Evaluation of the probability of crack initiation and crack instability for a pipe with a semi-elliptical crack

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ABSTRACT

For cracked components, the fracture mechanics theory provides deterministic relationship between the maximum permissible external loading and some parameters of the component : its dimensions, the material properties, the crack size and its location. However, due to uncertainties of some of those parameters (for instance the material properties, the applied loading ...), a purely deterministic approach provides an incomplete picture of the reality. Therefore, a probabilistic approach may be necessary on this field. Such an approach allows us to estimate the failure probability of a structure subjected to a certain external loading.

When dealing with ductile fracture, both initiation of the crack and its instability have to be examined. Moreover, studied defects for industrial cases are often semi-elliptical and loading conditions can sometimes be very complex. Therefore, it is very important for the development of probabilistic fracture mechanics and reliability methods to developed tools being able to calculate such complex configurations.

This paper presents some work conducted at EDF R&D to evaluate the probability that a semi-elliptical crack in a pipe not only initiates but also propagates when submitted to mechanical loading such as bending and pressure combined or not with a thermal shock. The first part of the paper is related to the description of the mechanical model : the simplified methods included in the French RSE-M Code used to evaluate the J-integral as well as the principle of the determination of the crack propagation. Then, the way this deterministic approach is combined to a reliability code is described. Finally, an example is shown : the initiation and the instability of a semi-elliptical crack in a pipe submitted to combined pressure and bending moment.

INTRODUCTION

The fracture mechanics theory provides deterministic relationship between the maximum permissible external loading and some parameters of a cracked component : its dimensions, the material properties, the crack size and its location. Such a purely deterministic approach provides only an incomplete picture of the reality since some of those parameters as for instance the material properties or the applied loading present some uncertainties. Therefore, a probabilistic approach may be necessary on this field to estimate the failure probability of a structure subjected to a certain external loading.

When dealing with ductile fracture of cracked pipes, both initiation of the crack and its instability have to be considered since the initiation of the crack does not conducts to the failure of the component. Moreover, studied defects for industrial cases are often semi-elliptical and loading conditions can sometimes be very complex. Therefore, it is very important for the development of probabilistic fracture mechanics and reliability methods to developed tools being able to calculate such complex configurations.

A first alternative is to combine directly the reliability code with a finite element code as described in different papers [1], [2], [3]. But for industrial problems, the finite element model is sometimes very time consuming because of the use of 3D model which has to be refined around the crack tip. Therefore, such combination can conduct to very large computing time since the finite element calculation has to be made several times. Another possibility is to use some analytical or semi-analytical models. Such work has already been published in the past when considering the initiation of a crack [4], [5], [6].

This paper presents some work conducted at EDF R&D to evaluate the probability that a semi-elliptical crack in a pipe not only initiates but also propagates when submitted to mechanical loading such as bending and pressure combined or not with a thermal shock. The first part is related to the quick description of the mechanical model used : the simplified methods included in the French RSE-M Code [7] used to evaluate the J-integral as well as the principle of the determination of the crack propagation. Then, the way this deterministic approach is combined to a reliability code is described. Finally, as an example, the initiation and the instability of a semi-elliptical crack in a pipe submitted to combined pressure and bending moment are studied.

MECHANICAL MODEL

Generality

The RSE-M Code [7] provides analytical or semi-analytical methods to evaluate the stress intensity factor K_I and the J integral in a pipe containing a circumferentially oriented surface crack submitted to mechanical loads (in-plane bending and torsion moments, pressure, tension), thermal loads as well as to the combination of these loads. These methods have been jointly developed and validated using finite element codes y CEA, EDF and Framatome for several years [8], [9], [10], [11]. Other cases as longitudinal surface cracks in straight pipes, surface cracks in elbows for example are still in progress and will be included in the future in the code.

