Earthquake induced damage quantification and damage state evaluation by fragility and vulnerability models

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ABSTRACT

In this paper we develop and extend some models and techniques for earhquake-induced structural damage assessment and prediction in terms of seismic fragility and vulnerability models (proposed in a couple of authors' earlier papers). In the first section we review several existing damage indices and functionals ; we identify which of them are most suitable for evaluating the damage levels induced by earthquake motions. We propose the use of a damage indicator vector that allows to identify the damage state of the structure in the space of damage parameters. A limit state function is then formulated which depends on the selected damage indices and other parameters that include some seismical load parameters. This limit state allows to evaluate the probability of failure / reliability of the structural system under analysis. This general approach is adapted by incorporating seismic fragility and vulnerability models. The latter ones allow for seismic damage assessment for already damaged structures.

INTRODUCTION

The analysis of the degree or state of damage is essential for the estimation of the remaining lifetime and also for rehabilitation/retrofitting decisions and programs. But – even in the design – the evaluation or predicition of potential structural damage during the expected lifetime is an important issue. Appropriate analytical and probabilistic damage models must be selected in order to perform a realistic analysis / evaluation. A large variety of damage indices or indicators have been proposed and used for damage quantification for specific categories of structures. In general, they are scalar functions whose values may be related to specific structural damage states or levels [1], [2], [3], [4]. Naturally, the models selected for the quantification basically depend on the nature of the structure and also on the expected adverse actions from the environment, at the site of the system under analysis, able to affect its stability or service capacity. The effects of earthquake actions are – probably – among the most relevant for the damage evaluation of structures, industrial facilities, bridges and dams, etc.

In order to define the collapse or another type of failure for components and structures, in is necessary to define and use damage parameters which are relevant for the strength and deformation capacity of structures under cyclic loads in the inelastic domain. These parameters are the arguments of a real scalar function that synthetically characterizes the degree of damage experienced by the structure. Such a function is – mathematically speaking – a damage functional. If only two damage states are taken into account, the failure and the absence of plastic damage, such a functional may be normalized so as to take value 1 in the former case and value 0 in the latter one. If several degrees of damage are admited, it may take any values in the interval [0,1]. The most commonly used damage parameter has been the ductility. The evaluation of the structural safety is evaluated by comparing the maximum plastic excursion and the allowable value, considering only one cycle with the largest plastic deformation and neglecting the others [5], [6]. Another class of models consider the hysteretic energy as the damage parameter and the collapse is assumed to occur when the structure dissipates an energy equal to (or higher than) a limit value. The methods based on such models take into account the contribution to energy dissipation of all cycles independently from the cycle amplitude. Conversely, it has been experimentally and practically proved that the cycles with limited plastic deformation are of no practical relevance to the degree of damage sustained. An approach that combines the knowledge on ductility and energy has been developed in several papers by E. Cosenza et al. [7], [8], [9]. In fact, earlier approaches of this type, due to Banon & Veneziano [2], respectively Park & Ang [3], are integrated in an extended formulation. However, not of all these methods for damage arrive to probabilistic evaluations for the damage levels of a structure induced by earthquake excitations. The most specific classes of models for the probabilistic assessment of earthquake effects on structural systems are those involving seismic fragility and vulnerability formats. The fragility of a component or structure is essentially its probability of failure conditional on a certain intensity of ground motion at its site. The vulnerability formats can also take into account the level of damage induced by previous earthquakes (or other factors like aging) for predicting the future damage events under expected earthquakes of certain intensities. Therefore the concept of vulnerability seems to be better suited for sesimic damage evaluation. In the next section we present an evaluation of the damage models which are compatible with the use of fragility and vulnerability concepts. A widely accepted ground motion parameter PGA (peak ground acceleration) is replaced by parameters like MMI and MSK intensities. A specific fragility model is thus obtained, in terms of events that consist in reaching a certain damage category over a scale of three to five damage levels. Then we present a way to enrich the vulnerability models by taking into account the magnitude and the maximum seismic intensity in the evaluation of the damage rate for certain classes of structures, also involving some attentuation laws.

SEISMIC DAMAGE INDICATORS AND DAMAGE FUNCTIONALS

The steps to be followed in defining a damage functional are formulated by Cosenza et al in [9] as follows :

(*i*) Definitions of parameters d_i which are considered as important / relevant to the definition of progressive collapse.

