Nonlinear Finite Element Analysis of Framed Shear Walls

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ABSTRACT

An extensive experimental and theoretical research has been carried out at the Structural Research Laboratory, University of Houston (UH), to study the behavior of RC framed shear walls. Fourteen one-third scale specimens were systematically tested under reversed cyclic loading. The envelopes of the load-deformation responses were accurately predicted by a non-linear finite element program called FEAPRC. FEAPRC was developed at UH by modifying the well-known finite element code FEAP for specific application to concrete structures. In FEAPRC, the analysis of cracked reinforced concrete elements utilizes the new smeared constitutive laws established in the Fixed-Angle Softened Truss Model (FA-STM). These constitutive laws include the smeared stress-strain relationships of concrete in compression and in tension, as well as the smeared stress-strain relationship of steel bars embedded in concrete. The smeared constitutive law of cracked concrete in shear, which was derived recently in a rational manner, was also verified.

INTRODUCTION

Observations of practical earthquake disasters in the past four decades showed conclusively that buildings with shear walls behaved much better than those without shear walls. Two types of shear walls are currently being used: the cantilever shear walls which are popular in the United States, and the framed shear walls which are used extensively in Japan and southeast Asia. The latter is particularly suitable for stiff, low-rise buildings, such as those used in nuclear power plants.

The framed shear wall system, which consists of frames in-filled with panels, offers an obvious advantage over the cantilever shear walls. The wall panels serve as sacrificial shear elements to absorb the energy during an earthquake, thus protecting the frame system and keeping the integrity of the whole structure. The damaged wall panels can then be rapidly repaired after an earthquake. With increasing interest in framed shear walls in recent years, it is necessary to have a clear understanding of the behavior of framed shear walls subjected to cyclic loading.

The specimens of framed shear walls tested at UH [1],[2] were designed to promote shear failures in the wall panels while protecting the frame elements (beams and columns) from flexural/axial load failures. The three main variables affecting the basic behavior of the framed shear walls were studied. They were the axial load ratio in columns, the percentage of steel in wall panels and the hoop spacing in frame elements.

TESTS OF FRAMED SHEAR WALLS

Fig. 1 shows the dimensions and steel arrangement of one typical specimen [1][2]. All the specimens are one story and single spanned. Their overall dimensions are 1397mm × 1397mm × 152.4mm. All the frame members, i.e. columns and beams, have identical cross sections of 152.4mm × 152.4mm. The columns and the beams are reinforced identically with 6 #4 longitudinal bars and with hoop steel of D3 or #2 bars at 63.5 mm spacing (three specimens FSW 9,10, 11 have reduced 31.75 mm spacing at plastic hinge regions). The thickness of the panels is 76.2mm. Two curtains of steel bar meshes in the panels are W2 or #2 bars at 152.4 mm spacing, or #2 bars at 76.2 mm spacing, arranged in the horizontal and vertical directions. Each specimen has two symmetrical axes (vertical and horizontal).

The specimens were tested in the universal panel tester at the University on Houston [3], using the close-loop servo-control system with mode switch functions [4]. The columns were subjected to a constant vertical compressive force simulating the gravity load. Reversed cyclic horizontal forces were applied at the ends of the top and bottom beams (top compression and bottom tension, and vice versa) to simulate the horizontal shearing action of earthquake loading. The cyclic loading or imposed deformation were applied in a slow and predetermined manner, such that the dynamic effects as well as the rate of deformation effects could be neglected [5]. A total of 36 LVDT (Linear Voltage Differential Transformers) were used to measure the deformation [1][2].

The three main variables studied are as follows:

1) Axial load ratio in columns R: R = P/P_0 where P is the applied vertical compressive force, and P_0 is the axial compressive capacity of the column. R is taken as 0.07, 0.2 and 0.4.
(2) Percentage of steel in wall panels ρ_p: ρ_p is taken as 0.23%, 0.55% and 1.1%.
(3) Hoop spacing in frame elements: standard and seismic hoop spacing.

