Rocking Response of Unanchored Body to Base Excitation

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ABSTRACT

The rocking response of rigid bodies with rectangular footprint, freely standing on a horizontal rigid plane is studied analytically. Rigid bodies are subjected to simulated single component of horizontal earthquake time histories. The effect of the baseline correction, applied to simulated excitations, on the rocking response is first examined. The sensitiveness of rocking motion to the details of simulated earthquakes and the geometric properties of rigid bodies (i.e., slenderness ratio and size parameter) is investigated. Because of the demonstrated sensitivity of rocking response of rigid bodies to these factors, prediction of rocking stability must be made in the framework of probability theory. Therefore, using a large number of simulated earthquakes, the effects of duration and shape of intensity function of simulated earthquakes on the overturning probability of rigid bodies are next studied. In the case when a rigid body is placed on any floor of a building, the corresponding overturning probability is compared to that of a body placed on the ground. For this purpose, several shear frames with fundamental natural period of vibration ranging from 0.5 sec (stiff) to 2.0 sec (flexible) are employed. Finally, the viability of the energy balance equation, which was introduced by Housner in 1963 and widely used by the nuclear power industry to estimate the rocking stability of rigid bodies, is evaluated. It is found that the energy balance equation is robust. This paper also gives examples to show how this equation can be used.

1. INTRODUCTION

There are many structures/bodies that are unanchored placed on the ground or on the floors of a building. These include oil or water tanks, mechanical or electrical equipment, art objects, computers, furniture and scaffoldings.

When the base on which an unanchored body is placed moves, the body may respond in different modes. Shenton, in 1995[1], identified these modes as rest, slide, rock and slide-rock. The body may leave the base momentarily if a vertical base excitation is involved.

The rocking mode of response was first studied analytically by Housner in 1963[2]. In addition to a limited amount of laboratory experimental work, the problem has since been studied analytically and numerically by solving the equation of rocking motion of a rigid body. This has been done either deterministically or probabilistically. When the problem is studied probabilistically, it is carried out either by using an ensemble of real or simulated earthquakes or by the theory of random vibration. The literature is too numerous to cite and the interested reader is referred to Aydin[3].

In 1998, Shao[4], following the approach of Yim et. al[5], used both simulated and real earthquakes to study the rocking behavior of unanchored bodies. There were, however, a number of issues not addressed by Shao. In this study, we use simulated earthquakes to investigate the following:

1. the effect of baseline correction of simulated base motion on rocking response,
2. the sensitivity of rocking response to details of earthquake motion, the intensity of earthquake motion and the size of the body,
3. the effects of the duration of earthquake and the shape of the intensity function on the probability of overturning of an unanchored body,
4. the difference between the probabilities of overturning of an unanchored body when the body is placed on the ground and on the floors of a building.

A very important part of this study is to verify the validity of the energy balance equation proposed by Housner in 1963. The energy balance equation enables one to determine the level of excitation that would overturn a rocking rigid body with an approximately 50% probability. It is simple in form and has been used by engineers, especially those in nuclear industry, in dealing with unanchored bodies such as scaffoldings, ladders, trash cans, cabinets etc. The energy balance equation was derived using heuristic arguments and has not been verified either analytically or by experiment. With a large number of simulated earthquakes, we are able to show that the energy balance equation is indeed very robust.

2. ANALYTICAL MODEL

The body under consideration is a rigid homogenous rectangular block of mass M, height H and width B (see Fig. 1). The size of the body is measured by R and the parameter $\gamma=H/B$ is called the slenderness ratio. It has a rectangular footprint, and the base excitation is horizontal in the x direction such that its motion is confined to the x-y plane. There is sufficient friction between the body and the surface on which it rests so that no slippage occurs under base excitation. The equation of rocking motion of the body governing the tilting of the body, θ, can be found in Aydin[3]. It is highly nonlinear and, under arbitrary earthquake-like base excitation, can only be solved numerically. In solving the equation of motion, an additional quantity, the coefficient of restitution denoted by e, must be provided. In this study, it is assumed to be constant (=0.925). The computer program used to solve the equation of motion is provided by Professor Solomon Yim of Oregon State University.