(*ii*) Definition of the values $\mathbf{d}_Y$ and $\mathbf{d}_U$ assumed by the state vector at the beginning of structural system yielding and at collapse under cyclic loads, respectively.

(*iii*) Introduction of a normalized damage functional D, defined on the space of the state vectors $\mathbf{d} = [d_1 \ldots d_i \ldots d_m]$; this function should assume a value of 0 at the beginning of yielding and a value of 1 at failure, and it should increase as damage increases.

(*iv*) Evaluation of the value assumed by the functional, with reference to the structure examined and for the seismic events expected ; the seismic checking will be satisfied if it results in $D < 1$.

It follows from (*iii*) that a damage functional would have to be analytically expressed as a function of the damage state vector $\mathbf{d}$, that is $D = \varphi(\mathbf{d})$ where $\varphi : \Delta \rightarrow [0,1]$ is a function from the space of (the possible values of) the damage parameters $d_1, \ldots, d_i, \ldots, d_m$ to the unit interval. This formulation is quite general, so far. Yet we may raise here the problem of the nature of the selected damage parameters from the probabilistic point of view. If at least one of them has a random nature then the damage functional itself becomes random. Clearly, all of the damage parameters may be random with specific distributional assumptions accepted for modeling their random behavior. Let us denote by F_i the cdf (cumulative distribution function) of the random parameter d_i. Then the *cdf* of the damage functional D will be defined by

$$F_D(\delta) = P(D \leq \delta) = P\big(\varphi(d_1,\ldots,d_i,\ldots,d_m) \leq \delta\big). \tag{1}$$

The evaluation of the probability in Eq. (1) will clearly depend not only on the distributions assumed for the random components of $\mathbf{d}$ (i.e., for the arguments of φ) but also on the analytic nature of this function. This problem will be examined (in more detail) in the next section.

Let us now see a couple of examples resulting for damage indicators or functionals considered in some references dealing with seismic damage modeling. The cinematic ductility is defined in [7-9], for a SDOF structure with EPP behavior, by

$$d = \mu_s = \frac{x_{\max}}{x_y} \tag{2}$$

where $x_{\max}$ is the maximum plastic excursion and x_y is the yielding displacement. The value of this damage parameter is equal to 1 at yielding (this value is denoted $d_y = 1$) while the value at collapse is d_u = the maximum allowable value of ductility $\mu_{u,\,mon} = x_{u,\,mon} / x_y$ where $x_{u,\,mon}$ is the maximum displacement determined by monotonic tests. If $x_{\max}$ is taken in absolute value then Eq. (2) gives the cyclic ductility. An analytic form for a normalized damage functional is proposed in [7-9] by

$$D = \begin{cases} 0 & \text{for } d \leq d_y, \\ \left(\dfrac{d - d_y}{d_u - d_y}\right)^{\alpha} & \text{for } d_y < d \leq d_u, \alpha > 0. \end{cases} \tag{3}$$

It is easy to see that this damage functional takes values in the interval [0,1], and it effectively depends on a single damage parameter d given by Eq. (2), but also on the parameters d_y , d_u and α. The first two of them are the characteristic values of d at yielding / collapse considered in step (*ii*) while α is a parameter involved in the analytic form of function φ. The domain of φ is $\Delta = [0, d_u]$. However, this simple example shows that a more exact formulation for the function φ would need to extend its argument(s) so as to include some parameters : $D = \varphi(\mathbf{d}, \theta)$. The corresponding form of the normalized damage functional in terms of kinematic or cyclic ductility is

$$D_\mu = \frac{\mu - 1}{\mu_{u,\,mon} - 1}. \tag{4}$$

Many hysteretic models for the damage evaluation are formulated in terms of the plastic dissipated energy E_h : the structural collapse is assumed to occur when the total dissipated energy reaches or increases beyond a limit value $E_{u,\,mon}$. Even in this case, the allowable amount of energy that can be dissipated can be obtained by means of a monotonic analysis which evaluates just the limit value $E_{u,\,mon}$. The hysteretic ductility is defined [7-9] as

$$\mu_e = \frac{E_h}{F_y x_y} + 1 \tag{5}$$

where F_y is the maximum force to which the structure can be subjected. The corresponding damage functional D_E has just the form in Eq. (4) with μ replaced by μ_e and $\mu_{u,mon}$ by $\mu_{e,u,mon}$.