The main considerations in this research are: (1) to study how these three variables affect the ductility, strength, cracking and failure modes of framed shear walls, and (2) to promote a shear failure in the wall panel while protecting the columns from flexural failure during the lateral reversed cyclic loading.

Nine out of a total of fourteen specimens are presented here to compare with the predictions of the nonlinear finite element program FEAPRC. These nine specimens vary systematically according to the first two variables, R and $\rho_{p.}$ A detailed comparison of all 14 specimens will be reported in a separate paper. The axial load ratio (R), the percentage of steel in the panels (ρ_p), and material properties of these nine specimens are listed in Table 1.

Table 1 Test variables and material properties of specimens

Speci-men	Axial Load Ratio	Percentage of Steel	f'_c (MPa)	$f_{y,\,panel}$ (MPa)	$f_{y,\,frame}$ (MPa)	E_c (MPa)	$E_{s,\,panel}$ (MPa)	$E_{s,\,frame}$ (MPa)
FSW-13	0.07	0.23%	56.91	419.2	424.9	36422	187,544	216,082
FSW-8	0.2	0.23%	48.29	419.9	424.9	33170	187,544	216,082
FSW-12	0.4	0.23%	57.07	419.2	424.9	35121	187,544	216,082
FSW-4	0.07	0.55%	49.51	419.2	424.9	34145	187,544	216,082
FSW-5	0.2	0.55%	56.34	419.9	424.9	37722	187,544	216,082
FSW-6	0.4	0.55%	49.75	419.2	424.9	39023	187,544	216,082
FSW-11	0.07	1.1%	56.99	419.2	424.9	36747	187,544	216,082
FSW-9	0.2	1.1%	50.24	419.9	424.9	43576	187,544	216,082
FSW-10	0.4	1.1%	55.85	419.2	424.9	33820	187,544	216,082

PROGRAM "FEAPRC"

The program FEAPRC (Finite Element Analysis Program for Reinforced Concrete) is a modified version of FEAP (Finite Element Analysis Program) for predicting the behavior of reinforced concrete structures. FEAP is described in the book "The Finite Element Method" [6]. It is a general-purpose program containing about 1,000 subroutines and more than 100,000 lines. As it stands, however, FEAP is not applicable to reinforced concrete structures because no constitute relationships of concrete and steel bars were included. The new computer code FEAPRC [7] was developed by installing the relevant material models of reinforced concrete and the related nonlinear solution technique. This extension of FEAP for application to reinforced concrete structures involves the modifications of 26 subroutines and the addition of 41 new subroutines.

MATERIAL MODELS OF REINFORCED CONCRETE USED IN "FEAPRC"

The four smeared stress-strain relationships of cracked reinforced concrete used in program "FEAPRC" was established for the Fixed-Angle Softened Truss Model (FA-STM) [8],[9]:

Concrete in Compression

The smeared (or average) stress-strain curve of concrete in compression is shown in Fig. 2 and the equations are given as

$$\sigma_2 = E_c \varepsilon_2 \left[1 - 0.5 \frac{\varepsilon_2}{\zeta \varepsilon_\circ} \right] \qquad \varepsilon_2 / \zeta \varepsilon_\circ \leq 1 \qquad (1)$$

$$\sigma_2 = \zeta f'_c \left[1 - \left(\frac{\varepsilon_2 / \zeta\varepsilon_\circ - 1}{4/\zeta - 1} \right)^2 \right] \qquad \varepsilon_2 / \zeta\varepsilon_\circ > 1 \tag{2}$$

where

σ_2, ε_2 = smeared (or average) stress and strain, respectively, in the principal compression directions.

f_c' = maximum compressive strength of standard 6×12 in. cylinders.

$\varepsilon_\circ$ = concrete strain at maximum compressive strength, taken as 0.002.

ζ = softened coefficient.

