

# Seismic vulnerability and fragility formats in damage risk assessment of RC structures

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## ABSTRACT

The evaluation of damages induced by earthquake motions and the prediction of expected seismic damages were intensively studied in the literature on earthquake engineering. Both seismic fragility and vulnerability models have been employed for the damage risk assessment of structures in areas with a high likelihood of earthquake motions. However, the former models evaluate the probability of failure (that may be assimilated to a severe damage) conditional on given levels of earthquake excitation, and they are more or less limited to the use of the lognormal format. The latter (vulnerability) models appear to be more suitable for the seismic damage assessment. In this paper we propose a couple of ways to integrate approaches based on fragility and vulnerability concepts for the damage evaluation of structures, with special attention paid to RC structures. The vulnerability of earlier damaged structures is quantified by a Bayesian updating approach. The number and the amplitude of the inelastic structural response peaks to earthquakes is also taken into account. A couple of examples for damage risk assessment of RC frame structures are presented.

## INTRODUCTION

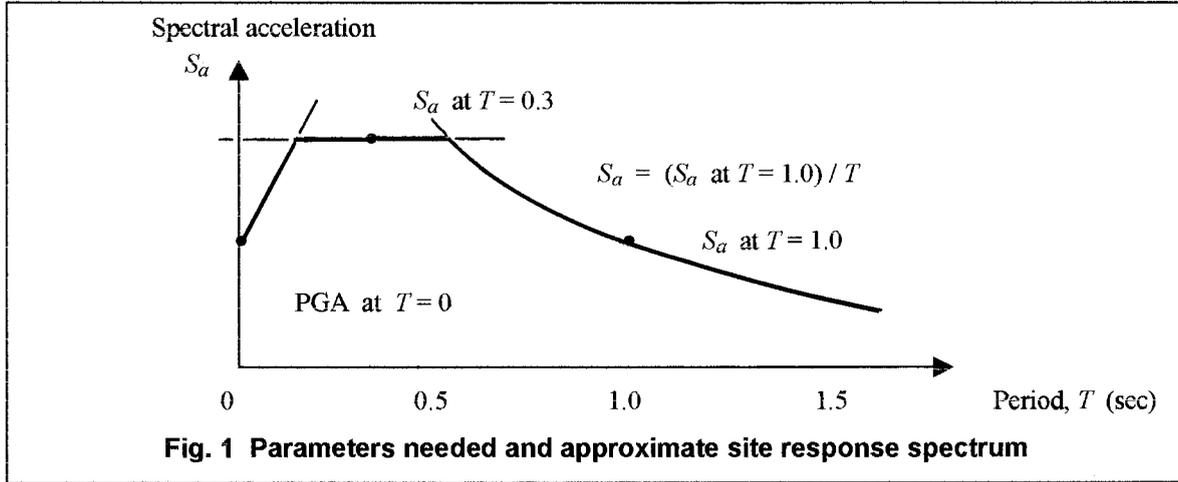
Among the various issues related to the structural reliability evaluation and seismic risk assessment, the risk to seismically induced damages to components and structures has been and is widely investigated. Many analytical models and a rather large variety of damage indices or damage functionals have been proposed in hundreds of articles, monographs and reports. Such topics are well covered by lectures and papers presented to WCEE and ECEE Conferences, as well as to SMiRT Conferences (Divisions K and M). Suitable mechanical models for the damage development under seismic ground motions in specific types of components and structures are quite necessary, yet not sufficient for a realistic prediction for the likelihood of damage occurrence during future earthquake in a given area. Probabilistic / stochastic approaches are unavoidable, and many models of this nature have been developed in the Reliability and Seismic Risk Analysis. Two seismically specific concepts and methods based on them are those based on seismic fragility and vulnerability. The former notion means the failure probability conditional on a certain earthquake intensity. Hence, two states of the component or structure are taken into account : failure / non-failure. In the damage analysis, a whole range of damage levels are usually considered, from “no damage” (or “insignificant damage”) up to “collapse” [1], [2]. Some ways to adapt fragility methods to this problem were recently proposed by A. Singhal and A.S. Kiremidjian [3], [4]. We have also presented some extensions of fragility models to seismic damage assessment in [5], [6]. The concept of seismic vulnerability [7], [8], [9], [10] appears to be better suited for the risk-to-seismic-damage evaluation.