Combinations of the ductility-based damage indicators with energy-based indicators were proposed. Among the best known are the methods due to Banon & Veneziano and to Park & Ang, respectively. Their original formulations may be expressed in terms of the ductility / energy parameters just presented above. The B&V method leads to the damage functional

$$D_{BV} = \sqrt{\left(d_1^*\right)^2 + \left(d_2^*\right)^2} \quad \text{with} \quad d_1^* = d_1 - 1 \;\&\; d_2^* = a(d_2)^b \tag{6}$$

where a and b are two parameters which are specific to the structural problem and have to be determined experimentally. The two damage parameters that occur in Eq. (6) are $d_1 = \mu_s$ (see Eq. (2)) and $d_2 = 2(\mu_e - 1)$ (see Eq. (5)). Therefore, the function φ corresponding to the Banon-Veneziano damage functional of Eq. (6) is

$$D_{BV} = \varphi(d_1, d_2) = \sqrt{(d_1 - 1)^2 + (a d_2^b)^2}\,. \tag{7}$$

In the (d_1^*, d_2^*) plane, the circles centered at the origin define the lines with equal collapse probability.

The Park-Ang damage functional is defined as a linear combination of the maximum displacement and the plastic dissipated energy ; its expression is given by

$$D_{PA} = \frac{x_{max}}{x_{u,mon}} + \beta \frac{E_h}{F_y x_{u,mon}} = \frac{\mu_s + \beta(\mu_e - 1)}{\mu_{u,mon}}\,. \tag{8}$$

The physical interpretation of the Park-Ang damage functional is based on the assumption that, under plastic dissipation of energy, the collapse does not occur when the the kinematic / cyclic ductility reaches the ultimate value of the monotonic test $\mu_{u,mon}$ but the fraction β of the hysteretic ductility μ_e must be added to the former ductility. This factor β may be considered as a free deterioration parameter that characterizes the structural elements. Since $\mu_{u,mon}$ is experimentally determined for a certain structure or component and it does not depend on the maximum EPP displacement x_{max} we may consider it as a structural parameter rather than a damage indicator ; if we denote it by η the analytic expression of the function φ corresponding to the Park-Ang damage functional of Eq. (8) is

$$D_{PA} = \varphi(d_1, d_2) = \frac{d_1 + \beta d_2 / 2}{\eta}\,. \tag{9}$$

The damage parameters d_1 and d_2 have been previously defined. It is remarked in [9] that D_{PA} is not a normalized damage functional since it does not provide the value 0 for $x_{max} = x_y$ and the value 1 in the case of failure.

Many other damage indicators (or parameters) are considered in the literature on seismic damage assessment. We do not intend to present and discuss very many of them. As we have already mentioned, different damage measures based on hysteretic models may be even not comparable since the differences in their formulations may follow from the different nature of the structures under analysis. However, a comparison between B-V, P-A functionals and the damage functional by the plastic fatigue method is presented in [9]. Let us remark, so far, that the general form of a damage functional with the function φ that occurs in Eq. (1) may be retrieved in the examples just presented ; however, its analytical expression may depend not only on the damage parameters like d_1 and d_2 but also on some parameters like $\theta_1 = a$ & $\theta_2 = b$ in Eq. (6), respectively $\theta_1 = \beta$ & $\theta_2 = \eta$ in Eq. (9). This gives a ground for the general form to be suggested for a damage functional, namely $D = \varphi(\mathbf{d}, \theta)$. It is clear that the damage parameters have a random nature since they are associated with the seismic response of the structure which is, in fact, the output to the random input consisting of the seismic ground motion at the site of the structure. But, besides this source of randomnes, some (or all) of the parameters in θ may be uncertain. Moreover, even the analytical form of the function φ would have to be considered as uncertain, even when it is derived on the ground of experimental results or by the analysis of response to actual or artificial earthquake accelerograms. This distinction between randomness and uncertainty is usual in the seismic fragility models. We approach such problems in the next two sections of the paper.

