

## Nonlinear control for a large air-gap magnetic bearing system

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### ABSTRACT

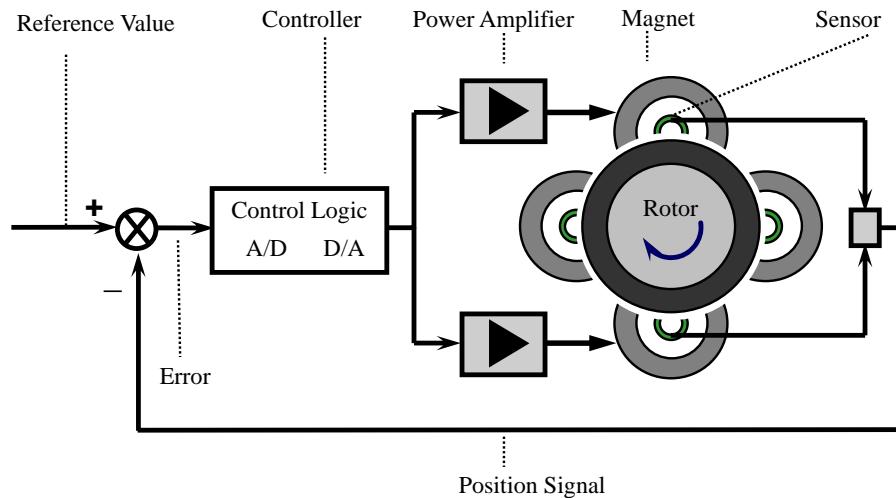
Active magnetic bearings are increasingly being utilized in rotating machinery applications as an alternative to the conventional rolling-element and fluid-film bearing types. In this paper three control schemes for a large air-gap magnetic bearing system are described. In the large air-gap situation, the magnetic bearing system operates in highly nonlinear regimes. The non-linearity in this system is due to the relationship between the forces generated in the electromagnetic actuator and the coil current and the air gap between the rotor and the stator. In most practical applications, The PID controller is utilized to ensure stable operation of the rotating machinery. However, The PID controller becomes not effective enough when the machine operates in highly nonlinear regimes. So this paper develops a fuzzy logic strategy and sliding mode to improve the performance of the magnetic bearing system operating in nonlinear regimes. The considered controller design procedures are PD controller, Fuzzy logic strategy, and sliding mode. For all three schemes simulations are presented in order to make a comparison. Compared to the PD controller, the nonlinear control schemes give better performance.

Key Words: nonlinear control, PD control, Fuzzy-PID, Sliding mode

### INTRODUCTION

Magnetic bearings are being increasingly used in industrial applications where minimum friction is desired or in harsh environments where traditional bearings and their associated lubrication systems are considered unacceptable. They have certain advantages over the conventional bearings especially in rotating machinery application that require oil-free operation. The non-contact operation between the rotor and the magnetic bearings reduces the frictional losses [1].

The typical AMB system diagram is illustrated in Fig.1. Besides the controller, the general control system also includes the sensor, A/D and D/A conversion and power amplifier.



**Fig.1. Typical AMB system working principle diagram**

The rotor's displacement along one of the axes is detected by the position sensors and converted into signals of standard voltage. Then compared with the setting value, the error signal enters the controller. After A/D conversion, the

controller processes this digital signal according to a given regulating rule (control arithmetic) and generates a signal of current setting. After D/A conversion, this current signal enters the power amplifier, whose function is to maintain the current value in the electric magnet winding at the current level set by the controller. Therefore, if the rotor leaves its center position, the control system will change the electromagnet current in order to change its attraction force and, respectively, draws the rotor back to its balance position.

## MATHEMATICAL MODEL

There are several sources of non-linearity in an AMB system, of which the most prominent is the relationship between the forces generated in the electromagnetic actuator and the coil current and the air gap between the rotor and the stator. The force is proportional to the current squared and inversely proportional to the gap squared.

In the derivation of the forces that are generated in the magnetic actuators, the following assumptions hold:

1. Leakage of magnetic flux is neglected.
2. Fringing effect i.e. the spreading of magnetic flux in the air gap is neglected.
3. The magnetic iron is operating below saturation level.

The expression for the electromagnetic force that is generated in a single-acting actuator is given in Eq. (1) where  $\mu_0$  is the permeability of free space,  $n$  is the number of coil turns on the magnetic actuator,  $A_l$  is the area of one magnetic pole,  $i$  is the coil current, and  $s_0$  is the length of one air gap.