In this paper we go further with presenting possibilities of applying vulnerability and fragility based methods for the seismically induced damage evaluation of RC components and structures. Some ideas of [5], [6] are developed, and a part of the analytical and probabilistic models of a complementary paper submitted to SMiRT 16 (Paper 1650 – Ref. [11]) are used. We give some details on the characterizations of ground motion in the next section. Possibilities to use certain parameters of the seismic structural response for damage evaluation, including the ordered peaks [12], are then considered. The vulnerability models are presented and discussed in more detail, and a proposal for the damage assessment through Bayesian updating for damage probabilities of previously damaged structures is formulated. Two examples involving multi-storey RC frame structures illustrate two of the proposed methods for damage prediction by use of vulnerability – fragility based methods. A couple of final remarks close the paper.

## GROUND MOTION MODELING

The problem of stating analytical and stochastic models for the seismic ground motion is not specific to the damage assessment of certain types of structures, like those built of reinforced concrete. However, specific models are preferred in the seismic risk studies formulated in terms of fragility / vulnerability methods. In order to characterize earthquake ground motions for the purposes of evaluating structural performance, it is necessary to describe the amplitude, frequency content, and duration of ground motion. It is difficult to select a single parameter that would capture the above main characteristics of the ground motion. The seismic characterization of the site, such as macroseismic intensity, peak ground acceleration (PGA) or velocity (PGV), accelerograms, etc. are often used in fragility and vulnerability models. Spectral acceleration values are used in [3] to specify different levels of ground motion for developing motion-damage relationship in the form of fragility curves. The spectral values at different periods are frequently represented by means of dynamic amplification factors which represent the normalized spectral values at specified damping ; such a factor is

obtained as the ratio of the spectral acceleration  $S_a$  to the PGA  $a$ . The spectral acceleration depends on the natural period  $T$  at the site. An example of this relationship between and the spectral acceleration is presented in [12] by means of the figure that follows :



**Fig. 1 Parameters needed and approximate site response spectrum**

Various intensity parameters of the seismic ground motion are related through specific relations, what makes possible to replace a parameter by another as required by the selected model for the seismic hazard. An attenuation relation for the PGA in terms of the magnitude  $m$  and the focal distance  $r$  is formulated in [13] as

$$a_g = c_1 e^{c_2 m} r^{-c_3} \quad (1)$$

where  $a_g$  is the value of the (random) PGA  $A$ ,  $m$  is the magnitude,  $r$  is the focal distance and the three coefficients are deterministic, with their values estimated as follows :  $c_1 = 15$ ,  $c_2 = 1.4$ ,  $c_3 = 1.7$ . The inverse function, expressing the magnitude in terms of the PGA, is

$$m(a_g) = (\ln a_g - \ln c_1 + c_3 \ln r) / c_2 \quad (2)$$

Certainly, the values for the three constant coefficients are site-dependent.

The spectral acceleration  $S_a$  is proportional to the PGA  $a_g$  and both of them are related to the MMI intensity  $MMI$  by an approximate logarithmic – linear relation (according to [13]) :

$$\ln a_g \approx \alpha + \beta MMI \quad \Leftrightarrow \quad a_g = e^{\alpha + \beta MMI} \quad (3)$$

Murphy and O'Brien estimated the two constants in Eq. (3) to  $\alpha = \beta = 0.25$ .

An earthquake intensity parameter related to the ground motion acceleration is the Arias intensity. While the PGA and the total duration provide only general information extracted from the earthquake accelerograms. A numerical processing of the accelerograms have resulted in specific parameters expressed in terms of time or of frequency content. The Arias intensity, the strong motion duration, the power and energy spectra are parameters which are better suited for seismic damage analysis by energy-based models. If  $t_e$  denotes the total earthquake duration and  $\ddot{x}$  is the ground acceleration then the Arias intensity is defined by

$$I_0 = \int_0^{t_e} (\ddot{x})^2 dt \quad (4)$$

The HUSID diagram is the time history of the seismic energy content scaled to the total energy content which is just the arias intensity. If the upper limit in the integral in Eq. (4) is replaced by  $t$  ( $0 \leq t \leq t_e$ ) and the corresponding integral is denoted by  $I(t)$  then the HUSID diagram is defined as the time-dependent function  $H(t) = I(t) / I_0$ . The strong motion duration  $T_{0.90}$  is defined by means of this HUSID diagram as

$$T_{0.90} = T_{0.95} - T_{0.05}, \text{ i.e., } T_{0.90} = H^{-1}(0.95) - H^{-1}(0.05) \quad (5)$$

