Vibration Control of Irregular Building-MTMD Systems Considering Soil-Structure Interaction Effect

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ABSTRACT

This paper deals with the applicability of multiple tuned mass dampers (MTMD) on the vibration control of irregular buildings modelled as torsionally coupled structures due to base motions considering the soil-structure interaction (SSI) effect. An efficient modal analysis methodology is used to systematically assess the combined soil-structure interaction and torsional coupling effects on asymmetric buildings. This method is implemented in the frequency domain to accurately incorporate the frequency-dependent foundation impedance functions. The performance index of MTMD is established based on the foundation-induced building floor motions with and without the installation of MTMDs. Unlike the traditional MTMD design criteria, the frequency ratio of each MTMD substructure to the controlled structural frequency is independent, in this paper, so that the MTMD with the optimal parameters can actually flatten the transfer functions of building responses. Numerical verifications show that the increase of height-to-base ratio of an irregular building and the decrease of relative stiffness of soil to structure generally amplify both SSI and MTMD detuning effect, especially for a building with highly torsionally coupled effect. With appropriately enlarging the frequency spacing of the optimal MTMDs, the detuning effect can be reduced. Moreover, the results of numerical investigations also show that the MTMD is more effective than single TMD as the SSI effect is significant.

INTRODUCTION

Due to recent intensive analytical and experimental research, vibration control in structures using Passive Tuned Mass Dampers (PTMDs) is gaining more acceptance not only in the design of new structures and components but also in the retrofit of existing structures to enhance their reliability against winds, earthquakes and human activities [1]. Since 1971, lots of PTMDs have been successfully installed in high-rise buildings and towers. Most of these retrofits were reported to be able to significantly reduce structural dynamic responses.

Basically, a PTMD is a device consisting of a mass connected to structures using a spring and a viscous damper. The PTMD damping effect depends upon the fact that the PTMD response delays the main structural response by a phase angle of 90°, so that the elastic force transmitted by the PTMD acts like a viscous force on the main structure. This condition will not occur unless the PTMD frequency is tuned to the frequency of the main structure and the excitation has this frequency content. Therefore, the structural property information is very essential for the optimum design of a PTMD. In previous studies about PTMDs, most of the researchers assumed that the controlled structural base is fixed, which is accurate only for structures built on rocks. In fact, many buildings are constructed on soft medium where the soil-structure interaction (SSI) effect may be significant. It is well known that the strong SSI effect would significantly modify the dynamic characteristics of structures such as frequencies, damping ratios and mode shapes [2-4]. Several researchers studied the SSI effect on the performance of PTMD for planar buildings subjected to wind and earthquake excitations [5-6]. They concluded that the PTMD effectiveness would rapidly decrease as the soil medium gets softer due to the increasing damping of the soil-structure system from soil material hysteresis and radiation effect.

Since the SSI effect is generally difficult to be assessed accurately, the PTMD detuning effect will occur because the PTMD does not tune to the right frequency. To solve the problem, using MTMD is one of the promising solutions.

The MTMD is a dynamic vibration control device that contains several parallel SDOF substructures. Each substructure has its own mass, damping ratio, and natural frequency. The original idea behind MTMD is to reduce the detuning effect through appropriately distributing the frequencies [7-9]. The objective of this paper is to investigate the influence of SSI effect on the performance of MTMD to suppress the excessive vibration of torsionally-coupled buildings. An appropriate set of MTMD parameters for considering both torsional coupling and SSI systems is presented. To compare the vibration control effectiveness between MTMD and single PTMD, the response spectra for a real earthquake are also illustrated to ensure the benefit and reliability of MTMD.

SYSTEM MODEL AND DYNAMIC EQUATIONS OF MOTION

The system model considered in this study, as shown in Figure 1, consists of a rigid floor of mass m, a rigid foundation of mass m_b, and axially inextensible columns of height h. The MTMD with p numbers of parallel SDOF systems is installed on the floor moving in the x-direction. Each SDOF system contains a mass m_{sk} which connects with the floor by a damper c_{s_k} and a spring k_{sk} and is located at a distance of d_{sk} from the x-axis, where $k=1,2,\ldots,p$. A uni-directional horizontal ground acceleration along the x-direction, $\ddot{x}_g$, is considered. One way eccentricity denoted by e along the y-direction as shown in Figure 1(b) is assumed. In addition, proportional viscous damping is assumed for the building such that the superstructure possesses classical normal modes.