Part of the estimation schemes existing in the RSE-M code including mechanical as well as thermal loads have been implemented in a software named OSTAND. In its actual configuration, this software allows the evaluation of the stress intensity factors for circumferential or longitudinal surface cracks in straight pipes (based on the influence coefficients given in the RSE-M Code), the temperature and the stresses in a pipe submitted to any kind of thermal (axisymmetrical) transient on its inner surface and the J-integral for a circumferential crack submitted to either mechanical loading, thermal loading or combined loading. One has to notice that the transient is described by the evolution of the fluid temperature and the heat exchange coefficient versus time and the thermal calculation can be non-linear.

In this paper we will concentrate on the case of mechanical loading. The corresponding model is described in the following paragraph.

Estimation of the J-integral under mechanical loading

For mechanical loads two methods - named CLC and CEP - are available in the Code. The CLC method, based on a corrected limit load, will be shortly presented here. A description of the CEP method can be found in [8]. The equation used to calculate L_r for a pressure combined with an in-plane bending moment is :

$$L_r = \sqrt{\left[\frac{m_2}{q_m \mu_{em} \mu_t}\right]^2 + \left[(1-\mu_{ti})\frac{p}{\mu_{ep}}\right]^2} + \mu_{ti}\frac{p}{\mu_{ep}} \tag{1}$$

where m_2 and p are non dimensional loading obtained by using Eq. (2) and (3) respectively and q_m, μ_{em}, μ_t, μ_{ep} and μ_{ti} are coefficients depending only on geometrical parameters and given in Table 1.

$$m_2 = \frac{M}{4 r_m^2 t S_y} \tag{2}$$

$$p = \frac{\sqrt{3}}{2}\frac{\Pr_m}{t S_y} \tag{3}$$

Then K_r is calculated from L_r by :

$$K_r = \left[\frac{E\,\varepsilon_{ref}}{L_r\,S_y} + \frac{1}{2}\frac{L_r^2}{L_r^2+1}\right]^{-\frac{1}{2}} \tag{4}$$

where ε_{ref} is the strain corresponding to the stress $\sigma_{ref} = L_r S_y$ on the true stress - true strain curve of the material (same approach as Option 2 of the R6 rule). Finally, an estimation of the elastic-plastic J-integral is obtained by :

$$J = \frac{J_e}{K_r^2} \tag{5}$$

where J_e is the elastic value of J which can be evaluated from the stress intensity factor K_I.

Table 1 - Coefficients for the CLC method

Coefficient	$\beta < 2\pi \frac{a}{t}$	$\beta \geq 2\pi \frac{a}{t}$
q_m	$\cos\left(\frac{\beta}{2}\frac{a}{t}\right) - \frac{1}{2}\frac{a}{t}\sin(\beta)$	$1 - \frac{a}{t}$
μ_{em}	$1 - 0.16\sqrt{\frac{a}{t}}$	1
μ_{ep}	1 (inner surface crack) 0.9 (outer surface crack)	
μ_t	$1 + \frac{1}{2}\frac{a}{t}\frac{\beta}{\pi}$	$1 + \frac{1}{2}\frac{a}{t}$
μ_{ti}	$\mu_{ti} = \frac{\sqrt{\mu_t^2 - 1}}{\mu_t}$	

COMBINATION OF THE DETERMINISTIC MODEL WITH THE RELIABILITY CODE

The combination of the reliability code PROBAN [12]] with the deterministic code OSTAND described in the first paragraph has been made in a direct way as it has already been done in case of a finite element code [2], [3]. In other word, each time the evaluation of J is necessary the reliability code creates the corresponding input file with the right values of the different probabilistic variables for the deterministic code. Then the deterministic code is executed. The output parameter (J in our case) is finally read from the output file of the deterministic code.

When considering the probability that a crack initiates, this combination is very simple since the limit state function is given by the comparison between the applied J-Integral (evaluated by OSTAND software) and the fracture toughness $J_{0.2}$. If the applied J is greater than the fracture toughness, the crack initiates.

When studying the crack instability an iterative method is necessary to evaluate the crack propagation and to determine whether this crack is stable or not. Moreover, for semi-elliptical crack, the crack propagation has to be considered not only at the deepest point but also all along the crack front. Important hypotheses have been used for dealing with this problem. Firstly, the J resistance curve is taken similar when considering the crack propagation at the deepest point or at the surface. Secondly, the crack remains semi-elliptical that is to say that the crack propagation can be defined by the crack propagation at the deepest point and the crack propagation at the surface. Thirdly, a stable crack growth greater than 3 mm is considered to be unstable since 3 mm is considered to be the validity limit of the J resistance curve determined using CT specimens.