Obviously, the HUSID function  $H(t)$  is monotonically increasing, hence its inverse is well defined and we used  $H^{-1}$  for a more rigorous definition of  $T_{0.90}$ . These seismic parameters  $I_0$ ,  $H(t)$ ,  $T_{0.90}$  are involved in damage potential description by energy-based models of [13]; a slightly different definition of the Arias intensity is given in [14], namely

$$I_A = \frac{\pi}{2g} \int_0^{t_e} (\ddot{x})^2 dt = \frac{\pi}{2g} I_0. \quad (6)$$

In the long-term studies for predicting the probability of a structure to keep its carrying or service capacity during its design lifetime, a relevant parameter is the (expected) total number of earthquake occurrences of a magnitude  $M$  greater than  $m$  in a source region. This number depends on the return period of the earthquakes of a given magnitude or intensity. We present, in Table 1, the numerical data on the return periods for the Vrancea main seismic source in Romania.

**Table 1. Return periods for Vrancea seismic source**

Magnitude $M$ (Richter-Gutenberg)	6.0	6.5	7.0	7.5
Return period $T_{\text{ret}}$ – years	6	14	32	125
MSK Intensity $I$	VII	VIII	IX	
Return period $T_{\text{ret}}$ – years	20	50	200	

The data for the MSK intensity were estimated for the Bucharest area. As regards the return periods for the major values of the magnitude, they were evaluated on the to a law due to Sandi and Staicu [7]. An earthquake severity parameter is used, namely  $k_s = \text{PGA} / g$ .

$$\lg T_{\text{ret}}(k_s) = \frac{14}{2.5 - \log_2 k_s} - 1.2 = \frac{4.2}{7.5 - \lg k_s} - 1.2. \quad (7)$$

In Eq. (7),  $\lg$  denotes the logarithm in basis 10. If it is assumed that the occurrence of earthquake of a given magnitude / intensity follows a Poisson law, it is possible to derive the expected annual rate of earthquakes of severity not less than 1 (i.e., with  $k_s \geq 1$ ). This rate, denoted  $n_1$ , is defined by  $n_1 = N_1 - N_{1+\kappa}$  where  $\kappa$  is the minimal step on an accepted scale of earthquake severity. If  $N_0$  denotes the total number of earthquake occurrences of magnitude greater than a lower bound magnitude  $m_0$  then the number of occurrences of magnitude greater than  $m$  then the annual number of earthquakes of magnitude  $m$  is expressed in [15] by

$$N(m) = N_0[1 - F_M(m)]. \quad (8)$$

The same assumption of a Poisson process for the occurrence process of earthquakes allows to express the probability distribution of the maximum magnitude  $F_{MX}(m)$ , that is the probability that all the earthquakes have a magnitude  $\leq m$ :

$$F_{MX}(m, t) = \sum_{k=0}^{\infty} \frac{[N_0 t F_M(m)]^k}{k!} e^{-N_0 t} = e^{-N_0 t [1 - F_M(m)]}. \quad (9)$$

Eqs. (8) and (9) allow to express this *cdf* and the corresponding *pdf* in terms of the number of earthquakes as

$$F_{MX}(m, t) = e^{-N(m)t}, \quad f_{MX}(m, t) = -\frac{dN(m)}{dm} e^{-N(m)t}. \quad (10)$$

The functions in Eqs. (10) get specific expressions if a model due to T.Utsu is used for  $F_M(m)$  and  $N(m)$ . They are presented in [15].

