RELIABILITY ANALYSIS OF PROJECTILE PENETRATION INTO GEOLOGICAL TARGETS

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ABSTRACT

The present study is dedicated to the reliability analysis of projectile penetration into a buried concrete target. The expressions for depths of penetration in the buried target have been derived for crater and tunnel regions separately. These penetration depths have then been employed for subsequent reliability analysis using First Order Reliability Method (FORM). Design points, important for probabilistic design, have been located on the failure surface. Sensitivity analysis has been carried out to study the influence of various random variables on projectile reliability. Some parametric studies have also been included to obtain some results of field interest.

INTRODUCTION

The area of *projectile impact* has been drawing attention of engineers and scientists the world over since World War-I. The recent nuclear tests conducted in the subcontinent have further awakened the investigators for undertaking extensive study on the subject of projectile impact upon different type of targets, particularly the geo-materials under which the strategic structures such as army bunkers and Nuclear Power Plants (NPP) may be buried. The success of these projectiles is measured through the damages caused to the targets. A full penetration of projectile into the target is usually considered as complete damage to the target. Investigators in the past have proposed various analytical, numerical and analytical formulas [1-8] for the estimation of penetration depths under a given projectile impact. These estimations and predictions, however, have significant uncertainties due to inherent variability involved in the associated variables. This shows that the projectile penetration is a probabilistic event, however, the review of past work [1] shows that almost all the researchers have considered it deterministic i.e. variability in the target material or in projectile properties are not given the due consideration. Further, perhaps no investigator has studied the reliability of these projectiles against target penetration. The present study is dedicated to the reliability analysis of projectile penetration into a buried concrete target. The expressions for the depth of penetration in the buried target have been derived separately for crater and tunnel regions using the methodology proposed by [2, 4 and 8]. These penetration depths have then been employed for subsequent reliability analysis using First Order Reliability Method (FORM). Design points, important for probabilistic design, have been located on the failure surface. Sensitivity analysis has been carried out to study the influence of various random variables on projectile reliability. Few parametric studies have also been included to obtain some results of field interest.

RELIABILITY ANALYSIS

In the present study the reliability analysis has been carried out using First Order Reliability Method (FORM) [9, 10]. In brief, in this approach of reliability estimation, the reliability is measured in terms of a *reliability index*, β, and it is related to the probability of failure or probability of limit state violation for any limit state as

$$\beta = -\Phi^{-1}(P_f) \tag{1}$$

where, P_f is the probability of failure and $\Phi^{-1}(\ )$ is the inverse of standard normal distribution function. The reliability index β is found from the solution of the constrained optimization problem:

$$\text{Minimize } \beta(y)=(y^T y)^{1/2} \qquad \text{subject to } G(y)=0 \tag{2}$$

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where **y** is a vector of basic random variables in the standard normal space and $G(\mathbf{y})$ is the limit state function. A limit state function is a mathematical representation of a particular limit state of failure.

The reliability index and the corresponding vector $\mathbf{y}^*$, usually referred to as a design point, obtained from the solution of Eq. (2) can also be used to estimate the influence of individual random variables on the projectile reliability in terms of the so-called *sensitivity factors*. For the jth random variable, the sensitivity factor, α_j, is defined as

$$\alpha_j = \left.\frac{\partial\beta}{\partial y_j}\right|_{y_j^*} = \frac{y_j^*}{\beta} \tag{3}$$

where y_j^* is the value of this variable at the design point.

Limit State Function

When a projectile travels through a geo-material, after traveling a certain depth hits the buried concrete target (Fig.1) it creates a conical shaped *crater region* with depth about two projectile shank diameter, (i.e. 4a), followed by a circular cylinder shaped *tunnel region* with diameter nearly equal to the shank diameter (i.e. 2a) [4]. The depths of penetration in the two regions have been obtained separately as follows:
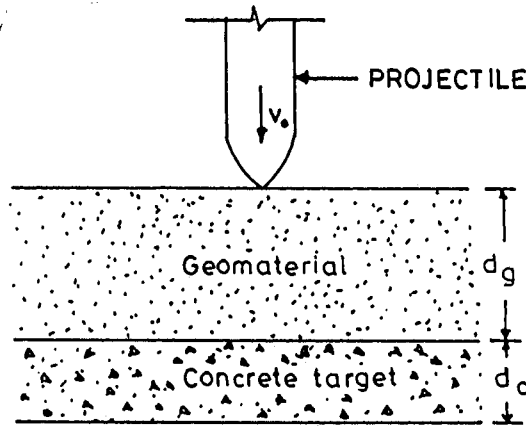

