A MARGIN – CONSISTENT PROCEDURE FOR CALCULATING THE B_2 STRESS INDEX AND A PROPOSED NEW DESIGN EQUATION

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ABSTRACT

In Section III of the ASME Boiler and Pressure Vessel Code, the design equation for primary stresses in piping contains stress equations for straight pipes that are modified by stress indices so that the equations can be applied to other piping components. In this paper we review the history of this equation in the context of elbows and then suggest a new procedure for calculating the B_2 stress index that applies to stresses for bending. The resulting stress index equation, which is the ratio of the collapse moment of a straight pipe to the collapse moment for any component, gives a B_2 value of 1.00 when applied to a straight pipe and a safety margin for the component that is always the same as for the straight pipe. This procedure is based on the Code-defined collapse moment, which we determine using nonlinear finite element analysis (FEA), supported by experimental data. We present B_2 values for elbows with a wide range of pipe bend parameters. The values of B_2 obtained using this equation are 14 to 48% lower than the values obtained using the current Code procedure when applied to stainless steel 90° butt-welding elbows.

INTRODUCTION

Most piping in nuclear power plants is designed by rules given in Section III of the ASME Boiler and Pressure Vessel Code (the Code)[1]. In the Code, stresses are divided into three categories: Primary which "is any normal stress or a shear stress developed by an imposed loading which is necessary to satisfy the laws of equilibrium..."[2]; Secondary which is "a normal stress or a shear stress developed by the constraint of adjacent material or by self-constraint of the structure. The basic characteristic of a secondary stress is that it is self-limiting."[3]; and Peak stress.

In piping design, Code Equation (9), which governs the primary stress intensity for design loads, has the following form:

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M_i \leq 1.5 S_m \tag{1}$$

where B_1 and B_2 are stress indices for internal pressure and bending. The equation has been described as assuring "against catastrophic membrane failure..."[4] or placing "bounds on loading such that necessary conditions for a collapse load will not exist anywhere in the piping system"[5].

The stress indices are used to modify nominal stress equations for straight pipes so that the behavior of piping components such as elbows can be controlled using the same basic stress limits as for straight pipe. Moore and Rodabaugh[6] state that "The B_1 and B_2 stress indices reflect the capacities of various piping products to carry load without gross plastic deformation."

Paragraph NB-3682 of the Code provides the following general definition of stress indices:

$$B, C, K, \text{or } i = \frac{\sigma}{S} \tag{2}$$

where, for the B indices, σ represents the stress magnitude corresponding to a limit load and S is a nominal stress associated with the limit load. Values for the B, C and K stress indices are given in Table NB-3681(a)-1 for a variety of piping components. For elbows the user is referred to equations in subparagraph NB-3683.7 where B_2 is defined as

$$B_2 = \frac{1.30}{h^{2/3}} \geq 1.0 \tag{3}$$

where h is the characteristic bend parameter, which is defined as

$$h = \frac{tR}{r_m^2} \tag{4}$$

and where the remaining parameters are defined in the Nomenclature Section.

"For piping products not covered by NB-3680, stress indices ... shall be established by experimental analysis (Appendix II) or theoretical analysis[7]." Appendix II, however, has the following section, titled Experimental Determination of Stress Indices for Piping: "In course of preparation. Pending publication, stress indices for piping shall be determined in accordance with the rules of NB-3680[8]." The appendix does contain a procedure for obtaining experimentally a value for the collapse load[9] although there is no guidance on how to use this information to obtain the stress indices.

In this paper we review the history of the B_2 index and suggest a generic procedure for obtaining values for the index for any type of component. The suggested procedure, which is consistent with the derivation for Eq. (3), relates the collapse moment of any piping product to the collapse moment of a straight pipe and hence will result in a B_2 value of 1.0 when applied to a straight pipe or in the limiting case of an elbow as the subtended bend angle approaches zero degrees. The procedure also guarantees that the margin of the component will always be the same as for a straight pipe with the same material and geometric properties. The suggested new procedure for obtaining the B_2 index relies on an accurate calculation of the collapse load – which we determine using nonlinear FEA methodology and our FEA procedures are verified by experiments.

HISTORY

The history of piping elbows and pipe bends, from von Karman[10] and Bantlin[11] in the early part of the 1900s to the present, has been documented in various papers (See for example Markl[12], Rodabaugh and George[13], Dodge and Moore[14], and Yu[15]). In this paper, we will discuss only those references related directly to the B_2 index; this includes not only B_2, but also C_2 and the stress intensification factor i.

The Foreword to the USA Standard B31.7 for Nuclear Power Piping, published in 1969, gives the following background for the use of stress indices: "Stress indices ... have been used in B31.1 since 1955 where they were called stress intensification factors. These factors are based for the most part on tests by Markl and George[16], and by Markl[17]. Another precedent use is in the ASME Boiler and Pressure Vessel Code, Section III, where the term 'stress index' is used. In the Boiler Code, the stress indices are for internal pressure loading and the stress indices given are based on test data. "Now, if a stress intensification factor is equal to 4, the user immediately visualizes that he has a component where a significant stress is four times as high as in a piece of straight pipe with the same applied moment.[18]" "In B31.1 the use of stress intensification factors is restricted to moment loadings."[19]

The stress intensification factor for elbows, used in the B31.1 Code, was first given by Markl [20] in 1952 as

$$i = \frac{0.9}{h^{2/3}} \geq 1 \tag{5}$$

and was based on his bending-fatigue data.