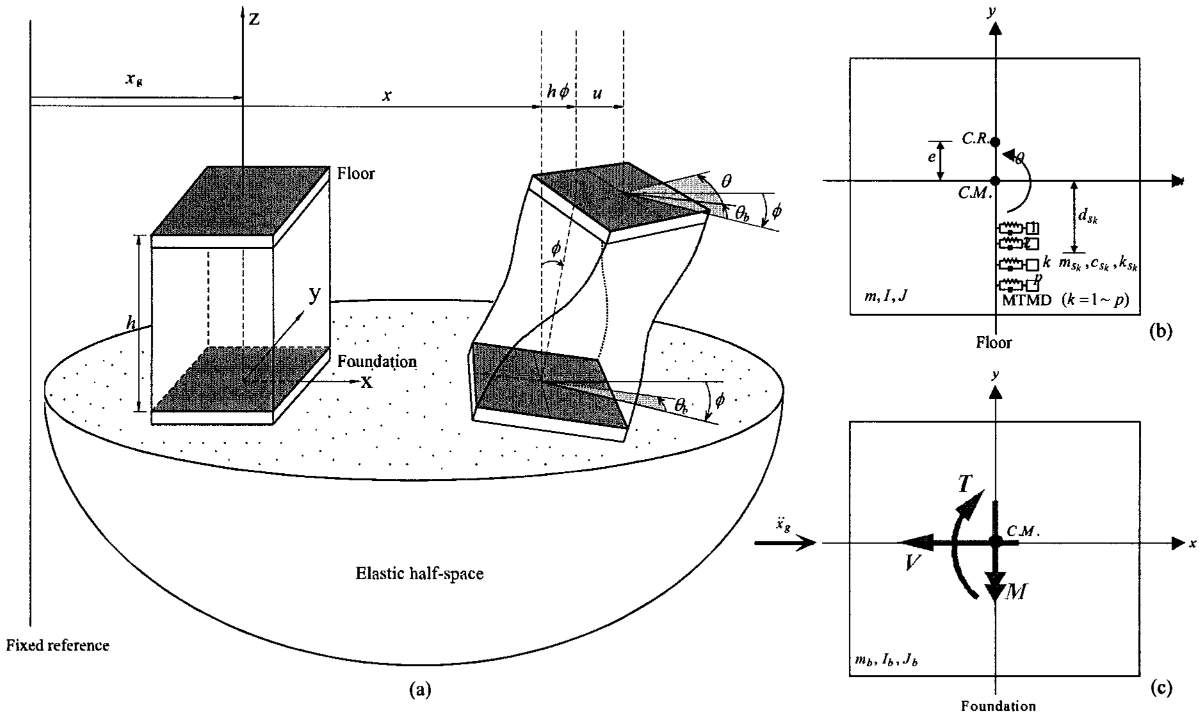

Figure 1. Irregular building-MTMD-soil interaction model

The soil is characterized by its mass density, ρ, shear velocity, v_s, and Poisson's ratio, υ. The dynamic behavior of the investigated system can be completely described by the following five degrees of freedom: u and θ represent horizontal translation and twist of the floor with respect to the foundation; x and θ_b denote horizontal translation and twist of the foundation; and ϕ is the rocking about the y-axis of the whole building. Applying the substructure method conventionally adopted in the SSI analysis, the response of the building-MTMD subsystem can be solved by using the interaction forces, the horizontal shear V, the overturning moment M, and the torque T, developed at the foundation-soil interface to replace the soil subsystem. Since the soil parameters are generally difficult to be accurately estimated, it is not appropriate to design the MTMD for an uncertain SSI system. In this paper, the MTMD is treated as a control device to alter the characteristics of the superstructure system. The undamped equations of motion for the building-MTMD superstructure system can be expressed as

