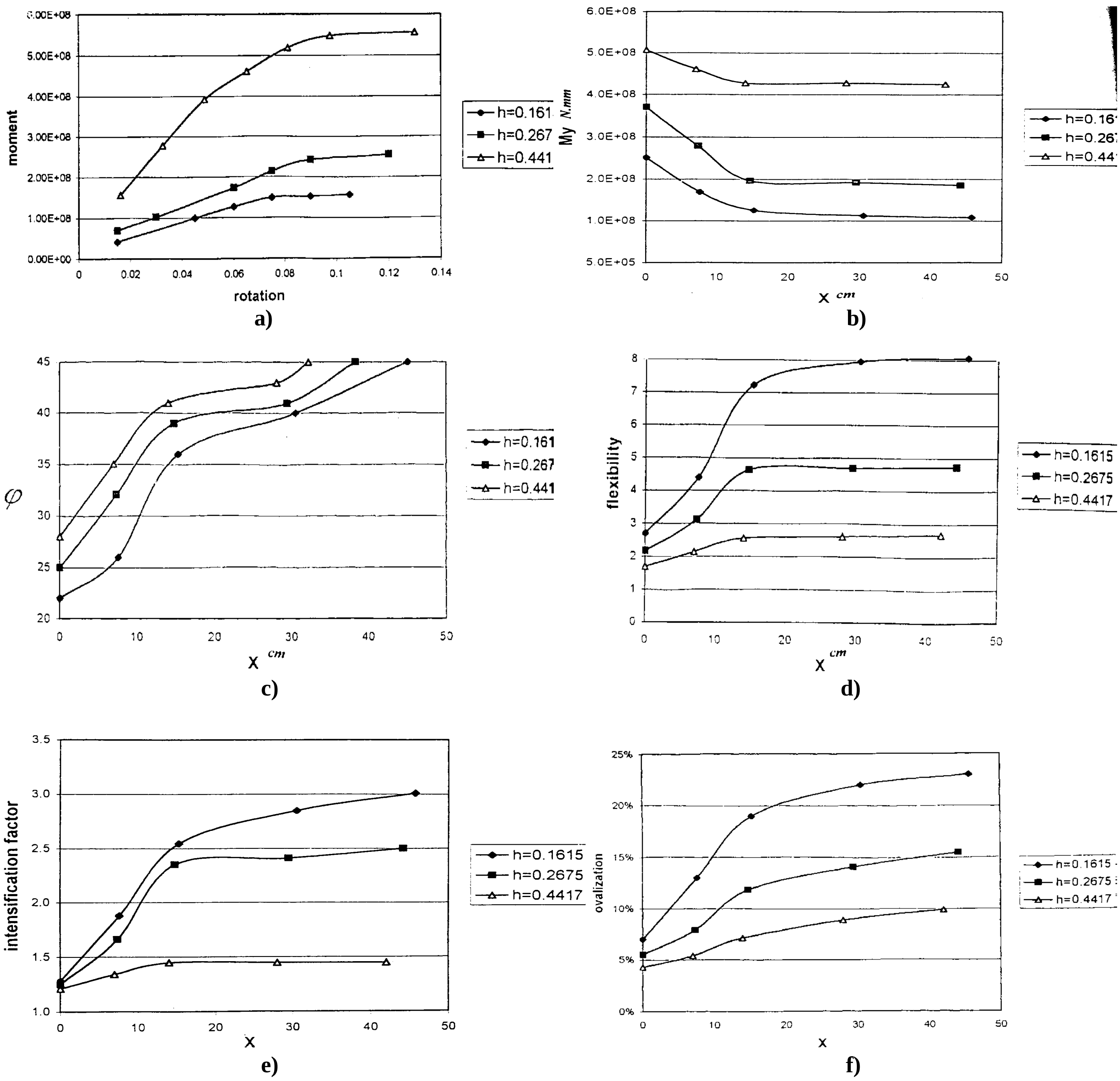