In 1969 the B31.7 Standard introduced the concept of stress indices as follows: "In B31.7 they [the stress intensification factors] have a wider scope; stress indices are used for three purposes. First, B-indices are based on limit load type of analysis. C-indices represent the Primary-plus-Secondary stresses and K-indices represent peak stresses which are involved in a fatigue evaluation. All three types of indices are used for internal pressure loads, moment loads and thermal gradient loads." The relationship between the B and C indices is described as follows: "From some limited test data [21],[22] probably a very conservative estimate is that the collapse moment is 4/3 of the moment to produce a local bending stress equal to the yield strength of the elbow material. This leads to a B_2 factor for elbows or curved pipes of 0.75 times the C_2 factor. This is an example of engineering judgment based on a few tests and some limited theory."[23] The Foreword also states that the stress indices of B31.7 are "quite close to double the stress intensification factors in B31.1."

Because the B_2 index was related to the C_2 index (and remains so today), it is worth while reviewing the origin of this index. The C_2 equation for elbows in the 1969 B31.7 Code,

$$C_2 = \frac{1.95}{h^{2/3}} \geq 1.5 \tag{6}$$

has not been changed and remains in the Code today. The relationship between C_2 and B_2 given above, which remained in the Code from 1969 until 1981, results in the following B_2:

$$B_2 = \frac{3}{4} C_2 = \frac{1.4625}{h^{2/3}} \geq 1.125 \tag{7}$$

The basis for Eq. (7) was given by Dodge and Moore[24] in 1972. This equation can be obtained from their more general equation

$$C_2 = \frac{2}{\lambda^{2/3}} \frac{1+1/4\gamma}{1+1/\lambda^{4/3}e^{1/j^{1/4}}} \tag{8}$$

by setting $\gamma=\infty$, $\varphi=0$ and $\nu=0.3$. To obtain this equation, Dodge and Moore used the theory given in Rodabaugh and George[25] to obtain the maximum stress intensity (which is, of course, related to the Tresca yield condition, which is the basis of the Code equations) in an elbow from three moments – in-plane, out-of-plane and torsion – and internal pressure. They used a computer program to evaluate the stress equations for a range of parameters and then plotted the stress index as a function of the bend parameter. Eq. (8) is "A conservative approximation for stress indices, which slightly over-estimates the tabulated values"[26].

In 1974 Mello and Griffin[27] showed, for perhaps the first time, the relationship between the collapse moment determined by limit analysis and the B_2 index:

$$M_{LL} = \frac{Z(1.2S_y)}{B_2} \tag{9}$$

where B_2 is as defined in the 1971 Code, and Eq. (9) is a conservative prediction of the limit moment. Mello and Griffin did not use this equation to calculate the B_2 index, but rather to calculate a Code-based limit moment. In the 1971 Code, the collapse load was taken as that in which the distortion is 2 times the value at the calculated initial departure from linearity. Mello and Griffin include a discussion of various alternatives to this definition.

In 1978 Greenstreet [28] published a report describing a series of 20 bending experiments on commercially available 6 inch 90 degree elbows. The objective of the tests was "to obtain in-plane and out-of-plane limit moments." He considered three existing methods for determining limit loads. The first, described by Demir and Drucker [29], defines the limit load as the load at which the measured deflection is three times the extrapolated elastic deflection. The second method is to use the point of intersection of a line drawn tangent to the initial (elastic) portion of the force-deflection curve and a line drawn tangent to the straight-line portion of the curve in the plastic region. The third method is to determine the load at the 0.2% offset strain from a load-strain diagram where the strains are measured by gages located in the high-strain region of the component. Greenstreet uses what he calls an equivalent method to compare the limit loads for his test specimens. Using the tangent method to determine the limit load, he calculates a parameter 'a' that is defined as the total deflection minus the extrapolated elastic deflection divided by the extrapolated deflection. These relationships are shown in Fig. 1. In Greenstreet's results, the values of the parameter 'a' ranged from 0.25 to 1.00 with an average of 0.38. It is interesting to note that a=2 for the Demir and Drucker method and a=1 in the current Code definition of collapse load. Two results from the Greenstreet report are relevant to our work. In his Table 6, the last column tabulates the ratio of $\frac{M_{CL}}{S_y Z}$ (after converting to our notation) which is the reciprocal of our Eq. (18) (given later in the paper). Although he does not relate this to the B_2 index, he does say that it "can be interpreted as indicating the margins of safety with respect to the onset of yield in straight runs of pipe." Also, in his Table 8, the last column contains the quantity $\frac{M_{CL,component}}{M_{CL,straightpipe}}$ (in our notation), which is the "ratio of the plastic collapse moment to the theoretical plastic collapse moment for straight pipe." Again, this is the reciprocal to our Eq. (24) (also given later in the paper.)