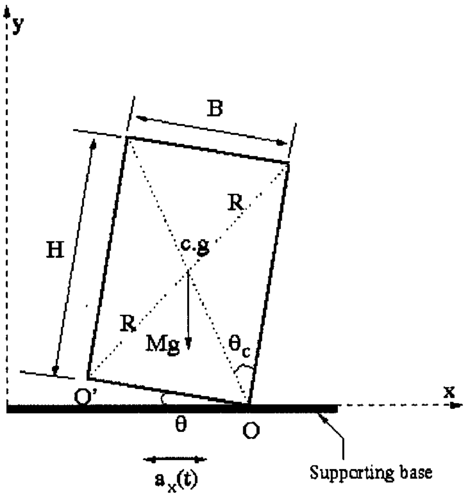

Fig.1 Rectangular rigid body with a response θ from the horizontal plane.

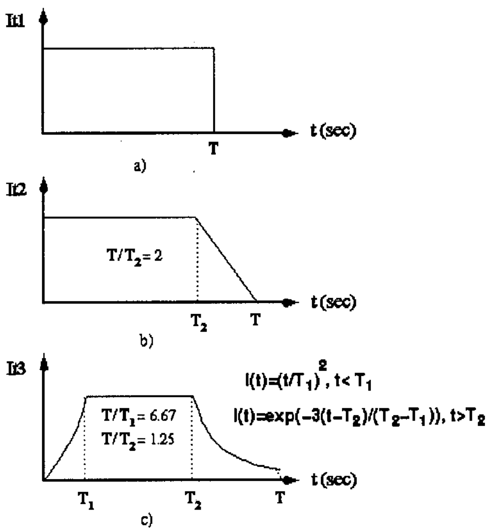

Fig.2 Intensity functions a) boxcar (rectangular), b) trapezoidal and c) compound.

3. SIMULATED EARTHQUAKES

The reason that simulated earthquakes are used in this study is because, to obtain statistically meaningful results, a large number (≥ 200) of earthquakes must be employed. The idea underlying the simulation of an earthquake is well documented. It consists essentially of representing the time function of ground acceleration as the sum of a large number of sine functions each with a random phase angle and whose amplitude is expressible in terms of the power spectral density function. To reflect the transient nature of a real earthquake ground motion, the above generated ground motion is multiplied by a time function referred to as the intensity function, which rises from zero to unity and, after remaining constant for a period of time, decays to zero. In this study, the intensity functions considered are the boxcar, trapezoidal and compound type functions (see Fig. 2).

A measure of the intensity of an earthquake time series is the peak acceleration (the maximum of the absolute value of the time function) denoted by $A_{gx}g$, g being the gravitational acceleration. When we deal with an ensemble of many earthquakes, the peak acceleration, also denoted by $A_{gx}g$, is the average of that of the individual acceleration time histories.

It has been common practice to represent the design ground acceleration by the design response spectrum. In order to simulate an earthquake, it is necessary to convert the design response spectrum to the power spectral density function, which is then used to generate a time history of ground acceleration. The manner in which such a conversion can be made has been well studied. The computer program used in this study is based on the work of Park[6]. The design spectrum used is that of Regulatory Guide 1.60[7], which has a damping ratio of 5% and A_{gx} of 1.22.

4. EFFECT OF BASELINE CORRECTION ON THE ROCKING RESPONSE

It is well known that the baseline of a real or simulated earthquake acceleration time history must be corrected[8]. It is done to filter out undesired long-period components, which cause the velocity and displacement time histories to have non-zero values at the end of the earthquake. The effect of baseline correction on the acceleration time history itself is imperceptibly small. Thus, it is of interest to find out how this small difference between the acceleration time histories with and without baseline correction would affect the rocking response of a rigid body. Using the simulation program and

Regulatory Guide 1.60 response spectrum, with a compound intensity function and a 20 sec duration, an earthquake is generated without correcting the baseline.

The intensity of this individual earthquake is specified by A_{gx}=0.6. The rocking responses of rigid bodies with a size parameter R=10.0 ft (3.05 m) and slenderness ratios γ=3.75, 4.0, 5.0 and 6.0 are shown in Fig. 3. It is seen that the response θ of the body increases with an increase in the slenderness ratio. This observation contradicts those of previous researchers such as Yim et. al[5] and Shao[4]. Fig. 4 shows the responses of the same rigid bodies as above subjected to the simulated earthquake with its baseline corrected. It is seen that, although the body with γ=6.0 does not overturn, the one with γ=5.0 does. In other words, the response of a rigid body does not necessarily increase with slenderness ratio. It is clear from this analysis that the rocking response of rigid bodies is affected by whether or not the baseline of the simulated earthquake is corrected. A paper by Shin[9] also states that the rocking response is sensitive to baseline correction. Because of this, all the simulated acceleration time histories used in the remaining parts of this paper are baseline corrected.