$$m(\ddot{u} + \ddot{x} + h\ddot{\phi}) + k_u(u - e\theta) = \sum_{k=1}^{p}(c_{s_k}\dot{v}_{s_k} + k_{s_k}v_{s_k}) \quad \text{(1a)}$$

$$J(\ddot{\theta} + \ddot{\theta}_b) + k_\theta\theta - k_u e(u - e\theta) = \sum_{k=1}^{p}(c_{s_k}\dot{v}_{s_k} + k_{s_k}v_{s_k})e_{s_k} \quad \text{(1b)}$$

$$m_{s_k}[\ddot{x} + \ddot{u} + h\ddot{\phi} + \ddot{v}_{s_k} + e_{s_k}(\ddot{\theta}_b + \ddot{\theta})] + c_{s_k}\dot{v}_{s_k} + k_{s_k}v_{s_k} = 0 \quad (k = 1,2,\ldots,p) \quad \text{(1c)}$$

where k_u and k_θ are the lateral and torsional story stiffness; J and J_b are the mass polar moments of inertia about the z-axis of the floor and the foundation, respectively. Defining $J = mr^2$ where r = radius of gyration of the floor; $\lambda_e = e/r$; $\lambda_{s_k} = d_{s_k}/r$; $\omega_u = \sqrt{k_u/m}$; $\omega_\theta = \sqrt{k_\theta/J}$; $\omega_{sk} = \sqrt{k_{sk}/m_{sk}}$; $\xi_{sk} = c_{sk}/2\omega_{sk}m_{sk}$, $\rho_{s_k} = m_{s_k}/m$, $u_\theta = r\theta$, $x_\phi = h\phi$ and $x_\theta = r\theta_b$, and applying structural damping, equations (1a) to (1c) can be rearranged in matrix form as

$$\begin{bmatrix}\mathbf{M}_b & \mathbf{0}\\ \mathbf{M}_{sb} & \mathbf{M}_s\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{U}}_b\\ \ddot{\mathbf{U}}_s\end{Bmatrix} + \begin{bmatrix}\mathbf{C}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_s\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{U}}_b\\ \dot{\mathbf{U}}_s\end{Bmatrix} + \begin{bmatrix}\mathbf{K}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{K}_s\end{bmatrix}\begin{Bmatrix}\mathbf{U}_b\\ \mathbf{U}_s\end{Bmatrix} = \begin{Bmatrix}\mathbf{F}_b^{\mathrm{MTMD}}\\ \mathbf{0}\end{Bmatrix} + \begin{Bmatrix}-\mathbf{r}_b\\ -\mathbf{r}_s\end{Bmatrix}\ddot{\mathbf{X}}_b \quad \text{(2)}$$

where $\mathbf{M}_b$, $\mathbf{C}_b$ and $\mathbf{K}_b$ are 2×2 mass, damping and stiffness matrices of building, respectively. And

$$\mathbf{M}_s = I$$

$$\mathbf{C}_s = diag.(2\xi_{sk}\omega_{sk})$$

$$\mathbf{K}_s = diag.(\omega_{sk}^2)$$

are $p\times p$ mass, damping and stiffness diagonal matrices of MTMD system. Further, $\mathbf{M}_{sb}$ and $\mathbf{r}_s$ are related to λ_{sk}. $\mathbf{U}_b^{\mathrm{T}} = \{u \quad u_\theta\}$, $\mathbf{U}_s^{\mathrm{T}} = \{v_{s1} \quad v_{s2} \quad \ldots \quad v_{s_p}\}$ and $\ddot{\mathbf{X}}_b^T = \{\ddot{x} \quad \ddot{x}_\phi \quad \ddot{x}_\theta\}$ are the building displacement vectors, MTMD stroke vector and the foundation excitation vector respectively. Substituting $\mathbf{U}_b = \mathbf{\Phi q}$ for equation (2) where $\mathbf{\Phi}$ is the 2×2 mode shape matrix of the fixed-base building and $\mathbf{Z}$ is the 2×1 modal displacement vector and multiplying $\mathbf{\Phi}^{\mathrm{T}}$ to the first row of equation (2), it becomes