The seismic hazard modeling also needs certain (site-specific) attenuation laws to be formulated. We only give an example of [16] for intermediate hypocentral depth earthquakes from Vrancea seismogene area. The law that follows

relates the magnitude, the hypocentral distance and a factor representing the variability in the PGA, to the horizontal PGA (denoted by  $Acc$ ) which is assumed to follow a log-normal distribution :

$$Acc = e^{b + b_M M} (R_h + C)^{b_R} \varepsilon \quad (11)$$

where  $M$  is the Richter magnitude,  $R_h = \|(R_e / \rho, h)\|$  is the hypocentral distance (in km) with  $h$  = the focal depth and the epicentral distance  $R_e$  corrected by the non-homogeneous attenuation factor  $\rho = [(1 + \tan^2 \alpha) / (a^{-2} + \tan^2 \alpha)]^{1/2}$ ;  $b, b_M$  and  $b_R$  are regression coefficients, while  $\varepsilon$  is the random factor with  $\sigma_{\ln Acc}$  = the standard deviation of  $\ln \varepsilon$ ;  $a$  in the expression of  $\rho$  is the ratio between the half-axes of the attenuation ellipse and  $\alpha$  is the azimuth to the instrument site with respect to energy propagation pattern and  $C$  is a constant. It is clear that the attenuation law in Eq. (11) is similar to Shibata-Utsu's law in Eq. (1), yet more general and with a probabilistic distributional assumption explicitly stated. The values of the coefficients that occur in this law were estimated (by regression analysis) for the last four major earthquakes in Romania (1977, 1986, 1990 - May 30 & 31).

Some of the formulas summarized in this section will be used for the seismic damage risk assessment by vulnerability and fragility formats.

## VULNERABILITY MODELS FOR SEISMIC DAMAGE EVALUATION

The concept of seismic vulnerability was developed since the mid-80's (with 8 ECEE, 1986) but mainly since 1990 (with several contributions to SEISMED Conf. held in Trieste and to 9 ECEE held in Moscow). It emerged from the necessity to separate the evaluation of seismic hazard from the assessment of the consequences of major seismic events. The hazard can be defined [8] as the probability of occurrence of an earthquake at a particular level of ground shaking within a certain period of time. The vulnerability is a measure of the probable effects of each possible level of ground shaking. This is obviously a very general definition. As we have seen in the previous section, the term of "level of ground shaking" can incorporate several relevant parameters of the ground motion. We suggest the term of "ground motion severity" as better suited for taking into account several such parameters (besides the magnitude or intensity, for instance). Each of them can have a higher or lower significance to the effects of an earthquake on a given structure in a certain area. A classification criterion of vulnerability-based approaches was reported by EAAE Working Group 3 to the 10<sup>th</sup> ECEE (1994) – Ref. [8]. This criterion is stated in terms of the nature of *input* data, the *method* employed and the *output* data. It deserves mentioning the basically probabilistic nature of both fragility and vulnerability models.

As we have remarked in [11], a difference occurs between the original fragility models (developed in the 80's) and the vulnerability models regarding the nature of what could be called the "output event" : the structural failure in the fragility concept, respectively reaching a certain damage level in the vulnerability approach. However, the difference is not so neat. Given a normalized damage measure  $D$ , taking values in the interval  $[0,1]$ , the structural failure can be defined as the event ( $D \geq 0.4$ ). As regards the fragility approaches, they have been adapted for the damage evaluation of buildings in urban areas by the projects like ATC-13, ATC-43 etc. Let us also mention that some of the vulnerability models use lognormal distributions, what is a common assumption in the seismic fragility models.

In view of the rather wide range of definitions and approaches to the seismic vulnerability concept it is hard to choose a sufficiently general formal statement of the vulnerability functions, and we do not aim to such a task. However, several definitions due to the Bucharest research group on seismic vulnerability [7], [9], [10], [17] give a basis for such an attempt. A useful seismic vulnerability or fragility damage measure for damage evaluation should be applicable not to a individual structure but to a whole class of structures with similar characteristics and located in a more-or less seismically homogeneous area. Let  $S^{(i)}$  denote a class of structures and let  $j$  denote a category of earthquakes likely to occur at a given site (among several possible categories). It is natural to consider this index as being site-specific. The severity (or intensity) of the earthquake is denoted by  $q$ . Then the average damage degree of a structure in category  $S^{(i)}$  induced by an earthquake of severity  $q$  or category  $j$  is defined by

$$d^*(q) = \sum_k k p_k(q) \quad \text{or} \quad d_j^* = \sum_k k p_{k|j} \quad (12)$$