The softened coefficient ζ in Eq. (1) and (2) is

$$\zeta = \frac{5.8}{\sqrt{f'_c}} \frac{1}{\sqrt{1 + \frac{400\varepsilon_1}{\eta'}}} \leq 0.9 \qquad (f'_c \; in MPa) \tag{3}$$

$$\eta = \frac{\rho_t f_{ty} - \sigma_t}{\rho_l f_{ly} - \sigma_l} \tag{3a}$$

where ρ_t, ρ_l = reinforcement ratios in the t and l directions, respectively; f_{ty}, f_{ly} = yield stress of steel in the t and l directions respectively; and σ_t, σ_l = applied stresses in the t and l directions, respectively. The symbol η' in the Eq. (3) is η or its reciprocal whichever is less than unity. η' is limited to the range of $0.2 < \eta' < 1$. To avoid the potential numerical problems during calculation, $0.2\zeta f_c'$ was taken as the lowest stress level in the descending branch. Equation (1) and (2) are plotted in Fig. 2 with various ζ values.

Concrete in Tension.

The smeared (or average) stress-strain curve of concrete in tension is shown in Fig. 3 and the equations are given as

$$\sigma_1 = E_c \varepsilon_1 \qquad \varepsilon_1 \leq \varepsilon_{cr} \tag{4}$$

$$\sigma_1 = f_{cr} \left[\frac{\varepsilon_{cr}}{\varepsilon_1} \right]^{0.4} \qquad \varepsilon_1 > \varepsilon_{cr} \tag{5}$$

where

σ_1, ε_1 = smeared (or average) stress and strain, respectively, in the principal tension directions.

E_c = modulus of elasticity of concrete.

f_{cr} = cracking stress of concrete, taken as $0.31\sqrt{f'_c}$ (f'_c and $\sqrt{f'_c}$ are in MPa).

ε_{cr} = cracking strain of concrete, taken as 0.00008.

Concrete in Shear

A rational shear modulus was recently derived for cracked concrete [9]:

$$G = \frac{\sigma_1 - \sigma_2}{2(\varepsilon_1 - \varepsilon_2)} \tag{6}$$

Mild Steel

The smeared (or average) stress-strain curve of mild steel bars embedded in concrete is shown in Fig. 4. The curve is expressed by two straight lines with the equations as follows:

$$f_s = E_s \varepsilon_s \qquad \varepsilon_s \le \varepsilon_n \tag{7}$$

$$f_s = f_y \left[(0.91 - 2B) + \left(0.02 + 0.25B \frac{\varepsilon_s}{\varepsilon_y} \right) \right] \qquad \varepsilon_s > \varepsilon_n \tag{8}$$

The f_s , ε_s are the average (or smeared) stress and strain of mild steel bars, respectively; f_y and ε_y are the yield stress and strain of bare steel bars, respectively; E_s is the modulus of elasticity of steel bars and ε_s = ε_y (0.93 – 2B). B = parameter defined as $(1/\rho)(f_{cr}/f_y)^{1.5}$, where ρ is the reinforcement steel ratio, f_{cr} is the cracking strength of concrete, and f_y is the yield strength of bare steel bars.

When a steel bar is embedded in concrete and starts to yield at the cracks, the stresses in the steel bars between the cracks will be less than the yield stress at the cracks, because part of the tensile force is resisted by the concrete. Using the smeared crack concept, the steel stresses are then averaged along the steel bar traversing several cracks. The resulting average (or smeared) steel stress at first yield will obviously be less than the local yield stress of a bare bar at the cracks. As shown in Fig. 4, the smeared (or average) stress of the steel bar at initial yielding, f_n, is lower than the yield stress of the bare steel bars, f_y. The difference between f_n and f_y depends on the parameter B which is a function of the steel ratio, ρ, and the ratio f_{cr} / f_y.