$$\begin{bmatrix}\mathbf{M}_b^* & \mathbf{0}\\ \mathbf{M}_{sb}^* & \mathbf{M}_s\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{q}}(t)\\ \ddot{\mathbf{U}}_s(t)\end{Bmatrix} + \begin{bmatrix}\mathbf{C}_b^* & \mathbf{C}_{bs}^*\\ \mathbf{0} & \mathbf{C}_s\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{q}}(t)\\ \dot{\mathbf{U}}_s(t)\end{Bmatrix} + \begin{bmatrix}\mathbf{K}_b^* & \mathbf{K}_{bs}^*\\ \mathbf{0} & \mathbf{K}_s\end{bmatrix}\begin{Bmatrix}\mathbf{q}(t)\\ \mathbf{U}_s(t)\end{Bmatrix} = \begin{Bmatrix}-\mathbf{\Gamma}_b\\ -\mathbf{r}_s\end{Bmatrix}\ddot{\mathbf{X}}_b(t) \quad \text{(3)}$$

Considering the orthogonality of $\mathbf{\Phi}$ and assuming proportional damping matrix, one can obtain $\mathbf{M}_b^* = diag.[1 \quad 1]$, $\mathbf{C}_b^* = diag.[2\xi_1\omega_1 \quad 2\xi_2\omega_2]$ and $\mathbf{K}_b^* = diag.[\omega_1^2 \quad \omega_2^2]$, where ξ_j and ω_j are the jth modal damping ratio and modal frequency of the building.

MTMD PERFORMANCE INDEX AND OPTIMAL PARAMETER DESIGN

Taking Fourier Transform for equation (3), the modal structural responses, $\mathbf{q}(\omega)$, can be extracted and take the form as $\mathbf{q}(\omega) = [H_{pp}(\omega)\mathbf{\Gamma}_p + H_{ps}(\omega)\mathbf{\Gamma}_s]\ddot{\mathbf{X}}_b(\omega)$ or

$$\begin{Bmatrix}q_1(\omega)\\ q_2(\omega)\end{Bmatrix} = \begin{bmatrix}H_{q_1\ddot{x}}(\omega) & H_{q_1\ddot{x}_\phi}(\omega) & H_{q_1\ddot{x}_\theta}(\omega)\\ H_{q_2\ddot{x}}(\omega) & H_{q_2\ddot{x}_\phi}(\omega) & H_{q_2\ddot{x}_\theta}(\omega)\end{bmatrix}\begin{Bmatrix}\ddot{X}(\omega)\\ \ddot{X}_\phi(\omega)\\ \ddot{X}_\theta(\omega)\end{Bmatrix} \quad \text{(4)}$$

In equation (4), if the amplitude of the transfer functions $H_{q_j\ddot{x}_i}(\omega)$ decreases, the structural modal response $q_j(\omega)$ can be reduced. It is well known that the frequency content of an earthquake excitation is generally wide-banded. Most of the dominant frequencies of building are smaller than the cutting-off frequency of the excitation spectrum. The mean square response which is related to the area of the transfer function becomes important. Therefore, the MTMD performance index, R_j, is defined as

$$R_j = \frac{\int_0^\infty \left| H_{q_j \ddot{x}_i}(\omega) \right|^2_{\text{MTMD}} d\omega}{\int_0^\infty \left| H_{q_j \ddot{x}_i}(\omega) \right|^2_{\text{NOMTMD}} d\omega} \tag{5}$$