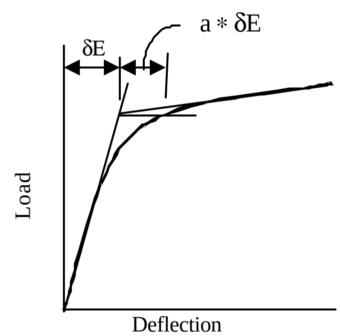

Fig. 1 Definition of 'a'

The relationship between C_2 and B_2 in the current Code was proposed by Rodabaugh and Moore[30] in 1978. In their paper, the authors discuss the design philosophy of the 1969 B31-7 Code, how limit load concepts might be introduced, and the Code definition of collapse as being the load or moment at which the displacement is twice the extrapolated elastic displacement. They begin by examining straight pipes because the "straight pipe serves as a reference geometry for more complex geometries such as elbows and branch connections."[31] This relationship between straight pipes and elbows is significant because it relates directly to the new procedure we describe later for computing the B_2 index. The straight pipe limit load theory was given in a paper by Larson, Stokey and Panarelli[32] in 1974 and is based on the lower bound theorem and the von Mises criterion. For elbows, they begin by summarizing the maximum stress ratios for in-plane bending ($1.8/h^{2/3}$, occurring in the hoop direction), out-of-plane bending ($1.5/h^{2/3}$), and torsion (1.0 for the shearing stress). They then state that "Considering all combinations of M_x, M_y, and M_z as represented by the vector moment $M_i = [M_x^2 + M_y^2 + M_z^2]^{1/2}$, the maximum stress intensity does not exceed $1.95/h^{2/3}$. This is the C_2 index given in the Code Table NB-3682.2-1 (now in subsubparagraph NB-3683.7 (b))." The authors point out that maximum or near-maximum stresses occur at four locations around the circumference, that the most significant stresses are related to through-wall bending, and that "if the moment is increased by 1.5 times the moment causing the maximum elastic stress to reach the yield strength, then four yield lines would form and the elbow could 'collapse'."[33] For in-plane bending, this leads to the following limit load equation

$$M_L = 1.5M_{Yield} = 1.5\left[\frac{h^{2/3}}{1.8}S_y Z\right] = 1.5\left[\frac{h^{2/3}}{1.8}S_y \frac{\pi}{4}D^2 t\right] \quad (10)$$

where the thin-walled pipe approximation has been used for the section modulus. This result is compared to a theoretical limit load analysis for elbows given by Spence and Findlay[34] and shown to be similar. The factor of 1.5 used above would mean that the relationship between B_2 and C_2 would be

$$B_2 = \frac{2}{3}C_2 = \frac{1.30}{h^{2/3}} \geq 1.0 \quad (11)$$

rather than the $3/4C_2$ used in the 1977 Code. The authors suggest that the reasons for this conservatism were related to the allowable stress used at the time and the dearth of test data for limit moments for elbows.

Eq. (11) is based on the assumption that there are no end effects, i.e. the ovalization is uniform along the elbow. The authors state that, because of this assumption, the equation would be conservative for 90^o or shorter elbows. The equation is also based on the assumption that pressure effects are negligible, although this is known to be incorrect in the elastic region, where the pressure reduces the stresses. "However, the pressure itself causes stresses, hence, the combination of moments and pressure may increase or decrease the limit moment. In principle, an elastic-plastic, large-deformation theory and computer program based hereon could be used to evaluate such combinations. However, we are not aware of any such analysis being done. We would guess that the cost of such an analysis would be several times the cost of a test."[35] Much has changed in the last 22 years, and the authors would no doubt revise this last conclusion if they were writing today. However, good experimental data are just as valuable today as they were then, although they might be used primarily for verification of finite element codes rather than for parametric studies.

Several revisions to the piping Code appeared in 1978 and 1981 and were discussed by Moore and Rodabaugh.[36] These changes involved modifications to the stress limits and the modification to the B_2-C_2 relationship that Rodabaugh and Moore had suggested in 1978 (i.e. from 0.75 to 0.67). The authors also include the following statement on the relationship between piping components and straight pipes: that Eq. (1) and the corresponding equation for Class II piping, are "highly simplified limit-load formulas expressed in terms *relative to the limit-load behavior of straight pipe*. For piping products with B_1=0.5 and B_2=1.0 ... these equations express the concept that the product is *at least as strong as the attached pipe*. (italics added)" These two aspects of piping design for primary loads are central to our proposed new procedure for calculating the B_2 index.

In 1982, Rodabaugh and Moore[37] summarized the literature on end effects on elbows subjected to moment loadings, and presented their finite element results on elbows with characteristic bend parameters from 0.05 to 1.5. As part of this study, they also investigated the effect of subtended bend angle on the C_2 index, and suggested equations for C_2 for three ranges of α_o ($\geq 90^o$, $= 45^o$, and $\leq 30^o$). They intuit "that, as α_o approaches zero, the maximum stress index should approach 1.00"[38]. In another part of their work, they suggest directional stress indices for primary and primary plus secondary stresses This work resulted in ASME Code Case N-319[39] giving an alternate procedure for evaluating stresses in butt welding elbows in Class 1 piping, Section III, Div. 1. This case was superseded in 1990 by Case N-319-2[40], which gives the following replacement for the B_2M_i calculation in Eq. (1):

$$\left.\begin{array}{ll}
C_{2x} = C_{2y} = 1.71/h^{0.53} \geq 1.0 & \\
C_{2z} = 1.95/h^{2/3} & \text{for } \alpha_o \geq 90^o \\
C_{2z} = 1.75/h^{0.56} & \text{for } \alpha_o = 45^o \\
C_{2z} = 1.0 & \text{for } \alpha_o = 0^o \\
B_2M_i = 0.67\left[(C_{2x}M_x)^2 + (C_{2y}M_y)^2 + (C_{2z}M_z)^2\right]^{\frac{1}{2}} &
\end{array}\right\} \quad (12)$$