Fig. 1 Problem formulation

Crater Region

The depth of penetration in crater region has been obtained as

$$z_{sc} = \frac{2}{\sqrt{\pi}a}v_0\left(1 - \frac{d_g\alpha\pi a^2\tan\theta(K\rho_g)^{1/2}}{mv_0}\right)\sqrt{\frac{m + 4\pi a^3 N\rho_t}{sf_c + N\rho_t v_0^2\left(1 - \frac{d_g\alpha\pi a^2\tan\theta(K\rho_g)^{1/2}}{mv_0}\right)^2}} \tag{4}$$

where,
z_{sc} = crater depth , measured from the concrete surface;
a = projectile shank radius;
v_0 = impact velocity of projectile;
d_g = depth of concrete target from the soil surface;
θ = equivalent cone angle;
K = geo-material bulk modulus;
S = a parameter which depends on unconfined compressive strength of concrete;

$\quad= 82.6\, f_c^{-0.544}$ from reference [8] $\tag{5}$

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N = dimensionless constant that depends on missile caliber radius head ψ;

$$= (8\psi-1)/(24\psi^2) \tag{6}$$

ρ_t = density of concrete target;
m = mass of the missile;
K = geo-material bulk modulus;
ρ_g = density of geo-material
α = slope of the linear fit [2]; and
f_c = unconfined compressive strength of concrete.

Therefore, if the depth of the concrete target is d_c then the limit state function G can be expressed as

$$G(\underline{X}) = \frac{2}{\sqrt{\pi a}} v_0 \left(1 - \frac{d_g \alpha \pi a^2 \tan\theta (K\rho_g)^{1/2}}{mv_0} \right) \left(\frac{m + 4\pi a^3 N\rho_t}{Sf_c + N\rho_t v_0^2 \left(1 - \frac{d_g \alpha \pi a^2 \tan\theta (K\rho_g)^{1/2}}{mv_0} \right)^2} \right)^{1/2} - d_c \tag{7}$$

where $\underline{X}$ denotes the vector of random variables given by

$$\underline{X} = (m, a, N, v_0, K, \rho_g, \alpha, d_g, \rho_t, S, f_c, d_c) \tag{8}$$

Tunnel Region
In this region the depth of penetration z_{st}, measured from the concrete surface, have been obtained as:

$$z_{st} = 4a + \frac{m}{2\pi a^2 N\rho_t} \times$$
$$\ln\left[1 + \frac{N\rho_t(m^2 v_0^2 - 2md_g\alpha\pi a^2 \tan\theta\sqrt{K\rho_g} v_0 + d_g^2\alpha^2\pi^2 a^4 \tan^2\theta K\rho_g - 4\pi a^3 mSf_c)}{(m^2 Sf_c + 4\pi a^3 mN\rho_t Sf_c)} \right] \tag{9}$$

Therefore the limit state function, G, for this region can be expressed as:

$$G(\underline{X}) = 4a + \frac{m}{2\pi a^2 N\rho_t} \times$$
$$\ln\left[1 + \frac{N\rho_t(m^2 v_0^2 - 2md_g\alpha\pi a^2 \tan\theta\sqrt{K\rho_g} v_0 + d_g^2\alpha^2\pi^2 a^4 \tan^2\theta K\rho_g - 4\pi a^3 mSf_c)}{(m^2 Sf_c + 4\pi a^3 mN\rho_t Sf_c)} \right] - d_c \tag{10}$$

where $\underline{X}$ denotes the vector of random variables given by Eq (8). The notations used in Eq.(9) and Eq (10) have the same meaning as given in the crater region.

DISCUSSION OF RESULTS

A projectile of 182 kg mass and 411 m/s velocity [2] has been chosen for the reliability analysis. Other statistical data that are needed for reliability analysis have been presented in Table 1. The analysis gives the projectile reliability index as 0.02 and corresponding probability of failure as 0.49. These values show that the projectile has almost 50% chances of not penetrating the target completely. Moreover, it is to be noted that any projectile having probability of failure above 10^{-4} (or reliability index less than 3) is not a *reliable projectile.* Therefore, projectile of [2] is very less reliable to serve its purpose.