COMPARISION OF FEM PREDICTIONS WITH TEST RESULTS

The normalized shear stress vs. total drift angle curves of the nine specimens are plotted in Fig. 5. The normalized shear stress is defined as the shear stress in MPa divided by the square root of the concrete cylinder compressive strength, also in MPa. The total drift angle includes both the shear drift angle and the flexural drift angle. The predicted curves are compared to both the positive envelope curves and the negative envelope curves obtained in the reversed cyclic loading. A comparison between the FEM predictions and the test observations shows that:

- The predicted curves of all nine specimens agree well with the test curves. The ultimate load of specimens increase with increasing axial gravity load.
- The initial cracks occur simultaneously in the frame and in the panel. The calculated initial cracking loads for all nine specimens are very close to the test values.
- Failure load analysis is an important part in the nonlinear finite element analysis. Three criteria were found to govern the failure loads.
  (1) Yielding of steel in a predominant portion of the panel (yielding in 70% of panel area);
  (2) Crushing of concrete in panel (ultimate concrete strain reaching $4\varepsilon_0$);
  (3) Sudden and significant changes of strains or displacements from previous steps or a change of sign from "+" to "-" or vice versa;

The loads at cracking of concrete, yielding of steel, crushing of concrete and large change of deformations are listed in Table 2. From these data it is obvious that the first criterion plays a major role in determining the failure load, because it governs the failures of four specimens FSW 13, 6, 5 and 11. In this steel yielding criterion, the "predominant portion of the panel" was found to be 70% of the panel area, i.e., when the yielding area reached 70% of the total panel area, a framed shear wall is considered to have reached its failure load. The second failure criterion is the concrete crushing, where the ultimate concrete strain is taken as $4\varepsilon_0$. This second criterion governs the failures of two specimens FSW 12, and 10. The third criterion concerns excessive deformations. This criterion governs the failures of three specimens FSW 8, 4 and 9. The actual failure load is the lowest value determined from these three criteria. A comparison between the FEM predicted failure loads and the experimental failure loads are listed in Table 3. The predicted failure loads are compared to both the positive and the negative maximum loads obtained in the reversed cyclic loading. It can be seen that the overall errors are about 5%.

CONCLUSIONS

1. A computer program FEAPRC was developed for specific application to concrete structures. FEAPRC incorporates a new set of smeared constitutive laws for concrete in compression, in tension, in shear, and for steel bars embedded in concrete.
2. The program FEAPRC was applied to analyze 9 framed shear walls, created by systematically varying the axial load ratio and the panel steel ratio. The predicted behavior agreed very well with the observed responses. In other words, FEAPRC can predict the trends created by these two variables.
3. In general, yielding of steel and crushing of concrete occur in the frame earlier than in the panel. However, failures are caused by the yielding of steel or crushing of concrete in the panels. The steel yielding criterion is defined as the yielding of steel in 70% of the panel area, and the concrete crushing criterion is found to be $4\varepsilon_0$.

Table 2 Loadings at cracking, steel yielding, concrete crushing and large changes

Spec-mens	Cracking (N)		Steel Yielding (N)			Concrete Crushing (N)		Large Changes of Deformations		Norm. Shear Stress
	Panel	Frame	Panel	>70% of Panel	Frame	Panel	Frame	Strains or Def.	Sign +, -	
FSW 13	200000	200000	440000	**490000** (80%)	440000	-	-	-	490000	0.80
FSW 8	252000	252000	490000	-	434000	-	-	**602000**	-	1.07
FSW 12	280000	280000	602000	-	504444	**700000**	448000	-	700000	1.14
FSW 6	200000	200000	405000	**445000** (78%)	425000	-	-	-	-	0.78
FSW 5	245000	245000	500500	**647500** (70%)	451500	647500	-	-	-	1.06
FSW 4	245000	245000	581000	-	525000	-	413000	**617400**	-	1.08
FSW 11	216000	216000	516000	**636000** (94%)	456000	636000	576000	-	-	1.04
FSW 9	280000	280000	732000	-	680000	-	584000	-	**732000**	1.27
FSW 10	315000	315000	729000	-	549000	**783000**	549000	-	783000	1.29