Even though Eq. (1) is usually thought of in terms of limit load analysis, primary stresses are also related to membrane stresses, and there have been attempts to use elastic analysis to calculate membrane stresses and use them to obtain the B_2 index. This was the approach taken by Rodabaugh and Moore[41] in 1983 for concentric nozzles. They state "Conceptually, [the B indices] are derived on a 'limit-load' basis. However, a tentative assessment of the adequacy of the B indices can be made by considering the calculated membrane stresses." In the second article of this 1983 WRC bulletin on eccentric reducers, Avent et al.[42] define B_2 as follows:

$$B_2 = \frac{\sigma_{mm}}{(M_i/Z)} \quad (13)$$

where σ_{mm} is maximum calculated membrane stress intensity in a model corresponding with M_i. Because this approach is based on linear behavior, any moment below the proportional limit will work. Williams and Lewis[43] have applied this approach to elbows.

In 1988 Touboul et al.[44] used limit load results from Spence and Findlay, Dodge and Moore and ASME Code Case N-317 to create a modified version of the B_2 index to account for internal pressure and subtended bend angle,

$$B_2 = \frac{1.6}{h^{2/3}}\left(\frac{\alpha}{\pi}\right)^{0.4}\frac{1}{1+\frac{0.7}{\lambda}\frac{Pr_m}{tS_y}} \tag{14}$$

where the notation has been modified to make the equation consistent with that used in this paper. Another useful feature of this paper is a discussion of margins, defined as the ratio of the collapse moment to the Code allowed moment, for different loading levels. This is the same approach we will take in assessing our proposed B_2 procedure.

Touboul and Acker[45] in 1991 referred to Eq. (14) as the index that would be used if the moment in Eq. (1) were based on actual component instability, and then present a modified version of this equation with the 0.7 replaced by 0.55 if the moment is to be based on Code-defined collapse. They also address the issue of buckling. They state "…it is usually difficult (and usually impossible) to experimentally distinguish between plastic instability and buckling…" and then propose an equation similar to Eq. (1), but with directional stress indices and modified stress limits.

The first paper to indicate how to use an experimentally determined limit moment to obtain the B_2 index was given in 1995 by Wais[46]. The application was to welded attachments (lugs), but the procedure he presents would seem to be applicable more generally. Using the same approach as Mello and Griffin, Wais states that "The limit load is defined as that load which results in a deflection twice that predicted based on elastic behavior. Once the limit load and hence the limit moment was determined experimentally the expression:

$$M_{CL} = \frac{S_y Z}{B} \tag{15}$$

was used to determine the primary stress B [index]."

In 1999, Liu et al.[47] use a procedure similar to Wais', but with a nonlinear finite element analysis to determine the limit moment instead of an experiment. Their equation takes the form

$$B_2 = \frac{S_{membrane,CL}}{\frac{M_{CL} D_o}{2I}} \tag{16}$$

where $S_{membrane,CL}$ is the maximum membrane stress intensity in a pipe bend corresponding to the collapse moment determined from elastic-plastic finite element analysis due to in-plane closing mode bending, in-plane opening mode bending, out-of-plane bending, or torsion. They also used the linear elastic approach with membrane stresses as a conservative but more economical method to obtain the B_2. For this, they used the following equation:

$$B_2 = \frac{S_{membrane}}{\frac{M D_o}{2I}} \tag{17}$$

where $S_{membrane}$ is the maximum membrane stress intensity as above, but for a moment that is below the collapse value. They conclude "...the B_2 determined from the limit analysis is about 40% lower than that obtained from the corresponding linear elastic analysis..."

Also in 1999, Yu et al.[48] describe their research on the Mello-Griffin-Wais approach for obtaining the B_2 index for elbows. They began by using the equation

$$B_2 = \frac{S_y Z}{M_{CL}} \tag{18}$$

where M_{CL} is obtained from either test data or finite element analysis. Because B_2 should be 1.0 when applied to straight pipes, they first applied this equation to straight pipes but found that B_2 was consistently less than 1.0. They tried various modifications of Eq. (18), including replacing S_y with the equivalent von Mises stress at the point of Code-defined collapse, (this resulted in slightly lower B_2 values) and using the plastic section modulus in place of the elastic modulus Z (this resulted in a B_2 of about 1.0 for the straight pipe.) They suggest that a procedure that will always result in a B_2 of 1.0 for a straight pipe is to normalize Eq. (18) for any component by dividing it by the equation evaluated for a straight pipe with the same material and geometric properties as the component, i.e.

$$B_{2,normalized} = \frac{B_{2,component}}{B_{2,straight\ pipe}} \tag{19}$$

Yu et al. also observed that, when Eq. (18) was substituted into Eq. (19), the result was just the ratio of the limit moments, i.e.

$$B_{2,normalized} = \frac{\dfrac{S_y \cdot Z}{M_{CL,component}}}{\dfrac{S_y \cdot Z}{M_{CL,straight\ pipe}}} = \frac{M_{CL,straight\ pipe}}{M_{CL,component}} \tag{20}$$

Note that, because the $S_y Z$ terms cancel, the question of whether Z is elastic or plastic is irrelevant. This work by Yu et al. is the basis for the procedure described later in this paper.