PROBABILISTIC ASSESSMENT OF SEISMIC DAMAGE IN HYSTERETIC STRUCTURES

We have just discussed the problem of the randomness of the damage indicators (or parameters, as they are termed in [9]), that is, of the components of the vector $\mathbf{d} = (d_1, \ldots, d_i, \ldots, d_m)$. It is clear that the evaluation of the probability in Eq. (1) cannot be accomplished without assuming certain probabilistic distributions for the random damage indicators. On another hand, the condition of the normalized damage functional to take only the $0 - 1$ values seems to be too restrictive since a rather widely accepted and used functional like D_{PA} (Park & Ang) has not been normalized. In order to avoid this limitation, we may let φ to take any positive value. As we have mentioned, the function defining the damage measure also depends (in general) on a set of parameters $\theta \in \Theta$. Therefore the general form of this function is $\varphi : \Delta \times \Theta \rightarrow [0,\infty)$.

There are many studies and even computer codes for the damage state assessment of a structure in terms of qualitative (or "linguistic") degrees of damage like, e.g., (no damage, moderate damage, heavy damage, collapse). These scales for damage levels vary from a reference to another. Other scales include an additional level, namely "slight damage" between "no damage" and "moderate damage". If a damage measure D taking values in $[0,1]$ is used then the five levels of damage severity may be assigned to the respective intervals described in Table 1 that follows.

Table 1. The intervals for a normalized damage measure over a five levels scale

Damage level	No damage	Slight damage	Moderate damage	Severe damage	Collapse
Interval for D	$[0, 0.1)$	$[0.1, 0.25)$	$[0.25, 0.4)$	$[0.4, 0.8)$	$[0.8, 1]$

This scale is the revised form (by Ang, in 1993) of the scale proposed by Park, Ang & Wen in 1985. The value of 0.4 is taken as the limit of damage beyond which the structure is not repairable.

From the probabilistic point of view, it is relevant to evaluate the probability that a certain structural member or structure will reach a certain damage level under an estimated structural response induced by an earthquake of an estimated intensity & duration. If a damage functional $D = \varphi(\mathbf{d}, \theta)$ is used, with its *cdf* $F_D(\delta)$, then the probability that the damage level of the structure will fall within an interval like those suggested in Table 1 will be given by

$$P(\delta_k \leq D < \delta_{k+1}) = F_D(\delta_{k+1} + 0) - F_D(\delta_k + 0). \tag{10}$$

The notation for the arguments of $F_D(\,)$ in Eq. (10) stand for the right lateral limits.

The evaluation of the probability in Eq. (10) would be not possible without some distributional assumption on the random parameters in $\mathbf{d} = (d_1, \ldots, d_i, \ldots, d_m)$. Indeed, the inequality under the probability in the left-hand side of Eq. (10) can be rewritten as

$$\delta_k \leq \varphi(d_1, d_2, \ldots, d_m; \theta_1, \theta_2, \ldots, \theta_\ell) < \delta_{k+1}. \tag{11}$$

If we assume that all the damage parameters are random variates with the respective *cdf*'s (cumulative distribution functions) $F_1, \ldots, F_i, \ldots, F_m$ or with the *pdf*'s (probability distribution functions) $f_1, \ldots, f_i, \ldots, f_m$ then the probability that d_i takes values in a specified interval $[a_i, b_i]$ can be expressed as

$$P(a_i \leq d_i \leq b_i) = \int_{a_i}^{b_i} f_i(x)\,dx = \int_{a_i}^{b_i} dF_i(x) = F_i(b_i) - F_i(a_i). \tag{12}$$

The probabilities of Eqs. (12) for $i = 1, 2, \ldots, m$ can be used for the evaluation of the probability in Eq. (10) provided the event in Eq. (11) can be assembled from the component events as those that occur in (12). Even when certain probabilistic distributions are accepted for $d_1, \ldots, d_i, \ldots, d_m$ this problem may be not too simple. Its solution also depends on the shape of function φ as well as on the values of the parameters θ which explicitly occur in Eq. (11). In many cases, these parameters are considered as deterministic but their values are estimated by regression analysis, as in Ref. [10] by Han and Wen. A limit state function can be easily obtained from a damage functional φ as in Eq. (11). If a failure threshold δ^* is selected (for instance $\delta^* = 0.4$) then the limit state function can be defined by

$$g(\mathbf{d}, \theta) = \delta^* - D = \delta^* - \varphi(\mathbf{d}, \theta). \tag{13}$$

The failure event will be characterized by $g(\mathbf{d},\theta) \leq 0$ while the survival of the structure will correspond to the converse inequality $g(\mathbf{d},\theta) > 0$. We include an example to illustrate a possible solution to the evaluation of probabilities as in Eqs. (10) & (13) at the end of this paper.