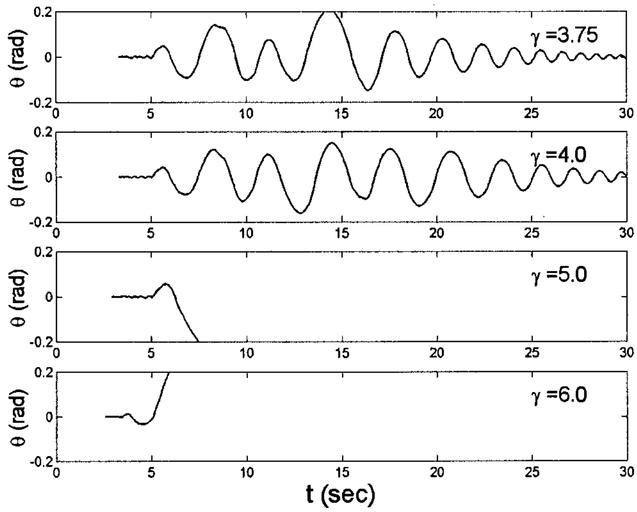

Fig.3 Responses of bodies with R=10.0 ft (3.05 m), e=0.925 subjected to baseline-uncorrected simulated earthquake (No.1) scaled to A_{gx}=0.6.

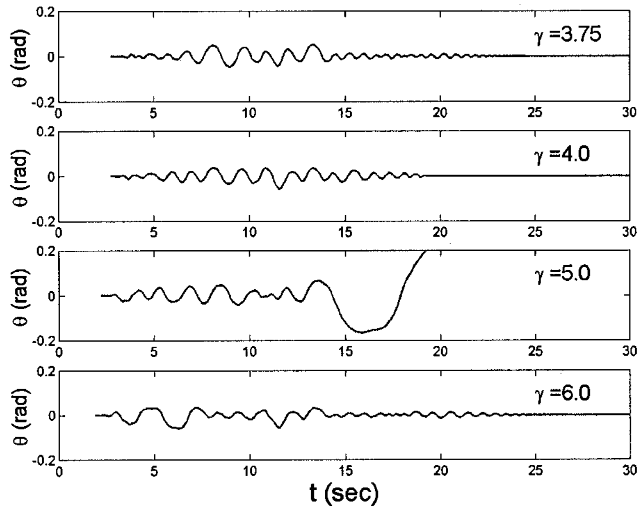

Fig.4 Responses of bodies with R=10.0 ft (3.05 m), e=0.925 subjected to baseline-corrected simulated earthquake (No.1) scaled to A_{gx}=0.6.

5. SENSITIVITY ANALYSIS OF THE ROCKING RESPONSE

The sensitivity of the rocking response to details of base excitation, intensity of earthquake motion α, size parameter R. It is already noted in the previous section that the slenderness ratio γ has no systematic effect on the rocking response of a rigid body.

To study how the details of base acceleration would affect the response of a body, 25 earthquakes are generated using the Regulatory Guide 1.60 response spectrum. Fig. 5 shows the influence of using three different members of the ensemble of 25 simulated earthquakes on the response θ. The rigid body is specified by R=12.0 ft (3.66 m) and γ=5.0. The intensity A_{gx} of each the three members (No.10, 15 and 23) of the ensemble is set at 0.5. The body overturns under the excitation of No.15 acceleration but remains stable under that of No.10 and 23 accelerations. It may be said, therefore, that the rocking response is sensitive to the details of base excitation.

Fig. 6 shows the effect of scaling factor α, defined as the value of A_{gx} of individual acceleration time history, on rocking response. A body with R=12.0 ft (3.66 m) and γ=5.0 is subjected to the No.25 simulated excitation with $\alpha=A_{gx}$=0.45, 0.60 and 0.75. It is observed that the body overturns when α=0.60 but remains stable when α=0.75.

Fig. 7 shows the effect of size parameter R. The rigid bodies with R=6.0, 10.0, 14.0 ft (1.83, 3.05, 4.27 m) and γ=5.0 are considered. The No.1 simulated earthquake with A_{gx}=0.5 is used as base excitation. The figure shows no systematic pattern on the response as R is varied.

It is due to this sensitivity of the rocking response that the following studies are carried out in the framework of probability. The rocking response of a body is studied using a large number of simulated base excitations and attention is focused on the probability of the event of overturning of the body.