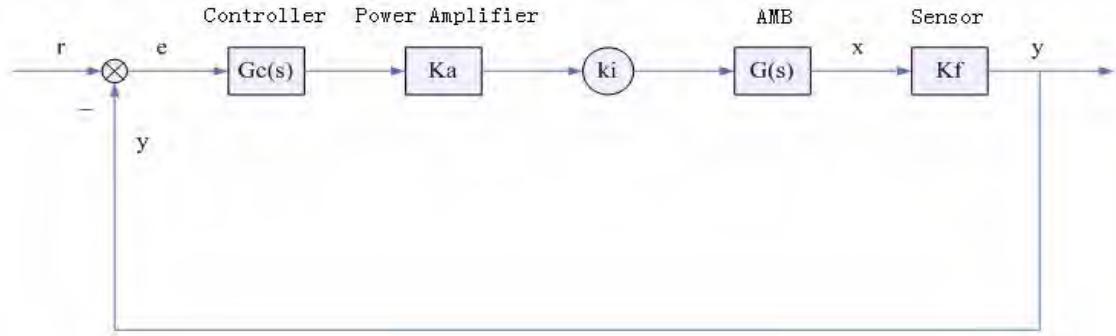
$$F = \frac{\mu_0 n^2 A_l i^2}{4 s_0^2} \quad (1)$$

In the actual operation of the magnetic bearing, a pair of magnetic actuators counter-acting each other is used. This configuration, known as the differential driving mode, makes it possible to generate both positive and negative forces. Therefore one magnetic actuator is driven with the sum of bias current and perturbation current ( $i_0 + i_x$ ), while the opposite one with the difference of bias current and perturbation current ( $i_0 - i_x$ ). So the expression for electromagnetic force of the counter-acting actuators is given in Eq. (2).

$$F = \frac{\mu_0 n^2 A_l}{4} \left[ \frac{(i_0 - i_x)^2}{(s_0 - x)^2} - \frac{(i_0 + i_x)^2}{(s_0 + x)^2} \right] \cos \alpha \quad (2)$$

## CONTROLLER DESIGN

It is a known fact that an AMB is an open-loop unstable system. It therefore requires a feedback control system to ensure stable operation.

**Fig.2 Control Plant of the AMB System**

The parameters of this AMB system are the diagram below:

**Table 1. Parameters of the Test Rig**

Mass of the rotor: m	13.2 Kg
Number of coil turns: N	2000
Area of one magnetic pole: $A_l$	$360mm^2$
Length of one air gap: $s_0$	1cm
Sensitivity of the sensor:	10V/cm

### Linear PD Controller

Consider the Eq. (2) above, use Taylor series expansion when  $x \ll s_0$  to linearize the equation. We can get:

$$m\ddot{x} = F = \frac{4ki_0}{s_0^2} i_x \cos \alpha + \frac{4ki_0^2}{s_0^3} x \cos \alpha = k_i i_x + k_s x \quad (3)$$

In that equation,  $k_i = \frac{4ki_0}{s_0^2} \cos \alpha$  is called force-current coefficient and  $k_s = \frac{4ki_0^2}{s_0^3} \cos \alpha$  is called force-displacement coefficient.

Use the Laplace transform to Eq. (3):

$$mX(s)s^2 = k_i I(s) + k_s X(s) \quad (4)$$

So the transfer function of the AMB system is:

$$G(s) = \frac{1}{ms^2 - k_s} \quad (5)$$

Consider about the PD controller, the transfer function is:

$$G_c(s) = K_p + \frac{K_d s}{1 + 1/2\pi f_s} \quad (6)$$

$$\text{In this equation, the parameters } K_p = \frac{k + k_s}{k_a k_f k_i} \text{ and } K_d = \frac{d}{k_a k_f k_i}.$$

In the PD controller, proportional gain determines the dynamic stiffness of the bearing, while derivative action is necessary for stabilization and damping. Make the stiffness of the system  $k = 1.0 \times 10^5 N/m$  and the damping ratio

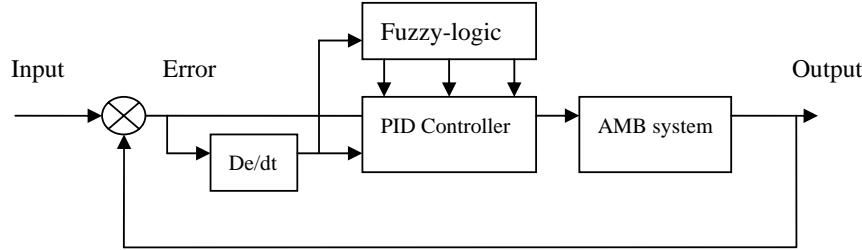
$\sigma = \frac{d}{2\sqrt{mk}} = 0.707$ . The exact value of  $K_p$  and  $K_d$  can be calculated. For this plant, combined with the

parameter above, after the calculation, we can get  $K_p = 0.4261$  and  $K_d = 0.0046$ . All the simulation results are in the simulation results section.

### Fuzzy-PID Controller

In the fuzzy control approach, the two winding currents derived by a linear controller are adjusted in an “imprecise” fashion to account for the force nonlinearity. The approach can be thought of as a type of feedback linearization [2].

The fuzzy controller described in this section was designed in two steps. First, fuzzy descriptions of the various operating points and possible control adjustments were chosen as described in the previous section. Operating point descriptions were the antecedents, and control adjustments were the consequents in the fuzzy control rules. Second, a set of rules for control adjustments was derived. Since control adjustments needed to vary over several orders of magnitude, the rules were designed to adjust a “fuzzy exponent” that is related to the final control adjustment.