Another way for a probabilistic evaluation of the nonlinear behavior of RC members consists in effectively using vectors of damage parameters $\mathbf{d}$ and not necessarily taking such a vector as the argument of the function φ. Such a vector is assumed to be time-dependent and it characterizes the state of the system in the space Δ of the damage parameters. An approach of this kind was considered (in [2]) by Banon & Veneziano. Six damage indicators were analysed, namely the rotation ductility μ_0, the curvature ductility μ_p, the damage ratio DR, the flexural damage ratio FDR, the normalized cumulative rotation NCR, and the normalized dissipated energy E_n. Among them, FDR and E_n were found to be the most relevant. They are defined and redenoted below, resulting in a vector damage function :

$$\begin{cases} \mathrm{FDR} = K_f \big/ K_r = d_1, \\ E_n = \int_0^t M(\tau)\theta(d\tau) \Big/ \frac{1}{2} M_y \theta_y = d_2(t), \end{cases} \tag{14}$$

where K_f = the initial flexural stiffness, K_r = the reduced secant stiffness, $M(\tau)$ is the yield moment (at time τ), $\theta(d\tau)$ is the rotational increment of the inelastic spring at one end of the member during the time interval $(\tau, \tau + d\tau)$, M_y = the yield moment, and θ_y = the rotation at yielding ; t is the time elapsed from the beginning of loading. The corresponding vector damage function is

$$\mathbf{d}(t) = [d_1(t) \;\; d_2(t)] \; . \tag{15}$$

The initial value of $\mathbf{d}$ is $\mathbf{d}(0) = [1 \;\; 0]$ and it corresponds to the point A (1,0) in the (d_1, d_2) plane. The current point P in this plane is considered to represent a damage state, that is the state of the system at time t. Two hazard functions are also considered in [2], namely $\lambda_1(d_1, d_2, d_2{}'_1)$ and $\lambda_2(d_1, d_2, d_1{}'_2)$, where $d_2{}'_1$ denotes the derivative of d_2 with respect to d_1, and similarly for the second function. A line integral from A to P, denoted by I_L, is involved in defining the survival failure probability of the structural member as follows :

$$I_L = I_{AP} = \int_A^P \lambda_1(d_1, d_2, d_1{}'_2)\, dd_1 + \lambda_2(d_1, d_2, d_1{}'_2)\, dd_2 \; ; \tag{16}$$

$$P(\text{survival}) = \exp(-I_L), \; P(\text{failure}) = 1 - \exp(-I_L). \tag{17}$$

In our paper [11] we presented evaluations for the integral in Eq. (16) without neglecting the third arguments of the two hazard functions (as it is done in [2]) ; in fact, this integral can be evaluated in the $(d_1{}^*, d_2{}^*)$ plane where the new variables are connected to the original ones according to the transformations in Eqs. (6). We also considered the progressive failure in a structure (consisting of several member with hysteretic behavior) ; the structural probability of failure / survival was evaluated on the basis of the assumption that the mean distance from the origin to the failure points is Weibull-distributed.

Other approaches to the probabilistic asseessment of the damage state of a structural component / structure consist in the use of stochastic processes. In fact, the deterioration of a structure during an earthquake is actually a time-dependent process. Many of the hysteretic models in use avoid to consider this dependence of time explicitly. However, the cumulative damage indices are closer to such a time-dependent approach, since the accumulation of dissipated energy grows with the time and the number of cycles in structural response. The same remark holds for the fatigue models. An intermediate approach consists in considering the number of peaks or zero crossing in the response process; alternatively, the number of hysteretic cycles may be taken into account. These numbers are, clearly, more or less proportional to the time T = duration of the structural response to a seismic ground motion.