The selection of $H_{q_j \ddot{x}_i}(\omega)$ is dependent upon the MTMD control goal. In equations (5), R_j is recognized as a function of structural parameters: ξ_1, ξ_2 and Φ, which should be known in prior, and the MTMD parameters: ρ_{sk}, ξ_{sk}, r_{fk} (the frequency ratio of the kth substructure to the controlled mode of the structure) and λ_{sk}. The MTMD mass ratio is assigned based on construction costs and structural capacity considerations before the MTMD design. Moreover, assuming that each MTMD substructure has the same mass ratio and damping ratio, ξ_{s0}, and they are uniformly distributed at the central location ratio λ_{s0} with known spacing, the optimal set of MTMD parameters, $r_{f1}, r_{f2}, \ldots, r_{fp}, \xi_{s0}$ and λ_{s0} can be obtained through solving the following simultaneous equations

$$\frac{\partial R_j}{\partial r_{f_1}} = \frac{\partial R_j}{\partial r_{f_2}} = \ldots = \frac{\partial R_j}{\partial r_{f_p}} = 0, \quad \frac{\partial R_j}{\partial \lambda_{s0}} = 0, \quad \frac{\partial R_j}{\partial \xi_{s0}} = 0 \tag{6}$$

OPTIMAL MTMD LOCATION

Theoretically, the MTMD optimal central location-ratio $(\lambda_{s0})_{\text{opt}}$ is found from equation (6). However, in fact, the primary structural mode-shape functions contain some information for determining $(\lambda_{s0})_{\text{opt}}$. From previous studies, it is well known that the optimal PTMD location is at the position where the mode-shape value of the controlled mode is maximal. Moreover, as stated earlier, one set of MTMD is used for reducing one of the structural modes. As the high structural modes are significant, another set of MTMD must be installed. In this situation, the MTMD location should be carefully determined to avoid interaction among the MTMDs and unexpected amplification for the uncontrolled modes. From this point of view, the optimal MTMD location should satisfy the following both conditions as far as possible: (1) at the position where the mode-shape value of the controlled mode is maximum (2) at the position where the mode-shape value of the uncontrolled mode is minimum. Therefore, the optimal MTMD central location for controlling the first mode should satisfy the equation, $\phi_{12} + \lambda_{s0}\phi_{22} = 0$, or

$$(\lambda_{s0})_{1\text{st}} = -\phi_{12}/\phi_{22}\,. \tag{7a}$$

Similarly, the optimal MTMD central location-ratio for controlling the second mode then satisfies the equation, $\phi_{11} + \lambda_{s0}\phi_{21} = 0$, or

$$(\lambda_{s0})_{2\text{nd}} = -\phi_{11}/\phi_{21}\,. \tag{7b}$$

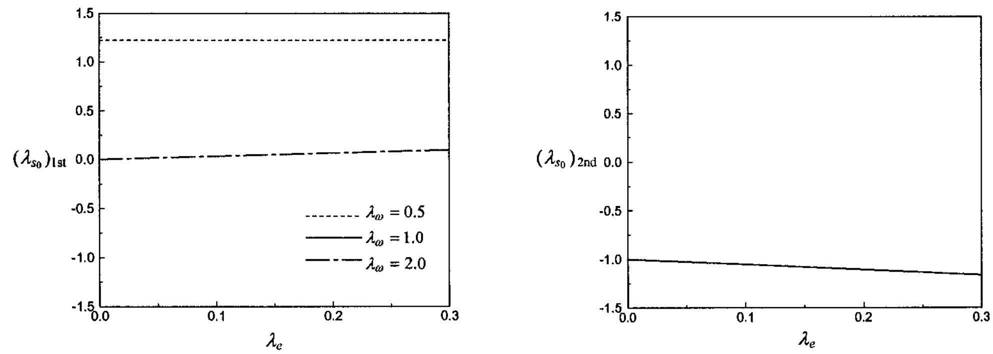

Figure 2. Optimal MTMD planar location for an irregular building with various λ_e and λ_ω.

Figure 2 shows the optimal MTMD location for three conditions. When $\lambda_\omega (= \omega_\theta / \omega_u) >> 1$, the translational stiffness is weaker than the torsional stiffness so that the translational motion dominates the first mode. Only one set of MTMD controlling the first mode located at the vicinity of the center of the floor mass is required. When $\lambda_\omega << 1$, one set of MTMD controlling the torsional motion (the first mode) located at the vicinity of the positive floor edge is required. When $\lambda_\omega = 1$, translational and torsional modes are equally important. Two sets of MTMD located respectively near the vicinity of $\lambda_{s0} = 1.0$ (which is at the opposite side of the center of floor rigidity) and $\lambda_{s0} = -1.0$ (which is at the same side as the center of floor rigidity) are required.