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Table 1 Statistical data (COV, coefficient of variation)

Random variables	Distribution	Mean	COV
Mass of the projectile, m	Lognormal	182 kg	0.05
Missile shank radius, a	Normal	0.0825 m	0.05
Dimensionless parameter, N	Normal	0.0355	0.05
Projectile impact velocity , v_0	Extreme type I	411.0 m/s	0.10
Geomaterial bulk modulus, K	Lognormal	9.52×10^3 MPa	0.15
Density of geomaterial , ρ_g	Lognormal	1.97×10^3 Kg/m^3	0.15
Slope of the linear fit, α	Normal	0.80	0.10
Depth of target in geomaterial, d_g	Normal	1.00 m	0.05
Density of concrete target, ρ_t	Lognormal	2000.0kg/m^3	0.10
Non dimensional parameter, S	Normal	15.56	0.15
Unconfined compressive strength of concrete, f_c	Lognormal	21.6 MPa	0.10
Thickness of concrete target, d_c	Normal	0.5 m	0.05

Design point or Most Probable Point (MPP)

In the probabilistic design of projectiles, for a given reliability index we locate a point, known as design point or most probable point, on the failure surface for which G($\underline{X}$) = 0. In the present study, for the reliability index of 4.0 this point has been located on the failure surface. The values of various random variables at this point are shown in Table 2.These values of different random variables are essential for reliability based probabilistic design of projectiles. In such designs, partial safety factors for load and resistance variables are determined for the required reliability (i.e. desired reliability index). These safety factors are separately defined for resistance and load variables. For resistance variables it is defined as the nominal, mean or characteristic value divided by the design value and for load variables as the design value divided by the nominal, mean or characteristic values.

Table 2 Design values for reliability index = 4.0

Random variables	Design values
Mass of the projectile, m	173.6 kg
Missile shank radius, a	0.11 m
Dimensionless parameter, N	0.036
Projectile impact velocity , v_0	794.9 m/s
Geomaterial bulk modulus, K	1.1×10^3 MPa
Density of geomaterial , ρ_g	1.97×10^3 Kg/m^3
Slope of the linear fit, α	1.01
Depth of target in geomaterial, d_g	1.04 m
Density of concrete target, ρ_t	1992.7 kg/m^3
Non dimensional parameter, S	16.21
Unconfined compressive strength of concrete, f_c	22.2 MPa
Thickness of concrete target, d_c	0.61 m

Sensitivity Analysis

Figure 2 shows that sensitivity factors for projectile mass and velocity are negative which indicates that these variables are resistance variables i.e. their increase in magnitude will improve the reliability of projectile against penetration. Other random variables are positive which show that they are the load variables and increases in their magnitude will increase the failure probability of projectile. Chart also indicates that the projectile impact velocity is more influencing to reliability than its mass. Moreover, shank radius of projectile adversely affects its reliability as it is the most influencing load variable.
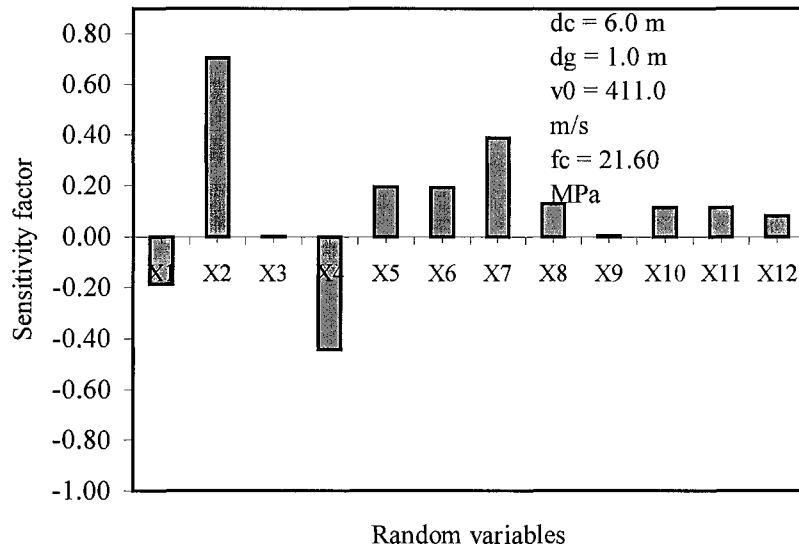
Fig. 2 Sensitivity diagram

where
  X1 - Mass of the projectile, m
  X2 - Projectile shank radius, a
  X3 - Dimensionless parameter, N
  X4 - Projectile impact velocity , v_0
  X5 - Geomaterial bulk modulus, K
  X6 - Density of geomaterial , ρ_g
  X7 - Slope of the linear fit, α
  X8 - Depth of target in geomaterial, d_g
  X9 - Density of concrete target, ρ_t
  X10- A parameter, S
  X11- compressive strength of concrete, f_c
  X12 - Thickness of concrete target, d_c