Figure 1 illustrates the case of a stable crack. The crack propagation in depth and in length directions (respectively Δa and Δc) have to be determined by the intersection of the applied J curve at the deepest point and at the surface point as a function of crack propagation and the J resistance curve. It has to be noted that the iterative process is not so simple since the applied J curve at the deepest point (or at the surface) is a function of the crack propagation at the deepest point and at the surface.

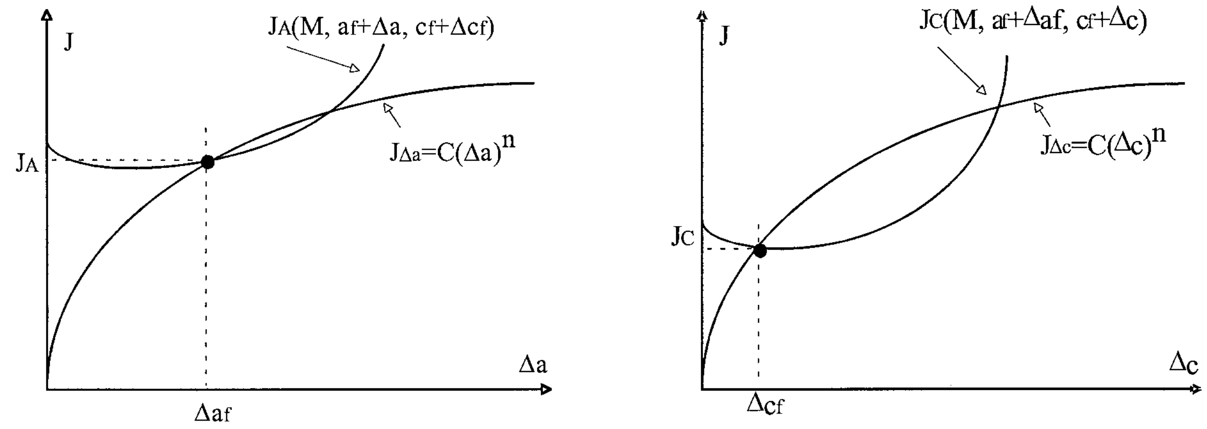


Fig. 1 : illustration of a stable crack growth

Figure 2 presents an illustration of the way the reliability code and the deterministic code have been combined to be able to deal with the crack initiation and the crack instability. The subroutine *JOSTAND.f* is able to write the input data of OSTAND, to execute it and to read in the output file of OSTAND the J value at the deepest point (Jfond) and at the surface point (Jbord). The subroutine *Amorcage.f* makes the link with PROBAN and takes the maximum of the J values at the deepest point and at the surface and compares it with the fracture toughness ($J_{0.2}$) The subroutine *PROPAG.f* corresponds to the iterative process for evaluating the crack propagation at the deepest point (Δafond) and at the surface (Δabord). This subroutine uses the *JOSTAND.f* subroutine. The subroutine *Propagation.f* makes the link with PROBAN and takes the maximum of the crack propagation at the deepest point and at the surface point.