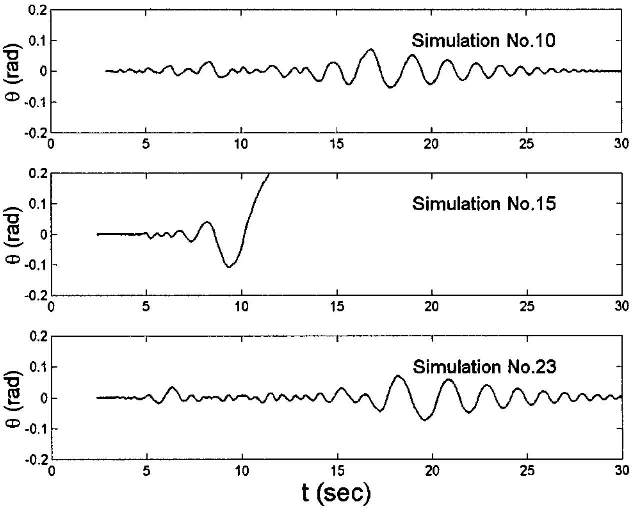

Fig.5 Responses of a body with R=12.0 ft (3.66 m), γ=5.0 and e=0.925 subjected to 3 simulated earthquakes scaled to A_{gx}=0.5.

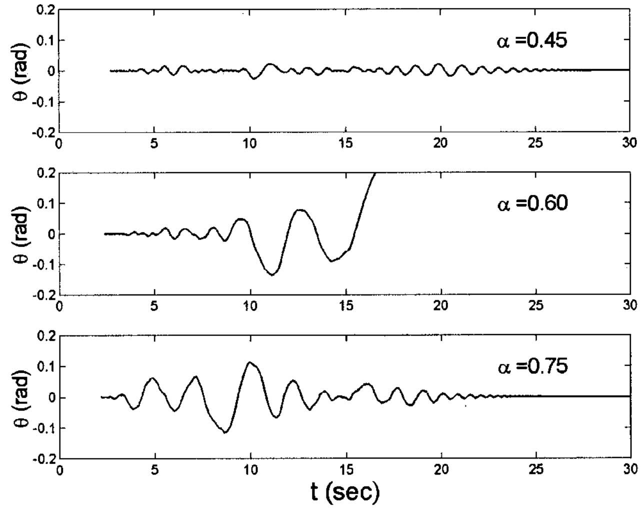

Fig.6 Responses of a body with R=12.0 ft (3.66 m), γ=5.0 and e=0.925 subjected to simulated earthquake No.25.

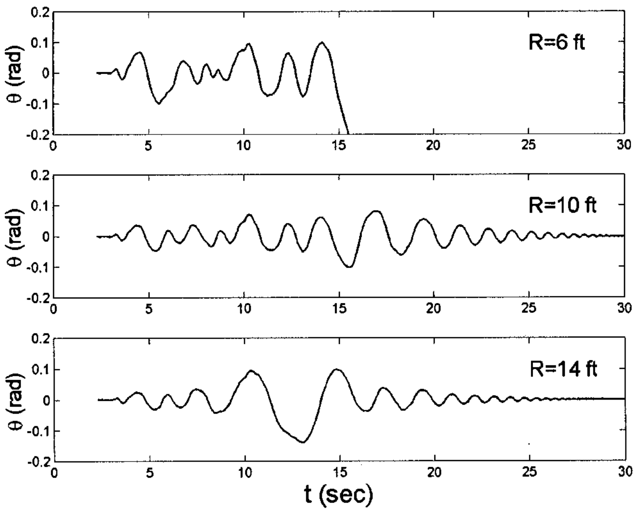

Fig.7 Responses of bodies with γ=5.0 and e=0.925 subjected to simulated earthquake No.1 scaled to A_{gx}=0.5 (1 ft=0.30 m).

6. THE EFFECT OF DURATION AND SHAPE OF INTENSITY FUNCTION ON THE ROCKING RESPONSE

An intensity function is needed in the earthquake simulation process to reflect the transient characteristics of a real earthquake. A variety of intensity functions have been proposed in the past. It is, therefore, useful to examine whether different intensity functions affect the rocking response of rigid bodies. Three commonly used intensity functions: boxcar (rectangular), trapezoidal and compound functions, identified as It1, It2 and It3 respectively, are investigated here (see Fig. 2). The duration of earthquakes is varied from 10 to 30 sec. To obtain statistically meaningful results, 500 earthquakes for each of the case are simulated. Fig. 8 gives the overturning probabilities versus A_{gx} computed using the boxcar intensity function with earthquake durations of 10, 15, 20, 25 and 30 sec for a body with R=4.0 ft (1.22 m) and γ=4.0. The figure shows that as duration increases, overturning probability also increases as expected but the difference between the overturning probability curves for the cases of 25 and 30 sec earthquake duration is rather small. The same conclusion is reached when computation is carried out for other body configurations using the other two intensity functions.