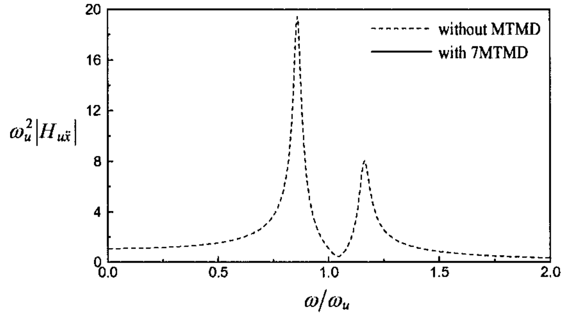

Figure 3. Transfer function of an irregular building with and without MTMD

OPTIMAL MASS-DISTRIBUTION RATIO

Figure 3 shows the displacement transfer function of an irregular building with high torsion-coupling effect ($\lambda_\omega = 1$, $\lambda_e = 3$). To control both the first and second modal responses simultaneously, the MTMD mass should be divided into two appropriate parts and distributed into two vibrational modes. The ratio of MTMD mass for second mode to that for the first mode, which makes the structural response minimum, is called the optimal mass-distribution ratio. To illustrate this concept, the building described in this study with a large eccentricity-ratio $\lambda_e = 0.3$, and two PTMDs with a total mass ratio equaling to 2% are used as an example. The mean-square-response ratios $R_{u\ddot{x}}$ and $R_{u\theta\ddot{x}}$ which are defined as

$$R_{u\ddot{x}} = \frac{\int_0^\infty |H_{u\ddot{x}}(\omega)|^2_{\text{MTMD}} d\omega}{\int_0^\infty |H_{u\ddot{x}}(\omega)|^2_{\text{NOMTMD}} d\omega} \quad \text{and} \quad R_{u\theta\ddot{x}} = \frac{\int_0^\infty |H_{u\theta\ddot{x}}(\omega)|^2_{\text{MTMD}} d\omega}{\int_0^\infty |H_{u\theta\ddot{x}}(\omega)|^2_{\text{NOMTMD}} d\omega} \quad (8)$$

with λ_ω=1.0 and 2.0 and various mass-distribution ratio $\rho_{s,2}/\rho_{s,1}$ (where $\rho_{s,j}$ means the MTMD mass ratio for controlling the *j*th mode)are plotted in Figure 4.

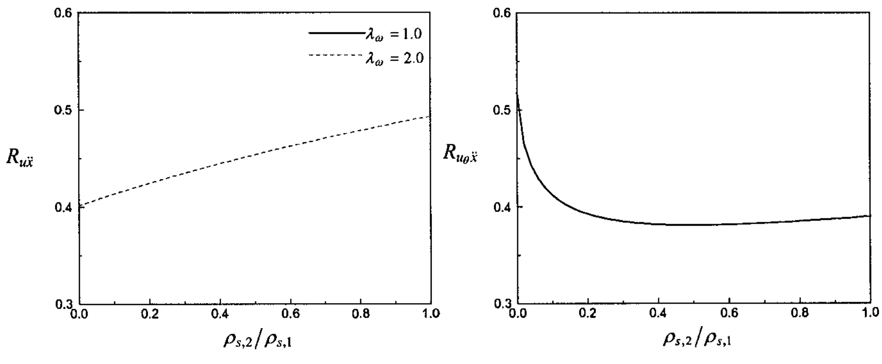

Figure 4. Mean-square-response ratios $R_{u\ddot{x}}$ and $R_{u\theta\ddot{x}}$ with different λ_ω for various MTMD mass-distribution ratios

Observing the $R_{u\ddot{x}}$ curves, it is found that the optimal $\rho_{s,2}/\rho_{s,1}$ is zero as λ_{ω}=2.0. It indicates that it is beneficial to use the total MTMD mass to control the first modal response. This is reasonable because the torsional stiffness is rigid and the translational motion dominates the first mode. Moreover, for a building with $\lambda_e = 0.3$ and $\lambda_{\omega} = 1.0$, both the first and second modes are important. Figure 4(a) shows that the optimal $\rho_{s,2}/\rho_{s,1}$ which makes $R_{u\ddot{x}}$ minimum is about 0.26. From Figure 4(b), we found that this is close to the optimal mass-distribution ratio for minimizing $R_{u\theta\ddot{x}}$.