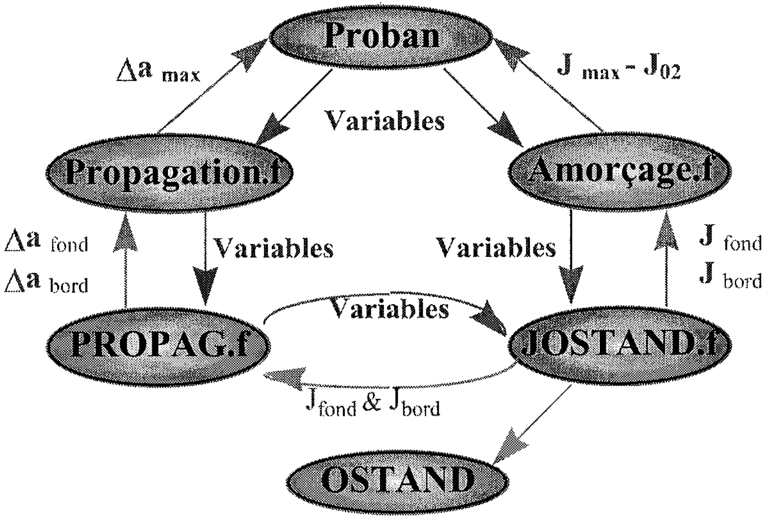


Fig. 2 : illustration of the combination between the reliability code PROBAN and the deterministic code OSTAND

CASE STUDY

Data of the studied case

The objective is to evaluate the probability of initiation and instability of a semi-elliptical crack in a pipe subjected to internal pressure and in-plane bending moment. The geometry of the cracked pipe is presented in Figure 3. The pipe is made of a Carbon Manganese steel and the crack is considered to be in the weld joint. Therefore, the stress-strain curve used is the one of the base metal and the resistance curve as well as the fracture toughness is from the weld metal. The materials properties are taken from Appendix 5.6 of RSE-M Code. Table 2 contains the deterministic and probabilistic data based on expert judgment or experimental measurement.

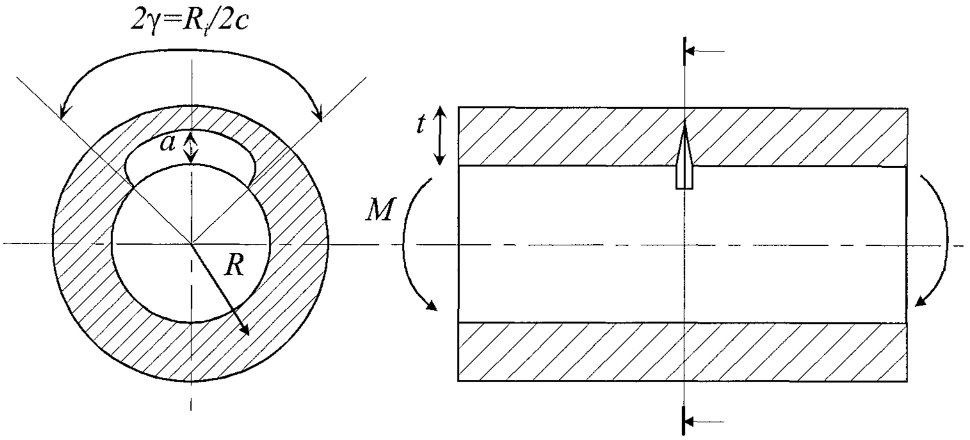


Fig. 3 : Geometry of the pipe considered

Table 2 : Data of the case presented

Parameter	distribution law	mean value	standard deviation or coefficient of variation
Young's modulus E, MPa	Lognormale	185 000	5 %
Yield strength Sy, MPa	Normale	213.57	5 %
Fracture toughness $J_{0.2}$, MPa.mm	Lognormale	74.1	20 MPa.mm
Exponent of the fracture resistance curve, n_J	deterministic	0.32	--
crack depth a, mm	Normale	8	1 mm
half crack length c, mm	Normale	24	1 mm
Poisson's ratio ν	deterministic	0.3	--
Pipe's internal radius r_i, mm	deterministic	374.4	--
Thickness t, mm	deterministic	32	--
Internal pressure P, MPa	Lognormale	15.5	5 %
Applied moment M, kN.m	Lognormale	1500-3000	20 %