Fig. 9 shows the overturning probabilities for the same body (with R=4.0 ft (1.22 m), γ=4.0) with a duration of 15 sec using the three intensity functions It1, It2 and It3. This figure and those produced using the other body configurations show that the overturning probability of a rigid body is not appreciably affected by the choice of the intensity function.

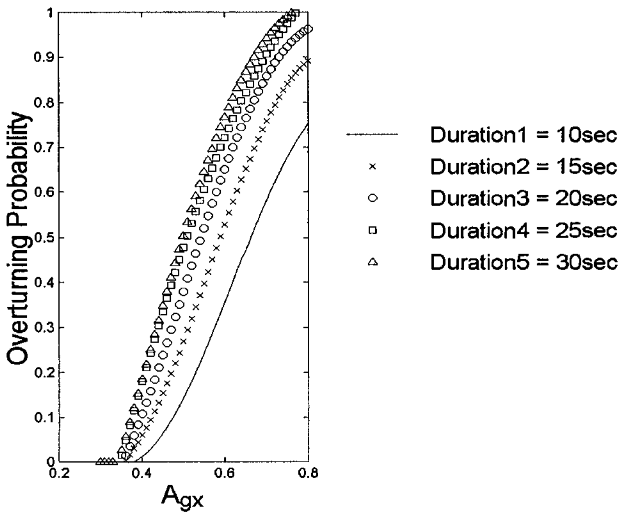

Fig.8 Overturning probabilities of a body with R=4.0 ft (1.22 m) and γ=4.0 (e=0.925), using the boxcar intensity function.

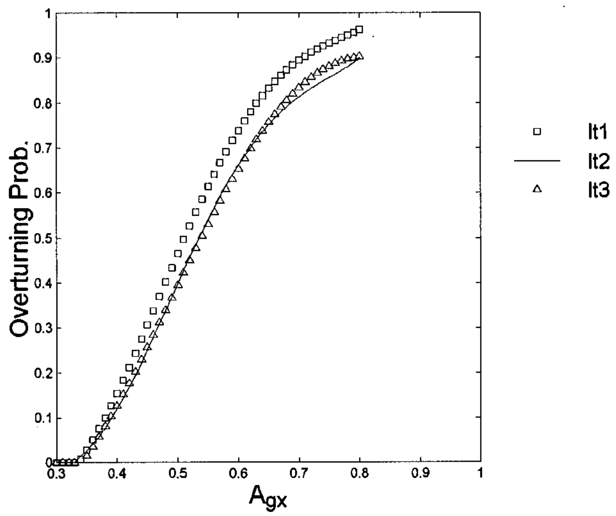

Fig.9 Overturning probabilities of a body with R=4.0 ft (1.22 m) and γ=4.0 (e=0.925), using a duration of 15 sec.

7. THE EFFECT OF FLOOR ACCELERATIONS ON THE ROCKING RESPONSE

A study is performed to determine how an unanchored body responds differently when placed on the ground and on the floors of a building. For this purpose, a 5-story shear frame of uniform floor mass and story stiffness (with a 5% modal damping) is used. It is expected that the accelerations on the floors would be amplified and the energy contained in various frequency bands would be altered. The second factor is investigated here, since it is already known, probabilistically, that the amplified earthquakes (i.e., greater intensity) increase the overturning probability of a rigid body. To compare the overturning probability of a rigid body on the ground and on the floors, the intensity A_{gx} of the floor acceleration time histories is scaled to be the same as that of the ground.

The uniform 5-story shear frame with a fundamental period of vibration T_n=2.0 sec is first considered. Six rigid bodies with R=4.0, 8.0, 12.0 ft (1.22, 2.44, 3.66 m) and γ=3.0, 4.0 are analyzed. Only the result for the body with R=4.0 ft (1.22 m) and γ=4.0 is presented. It is observed from Fig. 10 that the largest and the least overturning probabilities are obtained respectively when the body is placed on the fourth floor and on the ground.

It is felt that a stiffer shear frame should be considered since the value of T_n=2.0 sec is rather large for a typical 5-story building. Reducing the floor mass and increasing the story stiffness, a fundamental period of vibration T_n=0.5 sec is obtained. The same body considered earlier with R=4.0 ft (1.22 m) and γ=4.0 is placed on the floors of this shear frame. Fig. 11 shows that as the level of the floor on which the body is located increases, the overturning probability decreases. This trend is observed for all the rigid bodies (R=4.0, 8.0, 12.0 ft (1.22, 2.44, 3.66 m) and γ=3.0, 4.0) analyzed.