In the most frequently applied aseismic design practices, it is common to reduce the linear elastic design forces and thus to allow a few inelastic excursions in the structural response during a severe earthquake shaking. The structure is required to be sufficiently ductile to withstand these excursions without collapse. Anderson and Bertero [12] showed that the large amplitude accelerations may not always cause significant damage by driving the structure into the non-linear range, whereas many successive non-linear excursions of relatively smaller amplitudes may be more damaging during earthquakes of longer duration. A model based on the order statistics of the response peaks is used in [13] to consider the damage accumulation from several random nonlinear excursions and to associate the ductility with the total damage due to these excursions. The cumulative damage is defined as the sum of damages D_i which correspond to discretized damage levels μ_i. The expected damage is defined as conditional expectation of D_i on the previous $i-1$ ordered peaks $x_{(1)}, x_{(2)}, \ldots, x_{(i-1)}$, and its evaluation requires conditional *pdf*'s $p(x_{(i)} \mid x_{(1)}, x_{(2)}, \ldots, x_{(i-1)})$ to be known. These *pdf*'s show to verify Markov's property, that is they are effectively conditional on $x_{(i-1)}$ only. We use and adapt this model in another paper submitted to SMiRT 16 (by C.Robu, A.Vulpe & A. Carausu).

FRAGILITY AND VULNERABILITY MODELS FOR SEISMIC DAMAGE ASSESSMENT

Fragility models applied to seismic damage evaluation

We approached the use of fragility models in structural damageability assessment in our papers [14] and [15]. The models and formulas presented in the previous sections of this paper allow for more detailed characterizations of the seismic damage reached by a structure with hysteretic behaviour in terms of the severity classes, using adapted fragility models. Since the 80's when the fragility concept was introduced, mainly for the PSA (probabilistic safety assessment) of the NPPs, the fragility-based models have undergone a significant development and a wider use for the seismic risk evaluation of various classes of structures. In general, the relationship *ground motion intensity versus structural damage* induced thereby characterizes the level of damage to a particular class of structures as a function of a ground motion parameter.

A number of $n+1$ damage states are considered by A.Singhal in [16], which are the components of a damage state vector. We denote this vector by $\delta = (\delta_0, \delta_1, \dots, \delta_n)$, with the remark that is basically different from the vector **d** whose components were distinct damage parameters (or indicators); the components of the former vector, that is δ_i 's, may be taken as the points determining the partition of the range for the damage measure D ; for instance, they may be equal to the values in Table 1 if D is a normalized damage functional (in the terminology of [6], [7], [8]). This damage measure is assumed to be a random variable. A "classical" fragility curve describes the probability of failure conditional on a certain ground motion intensity. For the probabilistic damage evaluation, it has to characterize the probability of reaching a damage state at a specified ground motion level. Formally, we define the probabilities

$$p_{ik} = P(D \in [\delta_{i-1}, \delta_i) \,|\, M = m_k) \tag{18}$$

where D is a random (normalized) damage measure and M is a random ground motion intensity parameter. Eq. (18) gives a similar (but not identical) way to the to define fragilities proposed in [16]. The intervals that occur under the probability operator are just the same as in Eq. (10). These "damage fragilities" are clearly the probabilities of Eq. (10) conditional on the event that the seismic intensity M (denoted by Y in [16]) belongs to a certain intensity class. Clearly, it has been implicitly assumed a discretization of the range where this ground motion parameter takes its possible values. Hence, the event $M = m_k$ should be replaced by $M \in [\, m_{k-1} \,,\, m_k \,)$. Certainly, we may accept continuous values for m in $[0, \infty)$ or $[0, \sup M)$. In most of the typical fragility models the seismic input parameter is the PGA capacity A, that is the peak ground acceleration corresponding to the failure of the system. In the general case, the random variable M is represented as

$$M = \tilde{M}\, \varepsilon_R\, \varepsilon_U \quad \text{with} \quad \mathrm{Med}[M] = \tilde{M},\ \mathrm{Med}[\varepsilon_R] = \mathrm{Med}[\varepsilon_U] = 1,\ \sqrt{\mathrm{Var}[\varepsilon_R]} = \beta_R,\ \sqrt{\mathrm{Var}[\varepsilon_U]} = \beta_U. \tag{19}$$