Effect of Velocity

Velocity is a direct measure of projectile energy. As it increases its energy to penetrate the target also increases. Fig.2 shows that the missile reliability or chances of its penetrating the concrete barrier increases with the velocity. This is an expected trend. The figure also shows that the reliability index is 3-4, for the velocity range of 800–900 m/s. This range of reliability index is usually considered as the desirable range for probability based design. Therefore for the present study we can conclude that to achieve the reliability range of 3-4, the projectile should impact the geo-material with a velocity range of 800-900 m/s.

Effect of Uncertainty in Velocity

Figure 3 shows that as the uncertainty, measured in terms of Coefficient of Variation (COV), in the mean velocity increases there is corresponding continuous decrease in the reliability index magnitude. This shows that it is not only the mean velocity that controls the reliability of projectile but also the COV that plays a very significant role in determining its reliability. The figure shows that when COV is 5%, reliability is around 4.3 and it falls to about 3.4 when COV becomes 17%. This indicates that an increase of 12% in COV results in about 20% reduction in the projectile reliability index.
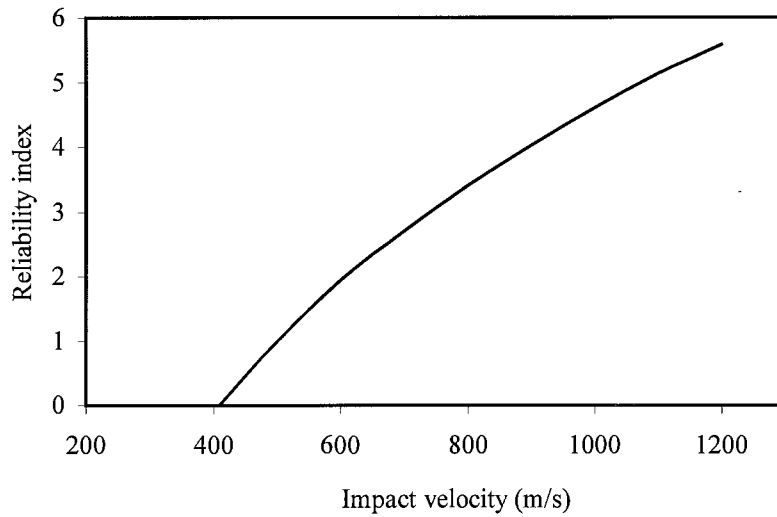
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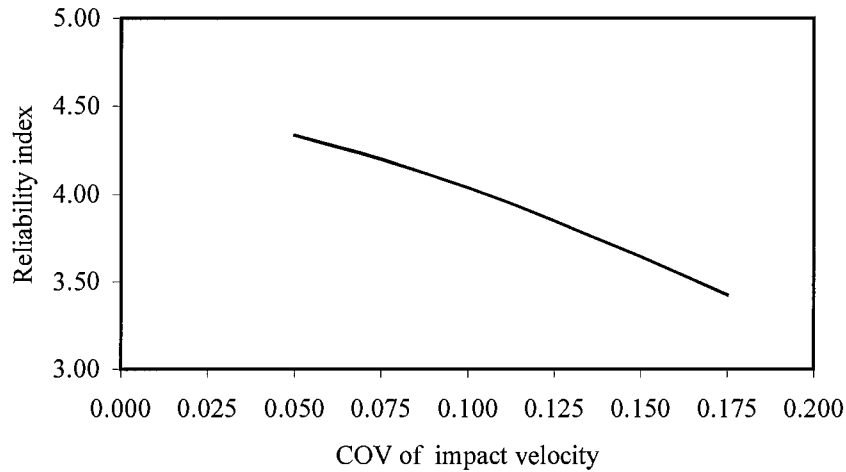Fig. 3 Effect of impact velocity on projectile reliability