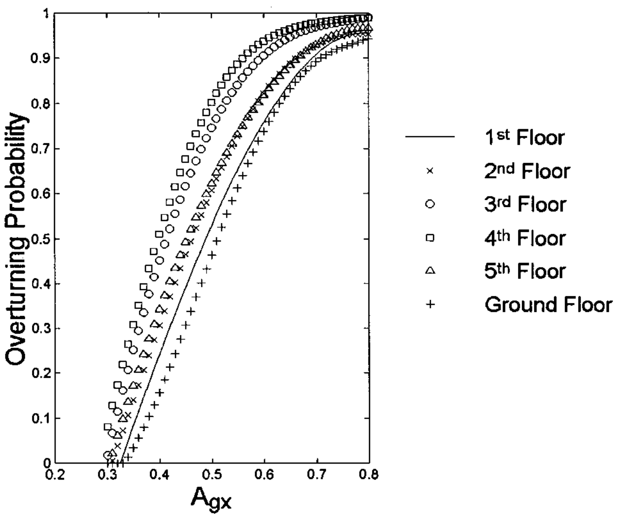

Fig.10 Overturning probabilities of body with R=4.0 ft (1.22 m) and γ=4.0 (e=0.925) placed on the shear frame with Tn=2.0 sec.

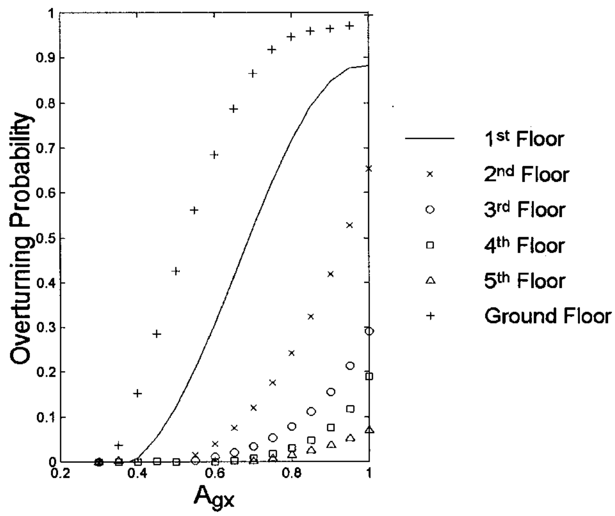

Fig.11 Overturning probabilities of body with R=4.0 ft (1.22 m) and γ=4.0 (e=0.925) placed on the shear frame with Tn=0.5 sec.

8. ENERGY BALANCE EQUATION

In 1963, Housner provided an equation for estimating the rocking stability of a rigid body. Roughly speaking, the energy required to overturn a body is equated with that input from the base to the body that is computed from the pseudo-velocity response spectrum S_v.

The energy balance equation takes the form

$$\frac{1}{2}\frac{W}{g}\frac{MR^2}{I_O}S_v^2\cos\theta_c^2 = W\Delta \tag{1}$$

in which W is the weight of the body, I_O=(4/3)MR^2 is the mass moment of inertia of the body about the point O. The quantity $\Delta=R(1-\cos\theta_c)$ is the vertical displacement of the center of mass of the body from the position $\theta=0$ to that when $\theta=\theta_c$, where $\theta_c=\tan^{-1}1/\gamma$.

From Eq. 1, the required value of S_v for overturning is

$$S_v = \frac{1}{\cos\theta_c}\sqrt{\frac{8}{3}gR(1-\cos\theta_c)} \tag{2}$$

Housner noted that when S_v satisfies Eq. 2, there is approximately 50% chance that the base motion will cause the body to overturn. He also noted that Eq. 1 was arrived at assuming that the ground acceleration is such that the average velocity response spectrum (undamped) is constant.