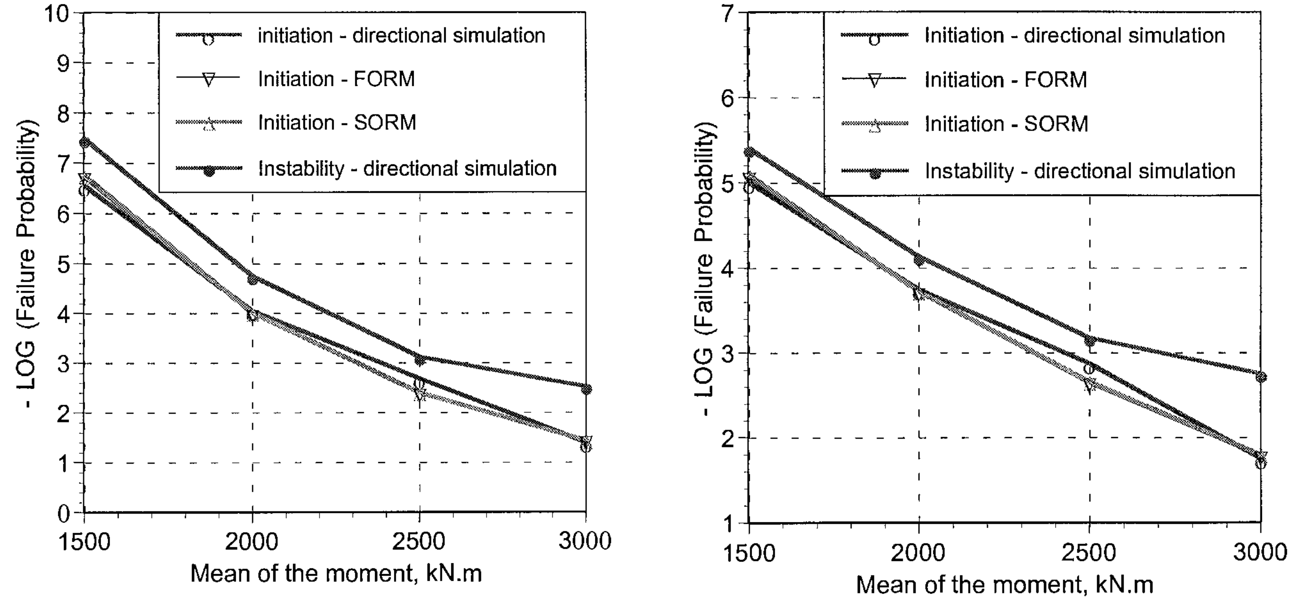
Results and discussion

The probability of initiation and the probability of instability of the crack have both been computed using different reliability methods such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and directional simulations. Such calculations have been conducted for four different levels of applied moment : 1500 kN.m ; 2000 kN.m ; 2500 kN.m and 3000 kN.m.

For the initiation case, 20 directional simulations have been used whereas only 5 directional simulations have been considered for instability problems because of the large computation time requested. Indeed, the evaluation of the crack instability needs about 30 hours on an Ultra60 workstation when using 5 directional simulations. This long duration for the calculation could be reduced by improving the convergence algorithm used for instability calculation and compiling the analytical models used in OSTAND with Proban instead of writing several data files and reading the results. The FORM and SORM methods did not converge correctly for the instability case due to the non regularity of the limit state function.

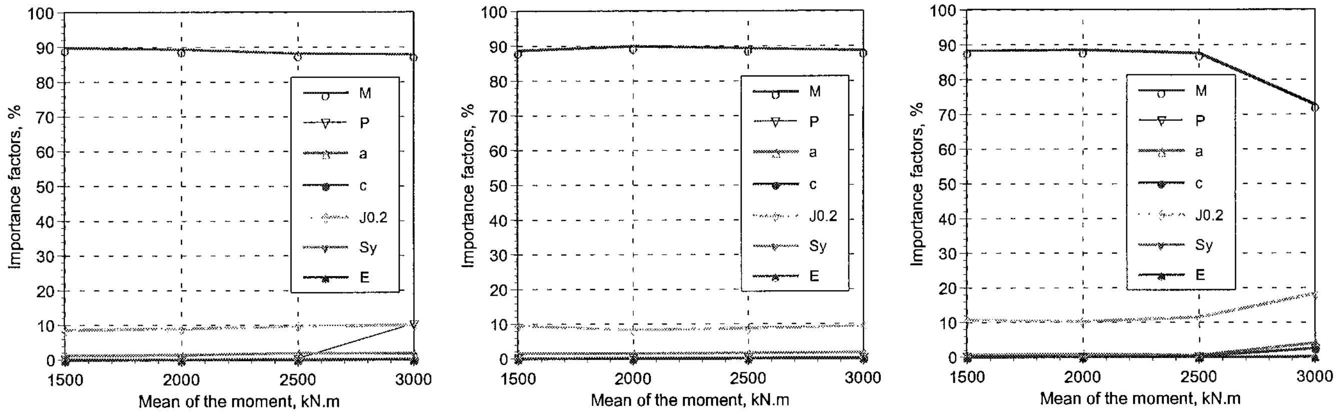
The results obtained in term of failure (initiation or instability) probability and reliability index for different values of the applied moment are compared in Figure 4. For initiation, all the different methods (FORM, SORM, and directional simulations) used give similar results. Moreover, there is about one decade between the probability that a crack initiates and the probability that it becomes unstable (crack propagation greater than 3 mm). The difference between the probability for the crack to initiate or to be unstable is not very important because the J resistance curve is relatively flat (depending on the value of the exponent of the fracture resistance curve, n_J).