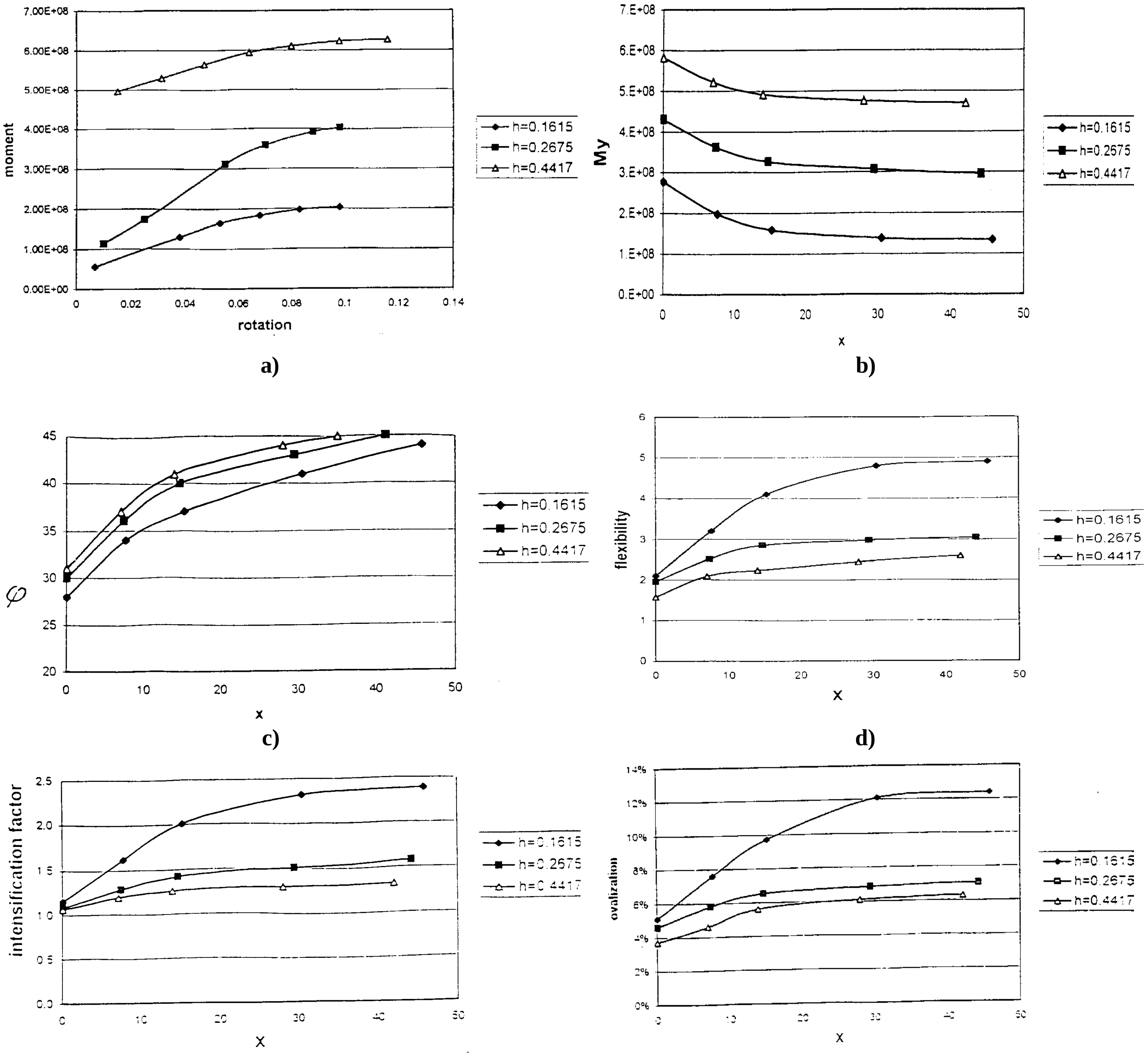