These Eqs. (12) are given in [17]; a subscript / superscript  $i$  to identify the class of structures is omitted. Let us also remark that the category of earthquake  $j$  is used as an alternative to the earthquake severity. As regards  $k$ , it would stand for the damage degree. However, the sum limits are not specified. A subsequent formula (due to M.Dolce) uses the same notation  $k$  as the lower index in a binomial formula; in fact, this  $k$  would be the current value in a binomial distribution, with  $0 \leq k \leq n$ , and the probability taken as  $p = d^*/n$ . Since the expected value of the binomial distribution is  $np$  it follows that the latter equation (giving  $p$ ) and the first of Eqs. (12) follow from each other and it is essential to define the probability  $p_k(q)$  which is, in fact, the conditional probability that a structure in the considered class will undergo a damage of level  $k$  under an earthquake of severity  $q$ . Thus, such a probability is – in fact – a

fragility expressed in terms of a damage scale with positive integer values :  $\{0, 1, \dots, n\}$ ;  $n = 4$  would correspond to the most usual range of five damage severity levels:

$$\{\text{none or insignificant, minor or slight, moderate, severe or heavy, collapse}\}. \quad (13)$$

These levels are expressed in terms of a normalized damage measure, over the interval  $[0,1]$ , in many papers on damage evaluation and also in [11] – Table 1. It appears that more realistic vulnerability models would have to make use of the rich “offer” of damage assessment models existing in the literature of the field published in the last decades.

Let  $D$  denote a random seismic damage measure taking values in the interval  $[0, \delta_{\max}]$  or  $[0, \sup D]$ . In a rather general formulation (proposed in [11]),  $D$  should be expressed as a function of the form

$$D = \varphi(\mathbf{d}, \theta) \text{ with } \mathbf{d} = (d_1, d_2, \dots, d_m) \text{ and } \theta = (\theta_1, \theta_2, \dots, \theta_\ell) \quad (14)$$

where  $d_i$  are damage parameters while  $\theta_j$  are structural & demand parameters related to the selected damage models. Certainly, the separation between the two types of parameters should not be taken *ad litteram*; some of the demand parameters  $\theta_j$  connected with the ground motion may be arguments to damage parameters. We gave some examples in [11]. From the probabilistic point of view, it is very important which of the parameters  $d_i$  and  $\theta_j$  are assumed to be random, and what distributional hypotheses are accepted for them. In any case, if uncertainty is assumed for least one argument of  $\varphi$  of Eq. (14) then  $D$  is uncertain. Let us exemplify such a dependence and the randomness aspect with Eqs. (4) or (6) of the previous section : acceleration  $\ddot{x}$  is a random (and time-dependent) function and the ground motion duration  $t_e$  is also uncertain, hence the Arias intensity is a random function, too. If the time variable is explicitly considered as one of the parameters  $\theta_j$  ( $= t$ ) then the model is effectively time-dependent and a proper treatment should involve random (opr stochastic) processes. Such an approach, with the seismic structural response spectrum  $X(t)$  as a stationary random process, is considered by Basu & Gupta in [12] and (earlier) by Jeong & Iwan [18]. We shall see, in Example 1, how the model of [12] can be reformulated for seismic vulnerability evaluation. In the early studies on vulnerability, this concept was specific to a single seismic event. In a broader approach, in the context of seismic risk studies, the vulnerability is related to the risk of damage occurrence at a certain level induced by a sequence of earthquakes of expected intensities.

The probabilities of the form  $p_{k|j}$  involved in the second of Eqs. (12) can be formulated in terms of the damage functional of Eq. (14) as follows :

$$p_{k|j} = P(\delta_{k-1} \leq D < \delta_k | \theta_q = q \in I_j) \quad (15)$$

where  $k$  identifies one of the possible damage severity levels ( $1 \leq k \leq n$ ),  $j$  corresponds to an earthquake intensity class (for instance the magnitude or the MSK / Arias intensity,  $1 \leq j \leq J$ ),  $\theta_q$  is the selected random earthquake intensity parameter and  $I_j = [q_{j-1}, q_j)$  is the interval defining an earthquake intensity of class  $j$ . Certainly, the practical use of the formula in Eq. (15) needs distributional assumptions on  $D$  and  $\theta_q$  expressed in terms of *cdf*'s or *pdf*'s. If we redenote  $\theta_q$  by  $Q$  and  $f_{D, Q}$ ,  $f_Q$  are the joint *pdf* (probability distribution function) of damage-intensity and the *pdf* of the intensity parameter respectively, then the probability in Eq. (15) can be written as

$$p_{k|j} = \frac{\int_{\delta_{k-1}}^{\delta_k} \int_{q_{j-1}}^{q_j} f_{D, Q}(d, q) dq dd}{\int_{q_{j-1}}^{q_j} f_Q(q) dq} \quad (16)$$