Table 3 Comparison of predicted and experimental failure loads

	Calculated Load(N)	EXPERIMENTAL LOAD(N)			Cal. Load / Pos. Exp. load	Cal. Load / Neg. Exp. load	Cal. Load / Ave. Exp. load
		Pos. Load	Neg. Load	Ave. Load			
FSW13	490,000	481314	−473307	477310	2%	4%	3%
FSW8	602,000	625886	−480869	553378	-4%	25%	9%
FSW12	700,000	694836	−657915	676375	1%	6%	3%
FSW6	445,000	438610	−463521	451065	1%	-4%	-1%
FSW5	647,500	651242	−613431	632336	-1%	6%	2%
FSW4	617,400	621438	−592079	606758	-1%	4%	2%
FSW11	636,000	592523	−579178	585851	7%	10%	9%
FSW9	732,000	754372	−707712	731042	-3%	3%	0%
FSW10	783,000	892344	−756669	824506	-12%	3%	-5%

ACKNOWLEDGMENTS

The research was supported by two grants CMS-9711084 and CMS-9713707 from the National Science Foundation. These generous supports are gratefully acknowledged.

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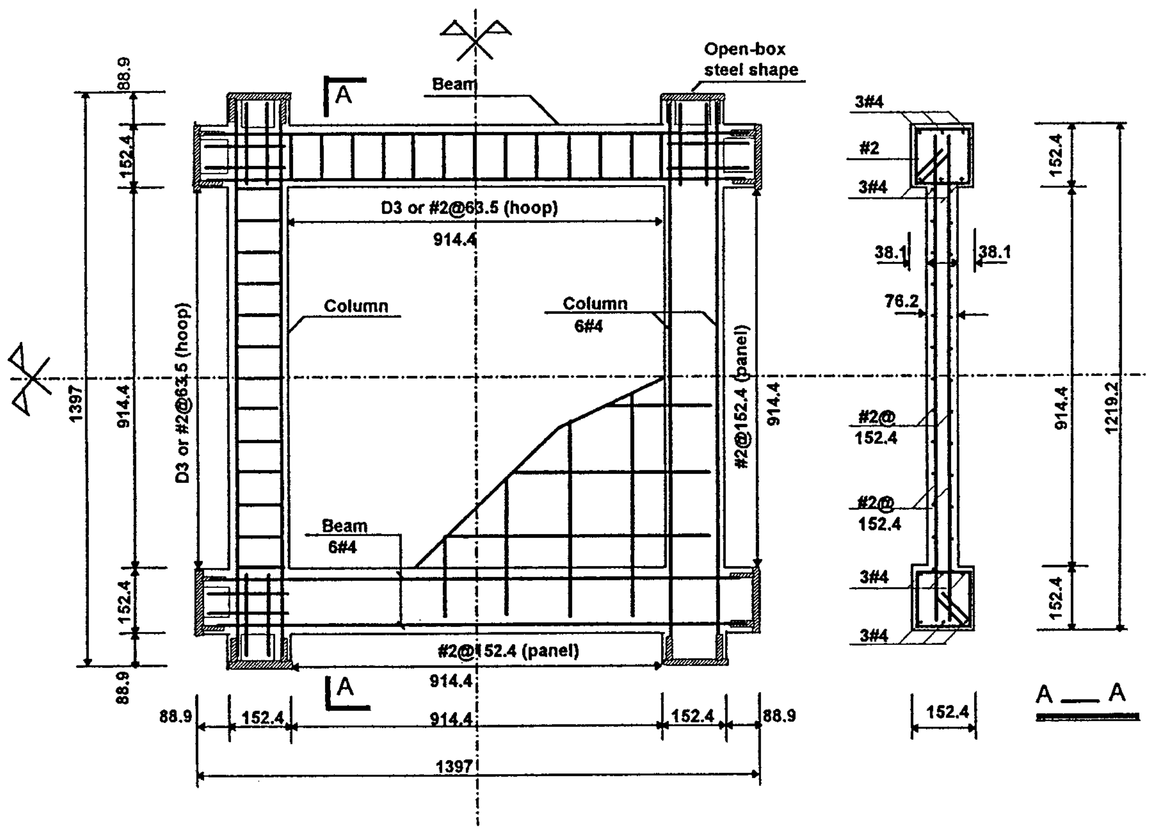