Figure 2. Comparison of results for the 90 degree elbows with different bend ratios subjected to in-plane moments

Presenting samples of all of these numerical results is not possible here because of lack of space and only a few samples are given. As the first sample of results those related to the 90 degree elbow subjected to in-plane moment are shown in figure 2. It is seen in figure 1-a that in all cases the behavior of elbow is highly nonlinear and that by increase in the bend factor, *h*, the rotation values decrease for a given moment value as expected. Figure 1-b shows that the yielding moment of the elbow decreases with decrease in *h* values. It also shows that for each value of *h* with in crease in the length of the straight segment of the pipe connected to the elbow, namely X value, at first the yielding moments decrease. But from a particular X value, called here the threshold value, the moment values remain relatively constant with increase in X. Variation of the angle of the maximum stress plane, ö, shown in figure 1-c is very interesting. As it is seen this value varies drastically with variation of X value. This can be an important point with regard to the effect of the combined loading such as internal pressure, thermal and seismic forces. Figures 1-d to 1-f show that variation of the elbow flexibility, the intensification factor and ovalization have similar trend versus X values. In all cases there is an initial increase and then from a particular value of X the curves remain essentially constant. On this basis the optimum value of X can be determined for any specific design case.



a) **b)**

c) **d)**

e) **f)**

Figure 2. Comparison of results for the 90 degree elbows with different bend ratios subjected to out-of-plane moments

As the second set of numerical results those related to the 90 degree elbows with different bend ratios subjected to out-of-plane moments are presented in figure 2. The results are generally similar to those of the elbows subjected to in-plane moments. But variations of ö values as well as the stress intensification are somehow different with the case of in-plane moments. In fact, in this case these two parameters are less sensitive to the variation of *h* values. The sensitivity of stress intensification to variation of X value is less comparing with the case of in-plane moments. Summary of the results relating to the 90 degree elbows with different bend ratios subjected to in-plane as well as out-of-plane moments are given in Table 2 for comparison. In this table the results are only for two values of X, namely zero and the threshold value to give a better picture of the effect of end conditions on the elbows' behavior. As it can be seen that applying a specific end condition can decrease the rupture or collapse moment as well as yielding moment, while increase flexibility, stress intensification and ovalization in the elbow. These variations are less for greater bend factor. It is also seen that in-plane moments are more critical for elbow than out-of-plane moments. Comparing the results given in table 3, which are related to 45 degree elbows, with those in table 2 shows that the 45 degree elbow has more critical situation than the 90 degree elbow similar end conditions and bend factor.

Table 2. Summary of the analyses results of the 90 degree elbow with weld connections subjected to in-plane as well as out-of-plane moments

		X(cm)	rotation rad	Mcollapse	My N.mm	flexibility	intensity	ovalization
inplane	h1	0	0.055	3.20E+08	2.51E+08	2.7	1.28	7%
		116.1	0.142	1.40E+08	1.09E+08	8.07	3.01	23%
	h2	0	0.7	4.72E+08	3.70E+08	2.176	1.25	5.50%
		104.7	0.129	2.29E+08	1.86E+08	4.733	2.5	15.50%
	h3	0	0.1	6.57E+08	5.07E+08	1.702	1.211	4.30%
		88.9	0.135	5.53E+08	4.28E+08	2.695	1.45	9.90%
out of plane	h1	0	0.0485	3.58E+08	2.78E+08	2.1	1.15	5.10%
		101	0.139	1.68E+08	1.35E+08	4.9	2.38	12.40%
	h2	0	0.067	5.45E+08	4.30E+08	1.95	1.081	4.60%
		89.7	0.126	3.69E+08	2.95E+08	3.05	1.57	7.05%
	h3	0	0.094	7.41E+08	5.82E+08	1.57	1.063	3.70%
		78.2	0.128	6.63E+08	4.69E+08	2.59	1.32	6.30%

Table 3. Summary of the analyses results of the 45 degree elbow with weld connections subjected to in-plane moments

	x	rotation	Mcollapse	My	flexibility	intensification	ovalization
h1	0.0	0.048	2.42E+08	2.00E+08	2.41	1.18	6.10%
	108	0.112	1.35E+08	1.06E+08	7.12	2.17	13.70%
h2	0.0	0.058	3.89E+08	2.92E+08	1.98	1.16	4.90%
	97.2	0.099	2.28E+08	1.67E+08	3.41	1.95	10.33%
h3	0.0	0.082	5.57E+08	4.38E+08	1.51	1.13	3.60%
	81.8	0.11	3.80E+08	3.39E+08	2.1	1.52	8.40%

CONCLUSIONS

Results show that in the case of welded elbows by increasing the length of the straight segment of the pipe connected to the elbow both yielding and failure moments as well as stress intensification factor increase, and ovalization occurs in higher moment values. In each case there is a limit value for the length of the straight part, more than which the increase has no significant effect on the ultimate sustained values of loading moments. This length can be called as the effective length in each case. Generally, the in-plane applied moments have greater effects on the elbow behavior, and the 45 degree elbows are less sensitive to the end conditions comparing with the 90 degree elbows and therefore are more critical when subjected to seismic loads. In the case of flange connections the effect of end conditions is much less and it can be said that in this case the flange makes the pipe to behave almost as a built-in beam. To obtain more reliable results it is suggested that some tests are performed on various elbows with different end conditions. The obtained effective lengths can be employed as very useful parameters in the seismic design of elbows leading to more reliable designs.

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