According to earlier definitions, the vulnerability was introduced as the probability of a structure in a certain class / of a certain type to undergo seismically induced damages beyond which the repair was not possible or too expensive. Thus, if  $D$  is a (synthetic) damage measure then a threshold  $\delta^*$  such that the event ( $D \geq \delta^*$ ) means that the structures enters a non-repairable state, then its seismic vulnerability due to an expected earthquake of category  $j$  can be defined as

$$V_j = \frac{\int_{\delta^*}^{\sup D} \int_{q_{j-1}}^{q_j} f_{D, Q}(d, q) dq dd}{\int_{q_{j-1}}^{q_j} f_Q(q) dq} \quad (17)$$

The overall seismic vulnerability can now be defined as the sum of vulnerabilities in Eq. (15) over the whole range of earthquake severity classes :

$$V = \sum_{j=1}^J V_j. \quad (18)$$

In Eqs. (16) and (17) the dependence on the parameters  $\theta$  has not been explicitly appearing. However, this dependence (stated in Eq. (14)) cannot be neglected. It is essential for a proper definition of a generalized vulnerability concept. A proposal for such a definition was formulated in [10] and [17]. The extended vulnerability is defined as the probability of the structure to reach a certain damage severity level conditional on the next (expected) occurrence of an earthquake of a certain category  $j$  but also on the updated damage state induced by the previous seismic event(s). Formally, we restate it as

$$V_j | d_{\text{pre}} = P(\delta_{k-1} \leq D < \delta_k | Q \in I_j, d_{\text{pre}} \in [\delta_{\ell-1}, \delta_{\ell}]), \quad (19)$$

where  $d_{\text{pre}}$  = the damage level reached by the structure due to the previous earthquake(s), assuming that no repair or strengthening operations have been applied until the current moment  $t$ . Such probabilities are denoted by  $p_{k'|j, k'}$  and (in [10,17]) several logical conditions on the successive damage states, conditional on earthquake intensities, are formulated. The main regards the damage severity expected to be induced by the next earthquake (of the same or higher intensity as the previous one) which will naturally be higher. The essential problem consists in selecting a method for estimating  $d_{\text{pre}}$ . We consider that the method based on Bayes theorem are the best choice. Theoretically, this damage level could be estimated by field inspections and various testing methods. Yet the results could not be of highest confidence. Alternatively, this previous damage level can be estimated on the basis of registered ground motion parameters at (or near) the site of the structure and also – if available – of registered features of the structural response like the EPA / EPV (effective peak acceleration / velocity), the characteristic period  $T_c$ , the strong motion duration  $T_{\text{str}}$  and others.

Before giving two examples using this approach on parameter updating, we illustrate the dependence between two ground motion parameters, namely  $\ln a_g$  and  $MMI$  (see Eqs.(3)), and the pre-event – post-event vulnerabilities, (respectively) in Fig. 1 below.