Fig. 1 Specimen Dimensions, Typical Steel Arrangement in Framed Shear Wall

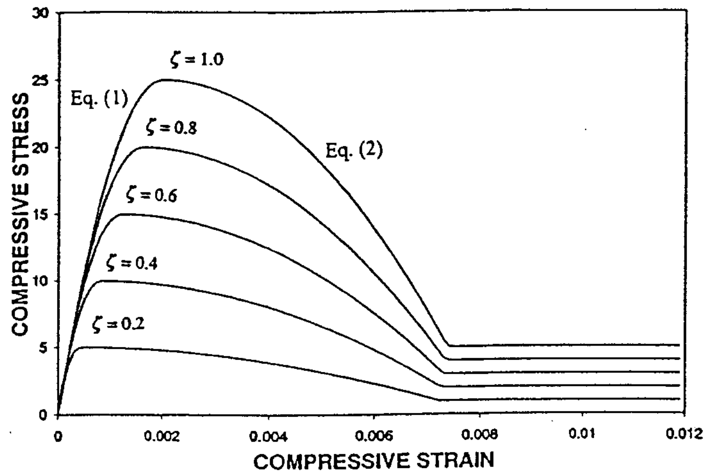

Fig. 2 Computational Softened Stress – Strain Curve of FA – STM Theory

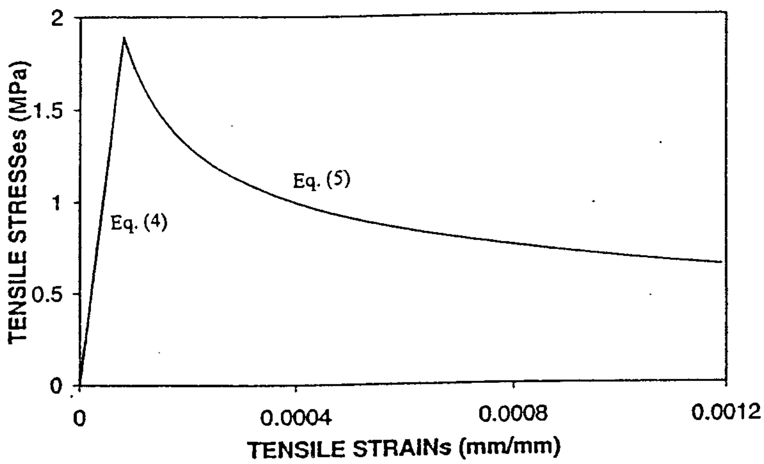