To verify the validity of this equation, 200 earthquakes with a compound intensity function and duration of 20 sec are produced from the response spectrum of Regulatory Guide 1.60. For a given body, say, with R=4.0 ft (1.22 m) and γ=2.0, a plot of overturning probabilities versus A_{gx} is constructed. From this plot, the A_{gx} value which corresponds to 50% probability of overturning (=1.02) is determined. The ordinates of 200 acceleration time histories are adjusted and the 200 pseudo-velocity response spectra (with a damping ratio of 5%) are computed, from which the average pseudo-velocity response spectrum is obtained (see Fig. 12). Since the pseudo-velocity response spectrum is not constant but varies with frequency, it is not possible to specify which value of S_v to use to compare it with the required S_v value given by Eq. 2. If we choose the maximum value of the average pseudo-velocity (=80.75 in/sec (205.11 cm/sec)) in Fig. 12 and compare it with required S_v value (=80.75 in/sec (205.11 cm/sec)) from Eq. 2, we see that these two values are exactly the same. The analysis is then extended to 72 rigid bodies with eight size parameter R values of 4.0 to 11.0 ft (1.22 to 3.35 m) (with 1 ft (0.30 m) increment) and nine slenderness ratio γ values of 2.0 to 6.0 (with 0.5 increment). Due to space limitation, the numerical results, given in [3], are not shown here. Suffice it to say that the agreement between the required S_v's and the maximum values of the average velocity response spectra is extremely good. In other words, the energy balance equation does what Housner claimed that it would do.

The applicability of the energy balance equation to a body placed on the floor of a building is similarly examined. For this purpose, the uniform 5-story shear frame (with fundamental natural period of vibration equal to 0.5 and 2.0 sec) is used. Details of the manner in which the study is done are not given here but may be found in [3]. The results show that the energy balance equation is equally applicable to a body placed on the floor of a building.

It should be mentioned that the average floor-velocity response spectra almost invariably have several spikes at certain frequencies such as those shown in Fig. 13. Using the maximum value of the spectrum may lead to erroneous or ambiguous conclusions. One should, however, realize that the frequency of oscillation of a rigid body tends to be rather small, especially when overturning is impending. This being the case, in this study, we have used the maximum values of S_v in the frequency range between 0.04 and 0.6 cps.

Having shown the robustness of the energy balance equation, the following examples are given to explain how it can be applied. The pseudo-velocity response spectra for the ground and for the floors (scaled to have the same A_{gx} value as that of the ground) of a building are first obtained. Using the Regulatory Guide 1.60 response spectrum with A_{gx}=0.6, such response spectra are given for the uniform 5-story shear frame with T_n=2.0 sec and 5% modal damping ratio in Fig. 13. Consider a body with R=4.0 ft (1.22 m) and γ=2.0 placed on the first floor of the shear frame. Fig. 13 shows that in the low frequency range (<0.6 cps), the maximum S_v value is approximately 45.00 in/sec (114.30 cm/sec). Eq. 2 shows that the required S_v is 80.75 in/sec (205.11 cm/sec), which is greater than 45.00 in/sec (114.30 cm/sec). Therefore, the body is safe against overturning. As a second example, consider a less stable body with R=4.0 ft (1.22 m) and γ=4.0 placed on the first floor. Eq. 2 gives 39.59 in/sec (100.56 cm/sec) for the value of the required S_v, which is smaller than 45.00 in/sec (114.30 cm/sec). The body will, therefore, overturn with a probability of more than 50%.

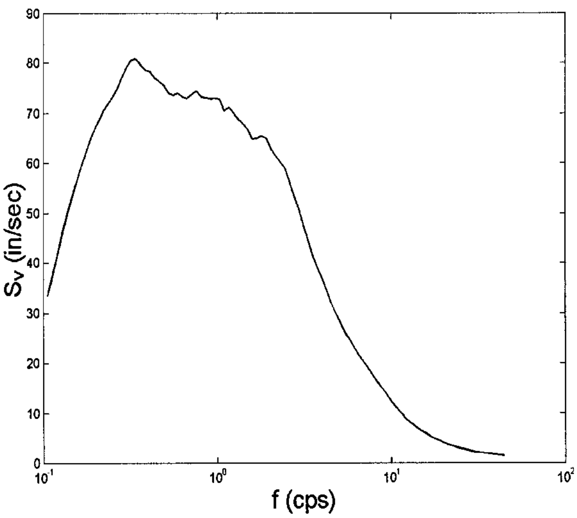

Fig.12 The average S_v spectrum corresponding to 50% overturning probability of a body with R=4.0 ft (1.22 m) and γ=2.0 (e=0.925), (1 in=2.54 cm).