These three parameters in Eqs. (19) are sufficient for expressing the seismic fragility of the structure in the double log-normal format by the *cdf* defining the failure probability conditional on the median seismic capacity C, and by the *pdf* that accounts for the random variability of C around its median :

$$P_f(m) = \Phi\left[\frac{\ln(m/C)}{\beta_R}\right] \quad \text{with} \quad f_C(c) = \frac{1}{\sqrt{2\pi}\,\beta_U\, c} \exp\left[-\frac{1}{2}\left(\frac{\ln(c/\tilde{C})}{\beta_U}\right)^2\right] \tag{20}$$

where Φ is the standard normal *cdf* and the variabilities due to randomness (of the response) and to uncertainty (in the theoretical model for C) are accounted for by the standard deviations in Eqs. (19). Fragility curves for a global damage index, defined as a weighted sum of element damage indices, were derived for RC structures of lower / higher rise by A.Singhal [16]. The damage functional there used was an equivalent form of Park & Ang damage index (similar to that of Eq. (9)). Its analytic expression is

$$D = \frac{\theta_m}{\theta_u} + \frac{\beta}{M_y\, \theta_u} \int dE \tag{21}$$

Using the general form of a damage functional (see Eq. (11)), we may write it as $D = \varphi\,(dE\,;\, \theta_m, \theta_u, \beta, M_y)$ where θ_m = the maximum positive or negative plasting hinge rotation, θ_u = ultimate hinge rotation capacity under monotonic loading, β is a model parameter (evaluated to be = 0.15), Q_y = the calculated yield strength, and dE = the incremental dissipated hysteretic energy. More details on this damage index may be found in [17]. On the basis of 100 artificial ground motions and using IDARC2D & DRAIN-2DX programs, a nonlinear dynamic analysis was performed by A.Singhal resulting in the fragility curves for the P&A damage index of Eq. (21), with the spectral acceleration taken for the seismic input parameter ($m = S_a$).

The observed building damage data from earthquakes makes possible to update the values of the parameters by the Bayesian method, on the basis of a vector $Z = (z_1, \ldots, z_n)$ of statistical evidence. More details on this Bayesian updating of fragilities are given in [18]. The probability that the damage to a structure exceeds a certain damage threshold δ_i at specified levels of the ground motion m_k is expressed by

$$Q_{ik} = P(D \geq \delta_i \,|M = m_k). \tag{22}$$

With reference to the *cdf* that occurs in Eq. (10), and also introducing the parameters θ, it is easy to see that the probabilities in Eq. (22) can be rewritten as

$$Q_{ik} = 1 - F_D(\delta_i - 0 \,|M = m_k, \theta) \tag{23}$$

As regards the probabilities for the damage measure D to fall exactly in the interval that defines a damage severity level, they are given by

$$q_{ik} = P(\delta_i \leq D < \delta_{i+1} |M = m_k, \theta) = F_D(\delta_{i+1} - 0 \,|M = m_k, \theta) - F_D(\delta_i - 0 \,|M = m_k, \theta). \tag{24}$$

These probabilities can be expressed in terms of the fragility model defined in Eqs. (20). The ground motion intensity m_k can be, for instance, the PGA, the PGV, or the MMI intensity at the site. We do not give explicit expressions for the damage fragilities in Eq. (24) ; we presented such expressions in [14]. Four fragility curves are plotted in Fig. 1, with the interval for the damage index partitioned into five severity classes ; they are drawn by means of the median D curve.