Fig. 3 Average Tensile Stress – Strain Curve of Concrete

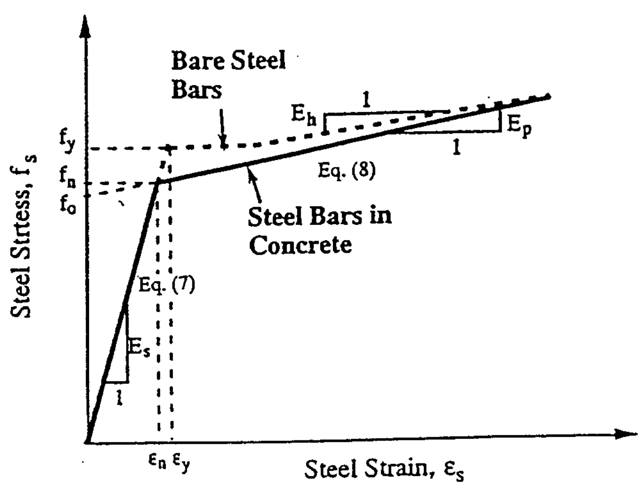

Fig. 4 Average Stress – Strain Curve of Steel Bars Embedded in Concrete

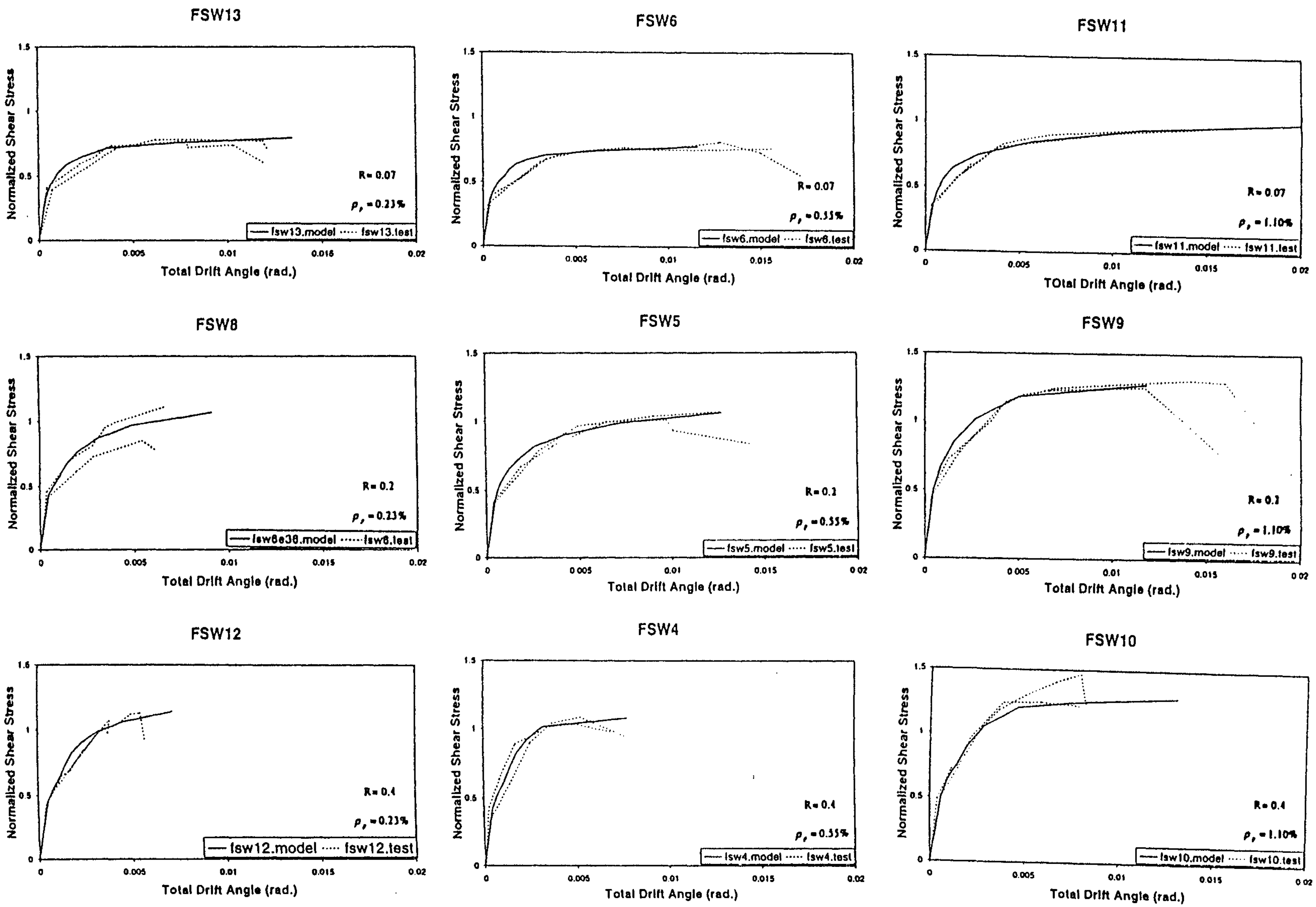

Fig. 5Normalized Shear Stress vs. Total drift Angle