In 1995, the Code was modified to accommodate reversing dynamic loading, and fatigue ratcheting as a failure mode. The following design equation for Level D loading was introduced:

$$B_1 \frac{P_D D_o}{2t} + B_2 \frac{D_o}{2I} M_E \leq 4.5 S_m \tag{21}$$

The interesting thing about this equation is that it still contains B indices, which are used elsewhere in the Code only for monotonic loading. Because the failure modes for monotonic loading will in general be quite different from cyclic loading it would seem as though the indices for this equation should have their own definition based on the failure mode associated with the equation. This part of the 1995 Code was not accepted by the NRC, and the ASME Code committees are still working on the appropriate form for design equations for reversing dynamic loading.

SUGGESTED PROCEDURE FOR CALCULATING A MARGIN-CONSISTENT B_2 STRESS INDEX

Eq. (20) was developed by normalizing the B_2 index obtained using Eq. (18), with respect to the B_2 index for a straight pipe [Eq. (19)]. This led to Eq. (20), which is the ratio of the collapse moments. This result can be derived in a more straight forward manner [49] by setting the stress limit for any component equal to the stress limit for a straight pipe, i.e.

$$\acute{o}_{component} = \acute{o}_{straight\ pipe} \tag{22}$$

Then, referring to Eq. (2), the definition of B_2 index, Eq. (22) can be written as follows:

$$B_{2,component} \cdot S_{component} = B_{2,straight\ pipe} \cdot S_{straight\ pipe} \tag{23}$$

Rearranging Eq. (23), letting $B_{2,straight\ pipe}$=1.0, and canceling the section modulii Z, we obtain the following:

$$B_{2,component} = \frac{S_{straight\ pipe}}{S_{component}} = \frac{M_{CL,straight\ pipe}/Z}{M_{CL,component}/Z} = \frac{M_{CL,straight\ pipe}}{M_{CL,component}} \tag{24}$$

From this perspective, Eq. (24) would seem to follow directly from the Code definition. It also has the advantage of guaranteeing that B_2 for a straight pipe will always be 1.0. This attribute was acknowledged in the 1982 paper by Rodabaugh and Moore in their work on C_2 indices for different bend angles. As the subtended bend angle approaches zero, the elbow approaches a straight pipe, and the authors suggested that, in the limit, the index should approach 1.0[38].

Another advantage in using Eq. (24) as the definition for the B_2 index is that the margin for the component always turns out to be the same as the margin for the straight pipe of the same geometric and material properties. This can be seen from the following proof, where the margin is defined, as it was in the paper by Touboul et al.[50], as the ratio of the collapse moment of the component divided by the Code allowed moment of the component.

$$\begin{aligned} Margin_{component} &= \frac{M_{CL,\ component}}{M_{code,component}} = \frac{M_{CL,\ component}}{\left(1.5S_m \cdot Z/B_{2,\ normalized}\right)_{component}} = \frac{M_{CL,\ component}}{1.5S_m \cdot Z} B_{2,component,\ normalized} \\ &= \frac{M_{CL,\ component}}{1.5S_m \cdot Z} \cdot \frac{M_{CL,\ straight\ pipe}}{M_{CL,\ component}} = \frac{M_{CL,\ straight\ pipe.}}{1.5S_m \cdot Z} = Margin_{straight\ pipe} \end{aligned} \tag{25}$$

RESULTS

We have applied our definition of B_2 (Eq (24)) to forty different elbow configurations, with values of h ranging from 0.048 to 0.997, subjected to in-plane-closing, in-plane-opening bending moments using the nonlinear FEA code ANSYS. Since the closing mode always controlled, data for the opening mode are not shown. These components are all butt-welding, seamless, 304L stainless steel, long and short radius, 90° elbows at room temperature. Nominal geometric and Code material properties were used. The FEA models utilized ANSYS SHELL 181 elements. See the Appendix at the end of this paper for examples of our FEA verification using experimental data. The details of 4 sets of calculations are tabulated in Table 1. The two rows labeled $B_{2,\text{straight pipe}}$ and $B_{2,\text{elbow}}$ show that using Eq. (18), which would seem to be a literal reading of the Code as given in Eq. (2), can lead to values that are less than one. The next row, $B_{2,\text{elbow, normalized}}$, has values that are all greater than one, as expected. The last row is the ratio of the B_2 value we calculate compared to the Code value. The values for $B_{2,\text{straight pipe, normalized}}$ would, of course, all be 1.00.