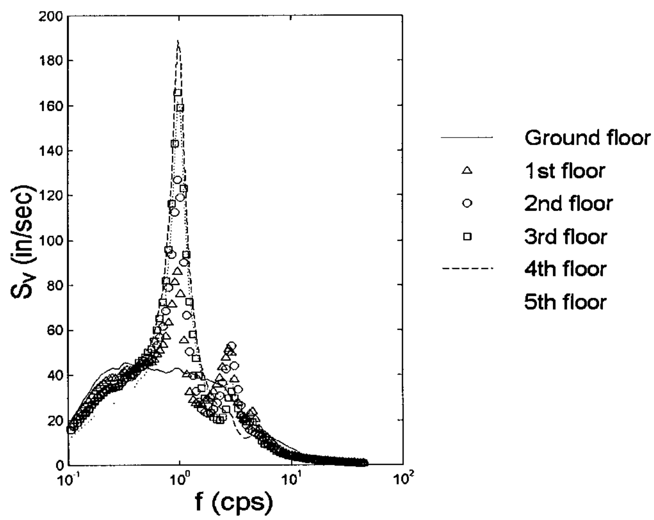

Fig.13 Pseudo-velocity response spectra of the floors of a shear frame with T_n=2.0 sec (1 in=2.54 cm).

9. CONCLUSIONS

By examining the rocking response of a body subjected to a single base excitation, it is found that

1. the rocking response is sensitive to baseline correction of the simulated acceleration time history,
2. the details of excitation time history, the intensity of an earthquake, the size of the body and its slenderness ratio, all affect the rocking response of a rigid body in a manner that no systematic pattern can be discerned,

However, when studied statistically, a clear trend of behavior emerges. It is found that,

3. as the peak acceleration increases, the overturning probability also increases,
4. as the duration of earthquake increases, the overturning probability increases,
5. as the duration of earthquake increases beyond 25 sec, the overturning probability does not increase appreciably,
6. the shape of intensity function has a little effect on the overturning probability,
7. in general, there is no clear correlation between the overturning probability of a body placed on the ground and on the floors of a building,
8. if the structure is stiff with natural fundamental period less than 0.5 sec, say, the overturning probability of a body placed on the ground tends to be larger than that placed on the floors of the structure,
9. Housner's energy balance equation has been verified: it is robust, easy to use, and can be applied to assess the overturning potential of a body placed on the ground and on the floors of a building.

In closing, it is emphasized that the work reported in this paper necessarily has its limitations. This stems from the fact that the model of the body considered is rather simple: it is rigid, homogeneous, rectangular and has a rectangular footprint. Also, the base excitations are horizontal and one-dimensional and are generated by simulation using a specific intensity function (and duration) and based on a specific response spectrum. The building model considered is a simple shear frame with uniform story mass and stiffness.

Using the method adopted in this study, the models of the body, the ground motion and the structure can all be modified.

10. ACKNOWLEGMENTS

This paper is based on the first author's Ph.D. dissertation. The support of the Department of Civil Engineering, the Center of Nuclear Power Plant Structures, Equipment and Piping of North Carolina State University and the Ministry of Education of Turkey is gratefully acknowledged.

REFERENCES

1. Shenton III, H. W., "Criteria for Initiation of Slide, Rock and Slide-Rock Modes", *Journal of Engineering Mechanics*, American Society of Civil Engineers, Vol. 122, No. 7, 1995, pp. 690-693.
2. Housner, G. W., "The Behavior of Inverted Pendulum Structures During Earthquakes", *Bulletin of Seismological Society of America*, Vol. 53, No. 2, 1963, pp. 403-417.
3. Aydin, K., "The Rocking Response of an Unanchored Body Subjected to Simulated Excitation", Ph.D. Dissertation, North Carolina State University, Raleigh, North Carolina, 2000.
4. Shao, Y., "Seismic Response of Unanchored Objects", Ph.D. Dissertation, North Carolina State University, Raleigh, North Carolina, 1998.
5. Yim, C-S., Chopra, A. K., and Penzien, J., "Rocking Response of Rigid Blocks to Earthquakes", *Earthquake Engineering and Structural Dynamics*, Vol. 8, 1980, pp. 565-587.
6. Park, Y. J., "New conversion method from response spectrum to PSD functions", *Journal of Engineering Mechanics*, American Society of Civil Engineers, Vol. 121, No. 12, 1995, pp. 1391-1392.
7. Regulatory Guide 1.60, Revision 1, US Atomic Energy Commission, Washington, D.C., 1973.
8. Jennings, P.C., Housner, G.W., and Tsai, N.C., "Simulated Earthquake Motions", EERL, California Institute of Technology, Pasadena, California, 1968.
9. Shin, T. M., "Effect of Input Baseline Correction on Sliding and Tipping", Transactions of the 15th Conference Structural Mechanics in Reactor Technology (SmiRT-15), 1999, pp. 249-256.