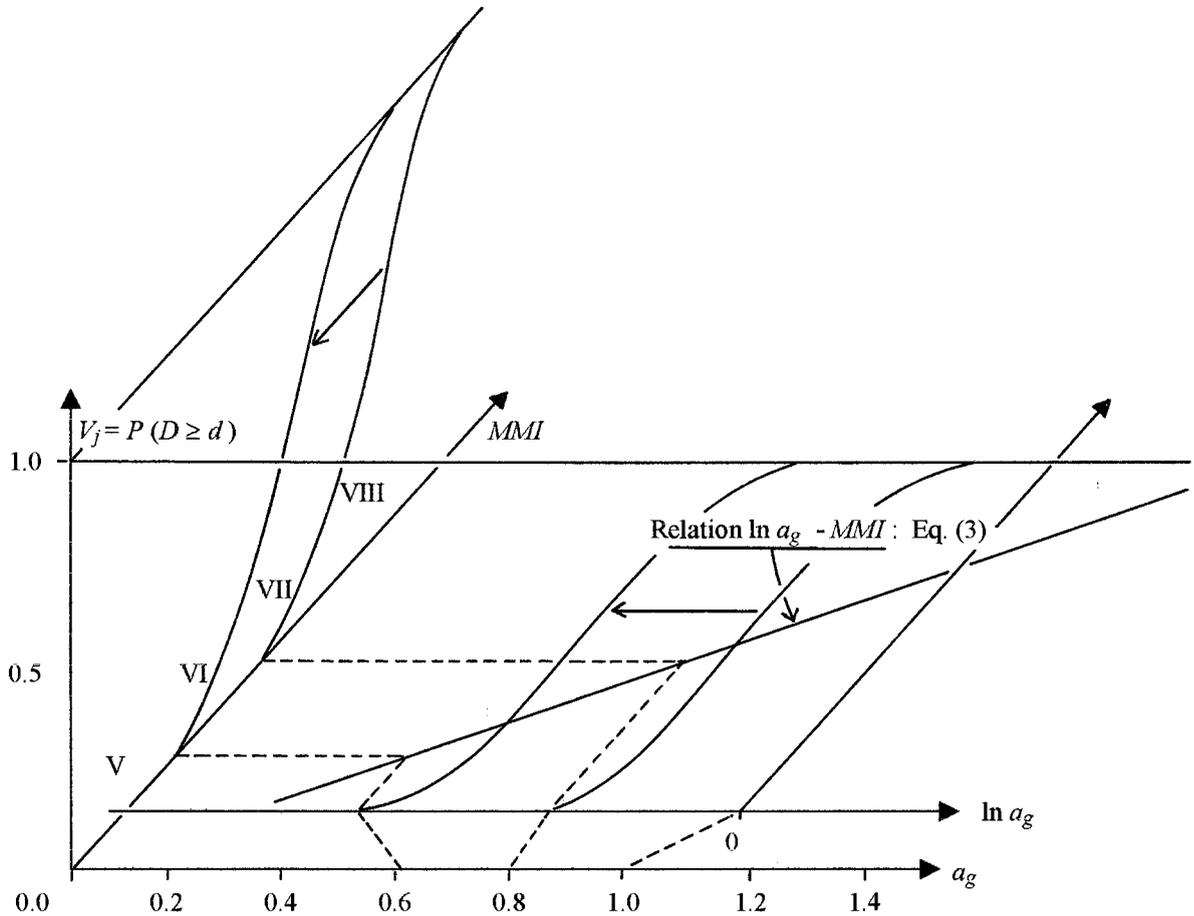


Fig.2 Vulnerability vs  $\ln a_g$  and  $MMI$

## EXAMPLES OF VULNERABILITY EVALUATION

### Example 1

One of the most widely accepted damage indices is the Park & Ang index [2], [3], [4]. We present it with modified notations for the ground motion level ( $Q, q$  instead of  $Y, y$  and  $D$  instead of  $I$ ):

$$D(\theta) = \frac{\theta_m}{\theta_u} + \frac{\beta}{M_y \theta_u} \int dE, \quad (20)$$

where the five parameters are random variables, namely:  $\theta_m$  = the maximum positive or negative plastic hinge rotation,  $\theta_u$  = the ultimate plating hinge rotation under monotonic loading,  $\beta$  = a model parameter,  $M_y$  = the calculated yield strength, and  $dE$  = the incremental dissipated energy. The integral in Eq.(20) is taken over the earthquake duration  $[0, t_e]$ . It was shown in [3] that the randomness in the damage parameter  $D(\theta)$  for RC frame structures can be well represented by the lognormal distribution. Hence  $\ln D \in N(\mu, \sigma)$  where  $\mu = \ln(\text{median of } D)$  and  $\sigma =$  the logarithmic standard deviation, that is the standard deviation of  $\ln D$ . Let us assume that the random parameter  $\theta$  has been estimated to a (vector) value  $\theta'$  but a set of observations on the damage index over a population of structures of the same class and in the same seismic area (ar subjected to similar earthquake excitations) becomes available; let this vector be  $\mathbf{d} = (d_1, d_2, \dots, d_n)$ . Then the value of  $\theta'$  can be updated to a new value  $\theta''$  by the maximum likelihood method. The likelihood function is