Fig. 4 Effect of uncertainty in impact velocity

Effect of depth of geo-material

The geo-material is lying above the concrete barrier. This overlying material acts as an energy dissipater to the projectiles. Thus it acts as a load variable to the projectile reliability (Fig. 2). The Fig. 5 shows that when the depth of overlying geo-material is 0.50 m, reliability index is 1.8 ($P_f = 3.54 \times 10^{-2}$) and it reduces to almost zero at a depth of 1.0 m. This indicates that for the projectile to perform its function successfully correct estimation of depth of overlying geo-material is very important.
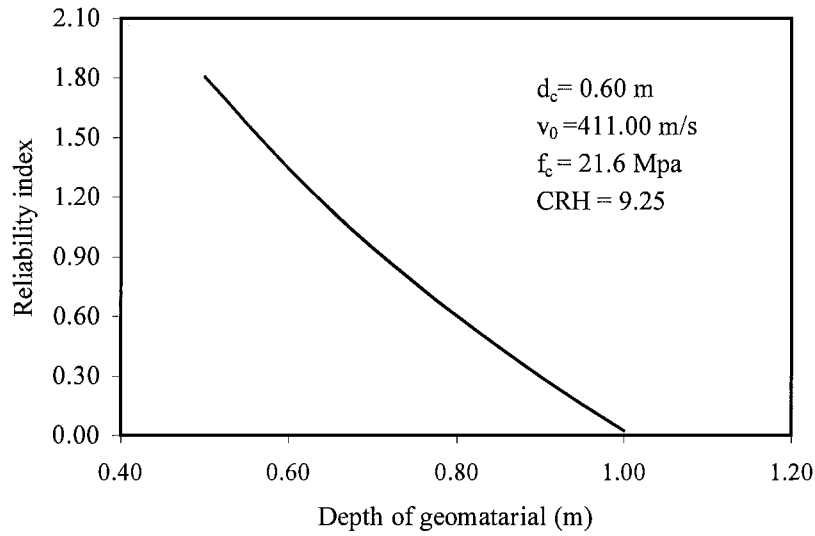
Fig. 5 Effect of depth of geomatarial on projectile reliability

Effect of Concrete Thickness

Thickness of concrete directly affects the projectile reliability. Since it is load variable (Fig. 2) obviously with the increase of its thickness projectile reliability will decrease.
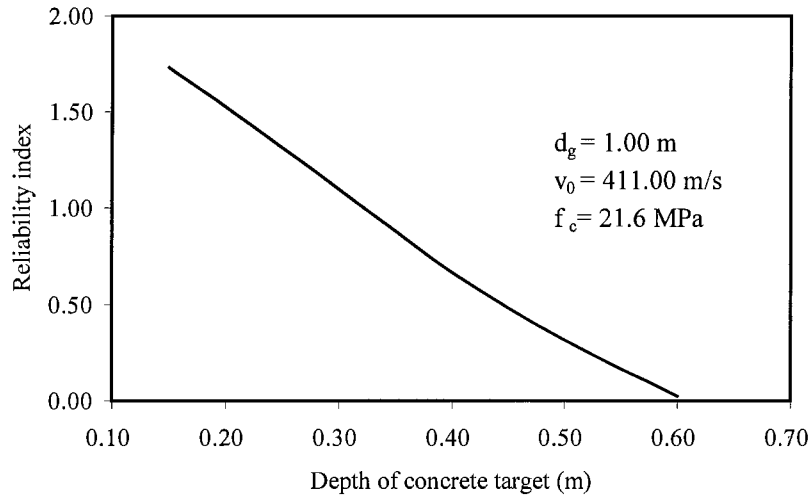

Fig. 6 Effect of concrete thickness on projectile reliability

Fig. 6 shows when the concrete barrier thickness is 0.15 m (i.e. 15 cm) reliability is around 1.8, however, it becomes almost zero when it is 60 cm. Keeping in view the desirable reliability range of 3-4 we may conclude through this study that the projectile of present study is reliable enough only when thickness of the concrete barrier is less than 15 cm.

CONCLUSIONS

Following conclusions may be drawn from the reliability analysis of an example given in the paper:

- The projectile is very less reliable to serve its purpose of causing damage to the concrete target.
- To achieve the reliability range of 3-4, the projectile should impact the geo-material with a velocity range of 800-900 m./s.
- Projectile impact velocity is more influencing to its reliability of causing damage to concrete target than projectile mass.
- Projectile shank radius adversely affects its reliability.
- An increase of 12% in COV results in about 20% reduction in the projectile reliability index.
- For the projectile to perform its function successfully correct estimation of depth of overlying geo-material is essential.
- The projectile of present study is reliable enough to damage the target only when thickness of the concrete barrier is less than 15 cm.

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