Table 1. Examples of New-definition B_2 Values vs. Code B_2 Values for Elbows

Size and Schedule			8" Sch5 SR(1)		2" Sch40 LR(1)		8"Sch160 LR(1)		2" Sch160 LR(1)	
h			0.048		0.375		0.730		0.997	
$B_{2,\text{elbow, code}}$ (Eq. (3))			9.83		2.50		1.60		1.30	
Z	(in^3)	(cm^3)	6.13	100.	0.561	9.19	38.5	631.	0.979	16.0
S_y	(ksi)	(MPa)	25.0	172.	25.0	172.	25.0	172.	25.0	172.
$B_{2,\text{straight pipe}}$ (Eq. (18))(2)			0.93		0.88		0.85		0.83	
$B_{2,\text{elbow}}$ (Eq. (18))(2)			4.72		1.49		1.05		0.93	
$B_{2,\text{elbow, normalized}}$ (Eq. (20))(3)			5.08		1.69		1.24		1.12	
Percent Reduction(4)			48%		32%		23%		14%	

(1) SR = Short Radius, LR=Long Radius

(2) Z is the elastic section modulus

(2 & 3) Collapse moments are obtained from FEA. For elbows, the moments are in-plane closing.

(4) Percent Reduction = $\dfrac{\left|B_{2,\text{elbow, normalized}} - B_{2,\text{elbow, Code}}\right|}{B_{2,\text{elbow, Code}}}$

The complete set of B_2 results for in-plane closing and opening mode bending are shown graphically in Fig. 2, where FEA Data stands for the evaluations of Eq. (24), LR stands for Long Radius elbows and SR stands for Short Radius Elbows. The line for "Minimum B_2" and "Code Equation" are self-explanatory, and the other curves will be explained below. We observe that there is very little difference between Long Radius and Short Radius elbow behavior.

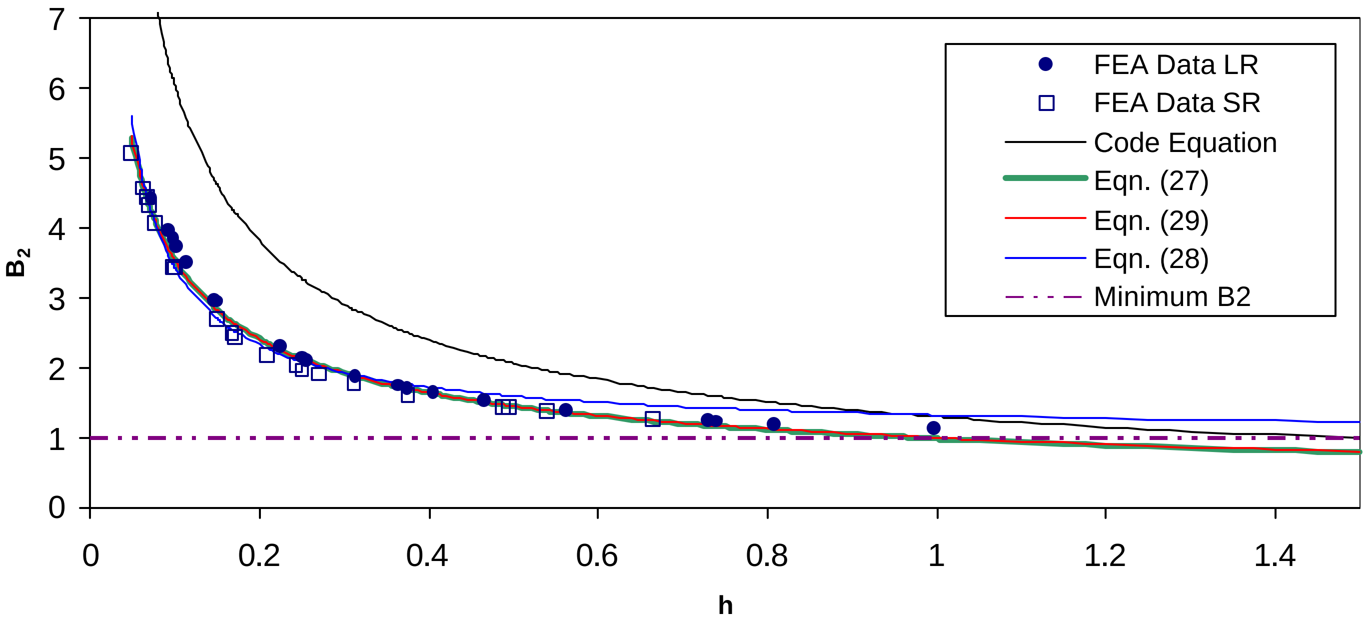

Fig. 2. B_2 Values vs. h for In-Plane Bending, Closing Mode

If we postulate an equation for the FEA data shown in Fig. 2, using an equation of the form

$$B_2 = \alpha + \frac{\beta}{h^{\gamma}} \tag{26}$$

then we can determine numerical values for the parameters α, β and γ by minimizing the sum of the-squared errors between the FEA data points and the corresponding values from the postulated equation. The minimization uses all of the results, i.e. both LR and SR elbows. If α is set to zero, then the form of the equation is the same as the Code equation, in which $\beta = 1.30$ and $\gamma = 2/3$. If α is set to 1, then the equation guarantees that B_2 will never be less than 1. Alternatively, we could leave α as a free parameter, and let the minimization algorithm establish the optimal value. We show below the results of these different equation possibilities:

Code Equation
$$B_2 = \frac{1.30}{h^{2/3}} \tag{3}$$

$\alpha = 0$
$$B_2 = \frac{0.986}{h^{0.554}} \tag{27}$$

$\alpha = 1$
$$B_2 = 1 + \frac{0.323}{h^{0.875}} \tag{28}$$

α = free parameter
$$B_2 = 0.047 + \frac{0.950}{h^{0.564}} \tag{29}$$

SUMMARY AND CONCLUSION

In our review of the history of Eq. (1) [Eq. (9) in the Code] and the B_2 index, we conclude that the developers of this equation in the Code were attempting to prevent gross plastic (and nonlinear) behavior of piping by relating the mildly plastic (and nonlinear) behavior of components to straight pipes and then using appropriate allowable stresses. The term 'mildly' is used here because the Code-defined collapse load is, for most elbows, well below the instability load, that is the load for which there is an actual physical collapse. The corollary to our conclusion is that stress index of a component should be related to the collapse behavior of a straight pipe which has the same material and geometric properties as the component. This is the basis of our suggested procedure for calculating the B_2 index as the ratio of collapse moments. We have shown that, using this procedure, (a) the B_2 for a straight pipe will always be 1.00 and (b) the margin for any component will be the same as for a straight pipe with the same material and geometric properties. Also, in the examples presented, the suggested procedure results values of B_2 which are up to 50% less than the values obtained from Eq. (3). Additional work needs to be done before a more definitive statement can be made regarding the B_2 stress index values for elbows, or before it is possible to determine how the results of this procedure would relate to Code equations for other types of components. Three possible equations are suggested to reflect the proposed definition of B_2, with Eq. (28) perhaps being preferred as it guarantees that the index will never be less than one.

FUTURE WORK

Before a definitive statement can be made about the usefulness of the suggested procedure, however, there remain several topics that need to be investigated. These include determining the effects of internal pressure, combined loadings, use of other materials, behavior at elevated temperature (covered in Code Subsection NH) and high strain rates, consideration of other sizes, schedules and bend angles of elbows and a consideration of other components such as tees and branches, and the relationship between Code-defined collapse and actual physical collapse. A study of cyclic loading would also seem to be worthwhile. In this case it would be necessary to redefine what is meant by collapse, and then use this definition in the calculation of a B'_2 stress index. If the beneficial attributes of the proposed procedure hold up under continued scrutiny, then it would seem appropriate to propose that it be included in the Code in some appropriate manner.

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NOMENCLATURE

a	=	ratio of plastic deformation to extrapolated elastic deformation in Greenstreet's report
B_1	=	primary stress index for pressure
B_2	=	primary stress index for bending
C_2	=	secondary stress index for bending
D	=	mean pipe diameter
D_o	=	outside diameter of pipe
E	=	Young's modulus
h	=	characteristic bend parameter, tR/r_m^2
i	=	stress intensification factor (SIF). Also a stress index for detailed analysis in NB-3200, given in tables NB-3685.1-1&2
I	=	moment of inertia
K	=	local stress index
M_{CL}	=	Code defined collapse moment
M_{LL}	=	limit load moment
M_E	=	amplitude of the resultant moment due to the inertial loading from the earthquake, other reversing type dynamic events and weight.
M_i	=	resultant moment due to a combination of Design Mechanical Loads
P	=	design pressure
P_D	=	pressure occurring coincident with a reversing dynamic load
R	=	nominal bend radius of elbow
r_m	=	mean pipe radius, $(D_o\text{-}t)/2$
S	=	nominal stress
S_m	=	allowable design stress intensity value
t	=	nominal wall thickness
Z	=	section modulus
α	=	bend angle of elbow
γ	=	R/r (unrelated to the circumferential stress-intensification factor given above)
γ_i	=	in-plane bending maximum circumferential stress-intensification factor
γ_o	=	out-of-plane bending maximum circumferential stress-intensification factor
λ	=	$h/\ (1\text{-}\nu^2)$
ν	=	Poisson's ratio
σ	=	stress magnitude corresponding to a limit load
σ_{mm}	=	maximum membrane stress intensity
φ	=	dimensionless pressure parameter, PR^2/Ert