$$L(\text{exp } \mu, \sigma | \hat{\mathbf{d}}) = \prod_{i=1}^n f_D(d_i; \mu, \sigma), \quad (21)$$

where the function under the product is the lognormal *pdf*. A simplified expression for  $\ln L$  and the posterior estimations of the Log-N parameters, that is  $\mu''$  and  $\sigma''$  can be obtained as functions of the prior estimates  $\mu', \sigma'$  and the theoretical parameters  $\mu, \sigma$ . The updated lognormal *pdf* allows to evaluate the extended vulnerability

$$V(j | \mathbf{d}') = \int_{\delta^*}^{\sup D} f_D(\delta; \mu'', \sigma'') d\delta. \quad (22)$$

Two pre-event and post-event vulnerability curves are represented in Fig 2 that follows.

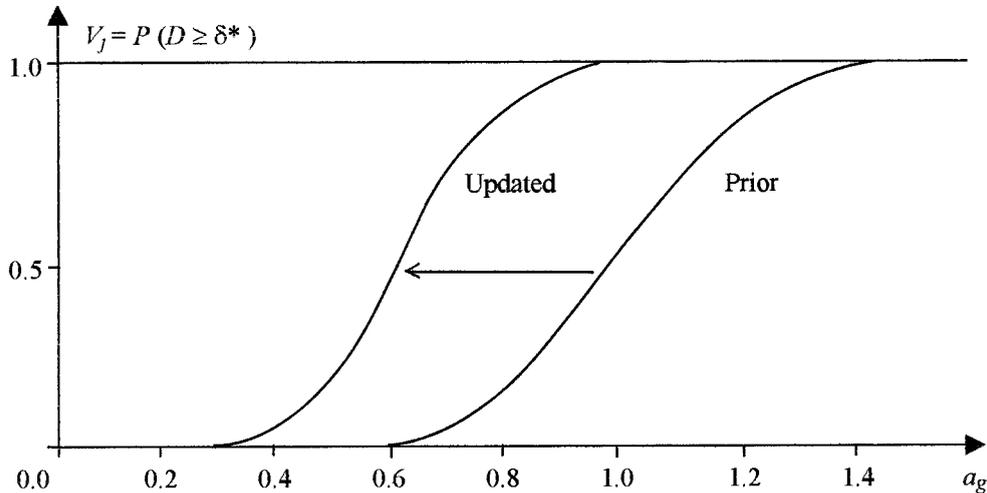


Fig. 3 Prior and updated seismic vulnerability curves

### Example 2

As we already mentioned, the explicitly time-dependent methods are less frequently used in damage evaluation and also in seismic risk evaluation by fragility and vulnerability models. A proper time-dependent approach should be formulated in terms of random processes or (alternatively) of time-dependent reliability models; an example for the latter ones can be found in [20]. One of the former models is presented by Basu & Gupta in [12], formulated in terms of the time history of the structural response  $X(t)$ . The  $n$  peaks of this response are decreasingly ordered, that is the first peak is just that with largest amplitude and the other peaks follow:  $x(1), \dots, x(k), \dots, x(n)$ . The distribution of ordered maxima was studied by Longuet & Higgins and a *pdf*  $p(\eta)$  was expressed in terms of the r.m.s of  $X$  and of  $\varepsilon$ , a measure of the width of the energy spectrum; the probability that the  $i$ -th peak is above  $\eta$  is expressed in terms of a binomial

distribution. This model is applied to a cumulative expected damage index. With no possibility to enter into details here, we only suggest that the failure law for a structure with repeated large strains in its members, namely  $N \mu^s = C$  is adequate for being used in seismic vulnerability models.  $N$  is the number of cycles to cause failure at the ductility level  $\mu$  and  $s, C$  are empirically determined constants. This rather simple model clearly allows for the Bayesian updating of parameters  $s, C$  and also for the formulation of a vulnerability model by selecting a threshold  $\mu^*$  corresponding to  $\delta^*$ .

## CONCLUDING REMARKS

Some methods developed in the damage fragility studies have been adapted for use in the seismic vulnerability evaluation. Vulnerability updating allows to account for damages due to previous earthquakes.

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