MTMD VIBRATION CONTROL EFFECTIVENESS CONSIDERING SSI EFFECT

In this paper, the methodology developd Wu and the authors[10] is employed to evaluate the building responses. The displacement transfer functions for two floor motions of a building with $\lambda_{\omega} = 1$ and $\lambda_e = 0.3$ without and with optimal 7MTMD (p = 7) of 2% mass ratio are calculated. Two conditions, only one set of 7MTMD designed for controlling the first mode (Case 1) and two sets of 7MTMD controlling both the first and the second modes (Case 2), are considered. For general buildings ($\lambda_h = h/r$=3) and slender buildings ($\lambda_h = 5$), the mean-square-response ratios $R_{u\ddot{x}_g}$ under ground excitation $\ddot{x}_g$, are illustrated in Figure 5 against σ ranging from 0.5 to 5.0. Here, the parameter σ, is defined as $v_s/h\omega_u$, which is regarded as a measure of soil stiffness relative to the structure. Observe that when the soil is soft relative to the building (i.e., when σ is small), STMD and 7MTMD become less effective. This results from the fact that the detuning effect occurs because the system properties change: the structural frequencies decrease and the damping ratio may increase or decrease due to SSI effect. In most conditions, 7MTMDs have better control effectiveness than STMD except when σ is less than about 0.75 and λ_h=5 in Case 2. This phenomenon indicates that the sensitivity of MTMD to the variations in system parameters is higher than that of STMD. In order to improve the undesired 7MTMD detuning problem, MTMD with 100% enlarged frequency spacing is employed and the corresponding $R_{u\ddot{x}_g}$ is also shown in Figure 5 (7MTMD*). It is seen that 7MTMDs* are less effective than optimal 7MTMDs when σ is large, but indeed have better control effectiveness when the soil is very soft, especially for buildings with large λ_h. The figure also shows that the Case 2 MTMD has better performance than Case 1 MTMD in most conditions.

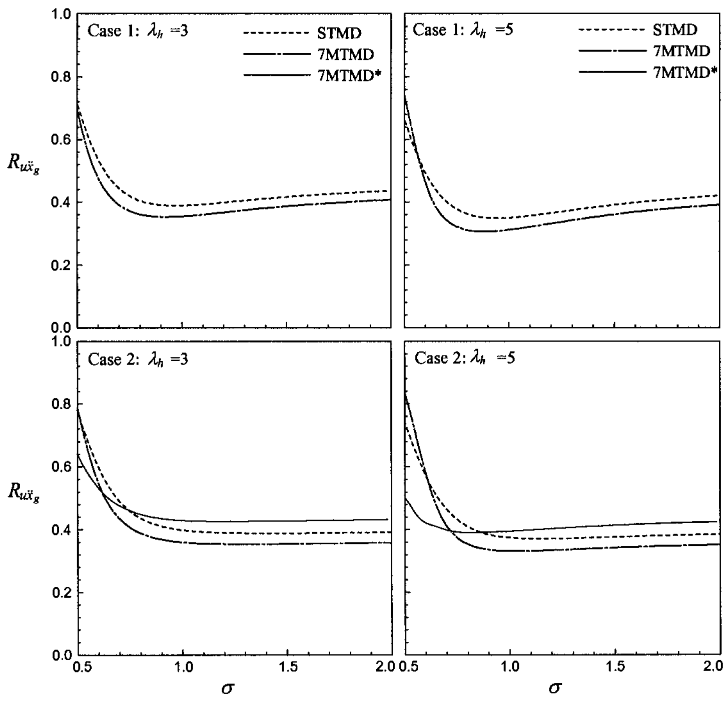

Figure 5. Mean-square-response ratios of floor translation for an irregular building with STMD, 7MTMD and 7MTMD*