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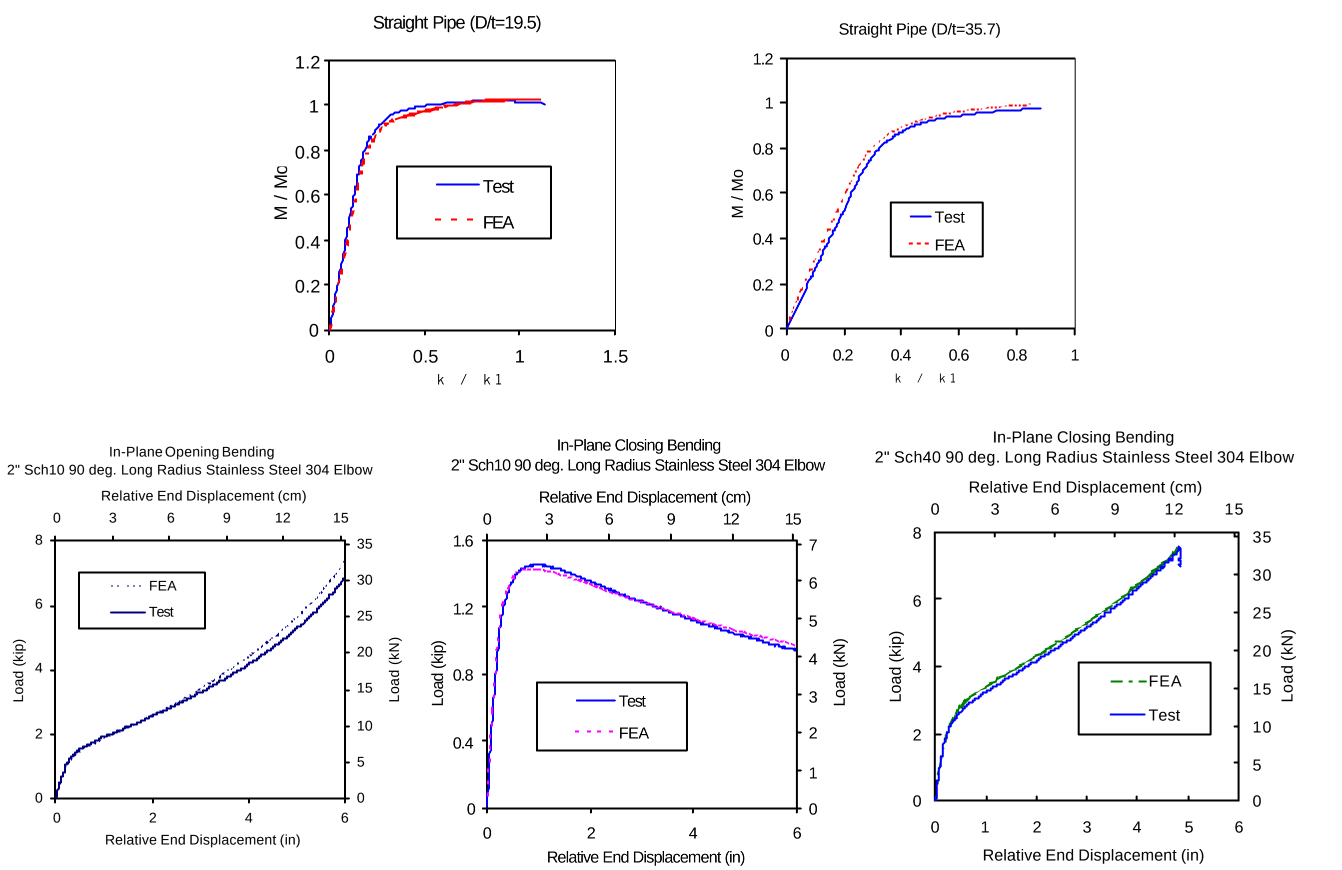
APPENDIX: CORRELATION OF FEA WITH EXPERIMENTAL DATA

The nonlinear FEA procedures used above to determine the collapse moments of straight pipes and elbows were verified by eight experiments, as demonstrated in Fig. 3 below. They are two four-point-bending tests on straight pipes (presented in Ju [51]) and six elbow tests, two for in-plane closing load, two for in-plane opening load and two subjected to out-of-plane load. All the FEA simulations show close agreement with test responses, which validates the FEA approaches we used to obtain B_2 values shown in Table 1 and in Fig. 2 above. The various issues associated in our finite element analysis such as element type, mesh size, reconciliation with test results and so on were discussed in Tan et al. [52].



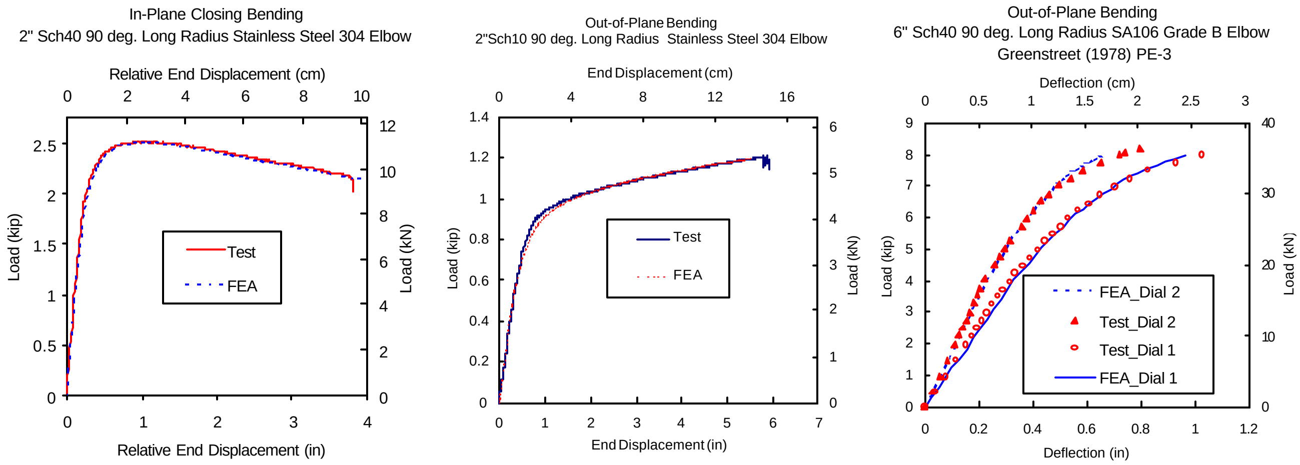


Fig. 3. FEA Results vs. Experimental Results