The importance factors obtained for both initiation and instability event are given in Figure 5. When comparing the importance factors obtained by directional simulation (Figure 5a) and FORM-SORM method (Figure 5b) for the initiation, we can note that the results are very similar. Moreover, for both, initiation and instability, the most important parameter is the moment applied to the structure with a value of about 80 %. The second most important parameter with a value of about 10 % is the fracture toughness. In other words, the probability for the crack to initiate or to be unstable is mostly governed by the variability of the applied moment. But one has to remember that the coefficient of variation for the moment used in this study is 20%, which corresponds for example to a standard deviation of 400 kN.m for a mean value of 2000 kN.m.



a) failure probability **b) reliability index**

Fig. 4 :Evolution of the probability of initiation and instability as a function of the mean value of the in-plane bending moment



a) initiation - directional simulation **b) initiation - FORM/SORM** **c) instability - directional simulatio**

Fig. 5 : Importance factors calculated for the initiation and the instability of the crack

CONCLUSIONS

A general software based on simplified methods in fracture mechanics has been developed to evaluate the applied crack driving force J for straight pipe with semi-elliptical crack. This software has been combined with the general probabilistic code Proban. This way the probability for a crack in a pipe to initiate or to be unstable can be evaluated for different loading combination including mechanical and thermal loading.

As an example, the case of a semi-elliptical crack in a pipe subjected to internal pressure and in-plane moment is presented in this paper. The results show a difference of a bout one decade between the probability of initiation and the probability of instability. This difference can vary quite a lot depending on the exponent of the crack resistance curve. The results obtained considering the importance factors give an example of the type of information such approach can furnish.

NOMENCLATURE

a = crack depth
c = half crack length
E = Young's modulus
J = elastic plastic crack driving force
J_e = elastic crack driving force derived from the stress intensity factor K_I
$J_{0.2}$ = fracture toughness
K_I = elastic stress intensity factor
K_r = plastic corrective term
L_r = plasticity level reached in the cracked structure
M = in-plane bending moment
m_2 = non dimensional loading related to the bending moment defined by Eq.(2)
n_J = exponential coefficient of the crack resistance curve
P = internal pressure
p = non dimensional loading related to the internal pressure defined by Eq.(3)
q_m = coefficient depending on geometrical parameters and given in Table 1
r_e = outer radius of the pipe
r_i = inner radius of the pipe
r_m = mean radius of the pipe
S_y = yield strength at 0.2% of plastic deformation
t = pipe wall thickness
β = half crack angle
Δa = crack propagation in depth
Δc = crack propagation in length
ε_{ref} = reference strain
μ_{em} = coefficient depending on geometrical parameters and given in Table 1
μ_{ep} = coefficient depending on geometrical parameters and given in Table 1
μ_t = coefficient depending on geometrical parameters and given in Table 1
μ_{ti} = coefficient depending on geometrical parameters and given in Table 1
ν = Poission's ratio
σ_{ref} s= reference stress

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