To evaluate the dynamic structural responses, the 1995 Kobe earthquake acceleration record is used as the ground excitation. From reference [11], it is known that for many practical types of building structure, $h\omega_u$ is approximately 60π. Therefore, in this study the values σ=1.5 corresponding to soft soil ($v_s \cong 280$ m/s) is used to represent the site conditions of the Kobe earthquake. The value $\sigma = \infty$ representing the fixed-base condition is also investigated to examine the influence of SSI effect. In order to consider the given irregular building with various $\omega_u (= 2\pi/T_u)$ values, the corresponding λ_h value should also be taken into account. In reference [11], the empirical relationship between λ_h and T_u is approximately assumed to be $\lambda_h = 2\,T_u$. According to these conditions, the dynamic responses are calculated using the Inverse Fourier Transform of the building response in frequency domain. Figure 6 shows the peak floor transverse acceleration of the irregular building with various values of T_u under the Kobe earthquake. Comparing the curves with and without considering the SSI effect, it is found that the building response would generally be overestimated if the SSI effect is ignored. These figures also show that the STMD and MTMD control effectiveness is strongly dependent upon the frequency content of the earthquake. Moreover, the SSI effect decreases the STMD and MTMD effectiveness since the detuning effect occurs. If the SSI effect is not considered, the vibration control effectiveness will be overestimated. In addition, controlling two structural modes is the better strategy.

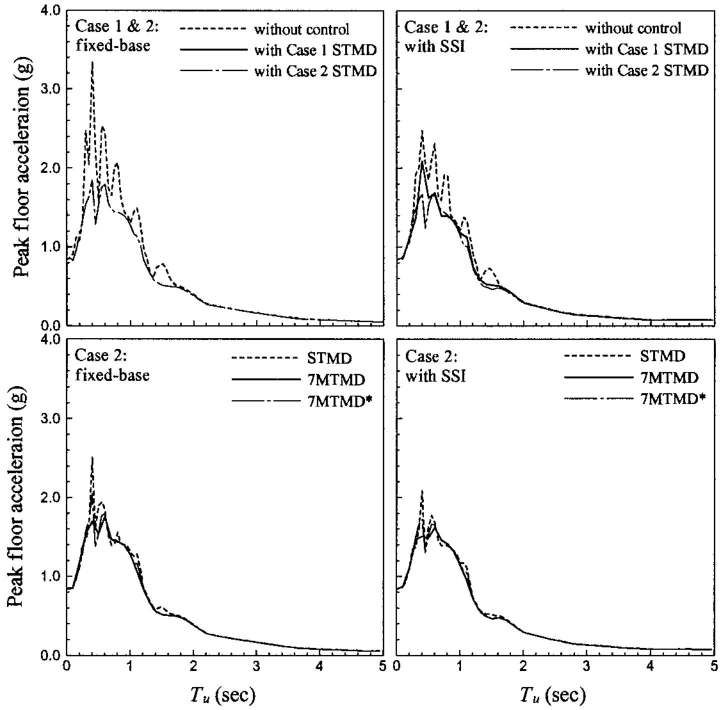

Figure 6. Response spectra of floor translation of building-STMD and MTMD systems with and without considering SSI effect

CONCLUSIONS

The SSI effect indeed deteriorates the MTMD vibration control effectiveness. When an irregular building is built on soft soils, the SSI and torsion-coupling effects should be considered to determine the optimal parameters of MTMD to avoid overestimating their effectiveness. Since a MTMD has many frequencies, its detuning effect can be reduced through appropriately adjusting the MTMD frequency spacing. It is the benefit which the STMD does not possess. The MTMD effectiveness is also dependent upon the characteristics of the external excitation. When the building dominant frequencies are located within the bandwidth of the external loading spectrum, MTMD can always reduce the building responses. It has been proven that the proposed MTMD is a useful vibration control device and more effective than STMD.

ACKNOWLEDGEMENT

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