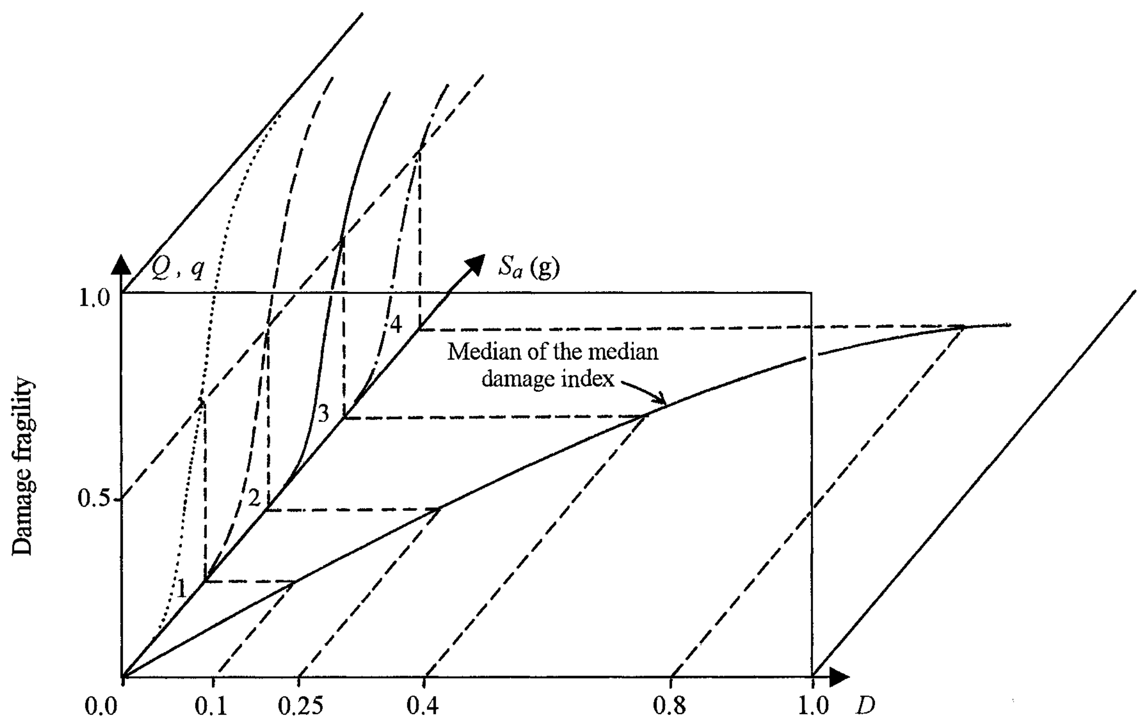

Fig. 1 Damage fragility curves for four damage severity classes :
minor: ········ ; moderate: ------ ; severe: ——— ; collapse: -·-·-·-

The probabilities in Eqs. (24) can be found as the lengths of the intervals on the $(O\ q)$ axis determined by the projections on this axis of the fragility curve segments cut by the segments of the median damage index line in the (S_a, D) plane.

Vulnerability models for seismically damaged structures

The concept of seismic vulnerability is similar to that of seismic fragility. Roughly speaking, it expresses a measure of the likelihood of failure (or severe damage) of a structure caused by strong earthquakes. As in the case of the fragility concept, the notion of vulnerability has not a unique and generally accepted definition. A classification and review of the vulnerability methods is presented in a report by M.Dolce *et al.* to the *10 ECEE* Conf. (Vienna, 1994). During the last decades, several investigations and results on the seismic vulnerability analysis have been accomplished by the research group led by H. Sandi at INCERC – Bucharest. In his paper [19] it is emphasized the importance of the damage probability assessment for already damaged structures in seismically active areas (like Romania with its major seismic source in Vrancea).

The pre-event damage is denoted by d' while the post-event damage by d'' ; a similar conventional notation is used for the subscripts when the scale of the damage values is discretized. The "classical" vulnerability of a structure can be quantified in terms of a conditional *pdf* (probability density function) of the form $f_V(d \mid y)$, or of a damage probability matrix whose entries are probabilities form $p_V(i \mid j)$ where d is a damage indicator that can be discretized in n damage states (that is $1 \le i \le n$) while j identifies an earthquake intensity (a discrete value for the ground motion parameter y). In the generalized vulnerability format of [19] the probabilities are replaced by $p_V(i'' \mid j, i')$ = the conditional probability that the structure enters the damage state i'' under a ground motion of intensity j and knowing its previous damage state i'. A suitable way to evaluate these probabilities (hence the corresponding 3D vulnerability matrix) consists in a modified version of the Bayesian updating method of fragilities due to A.Singhal & A.S.Kiremidjian [18]. We present this approach in the (earlier mentioned) paper by C. Robu *et al.* (Paper 1932 submitted to SMiRT 16, Div. M), which is complementary to this paper and also includes some examples of seismic damage evaluation in RC structures by use of fragility and vulnerability models.

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