**Fig.3 Fuzzy-logic Control Block Diagram**

The 2 inputs of the fuzzy controller are the error  $e$  and the differential of the error  $e_c$ . According to the fuzzy rules, calculate the control parameters using the fuzzy logic to get the precise parameters  $K_p, K_i, K_d$  of the PID controller. So the parameters of the controller vary when the working environment changes to gain a better system performance.

To guarantee the precision of the result, we define seven variable quantities as NB,NM,NS,ZO,PS,PM,PB on the parameters  $e, e_c$  and  $K_p, K_i, K_d$ . For convenience we use triangle membership function.

The fuzzy state domain and scaling factor of  $e$  is [-4,4] and 5.3; The fuzzy state domain and scaling factor of  $e_c$  is [-4,4] and 2.3; The fuzzy state domain and scaling factor of  $K_p$  is [0,8] and 0.5; The fuzzy state domain and

scaling factor of  $K_i$  is [0,8] and 0.1; The fuzzy state domain and scaling factor of  $K_d$  is [0,8] and 0.01.

### Sliding Mode Controller

Establish the state equation of the AMB system:

$$\dot{X} = AX + BU \quad (7)$$

In this equation  $X = [x \ \dot{x}]^T$ ;  $A = \begin{bmatrix} 0 & 1 \\ \frac{k_s}{m} & 0 \end{bmatrix}$ ;  $B = \begin{bmatrix} 0 \\ \frac{k_i}{m} \end{bmatrix}$ ;  $U = i$

Consider about linear or nonlinear system affine system:

$$\dot{x} = f(x) + bu \quad (8)$$

Which  $x \in R^n, b \in R^{n \times m}; u \in R^m$ .

In sliding mode controller design, we have to identify switching function vector:

$$u_i(x) = \begin{cases} u_i^+(x) & s_i(x) > 0 \\ u_i^-(x) & s_i(x) < 0 \end{cases} \quad (9)$$

To make the system reach the  $s(x) = 0$  plane within limited time and keep the sliding stability of the progressive movement to get a good dynamic quality. Variable structure controller design generally uses the Lyapunov method to find that can satisfy the conditions above and meet the global Lyapunov stability control [3].

In this paper, index reaching law below is adopted:

$$S = -qs - \varepsilon \operatorname{sgn}(s); q > 0, \varepsilon > 0 \quad (10)$$

The effect of  $q$  is to adjust the transient process which tends to the sliding plane. The effect of  $\varepsilon$  is to control the gain. It adjusts the control force during the sliding process which influence the anti-interference of the controller directly [4].

Combine the index reaching law of Eq. (10) and the state equation of Eq. (7)

$$U = -(CB)^{-1}(\varepsilon \operatorname{sgn}(s) + qs + C^T AX) \quad (11)$$

Which switching surfaces function is  $S = CX = c_1x_1 + c_2x_2$ .

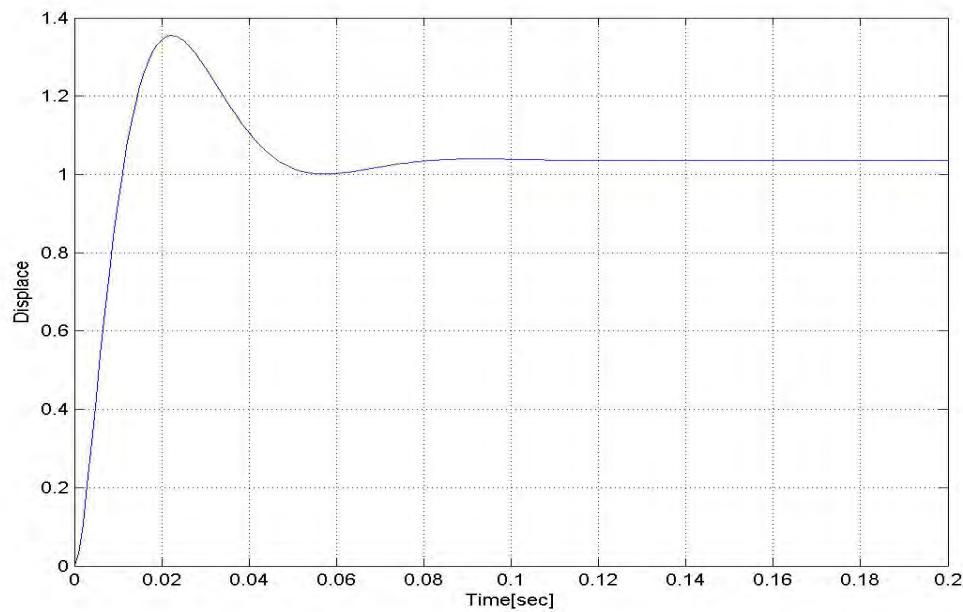
Using pole configuration method to verify the stability, choose Lyapunov function:

$$v(x) = \frac{1}{2}S^2 > 0 \quad (12)$$

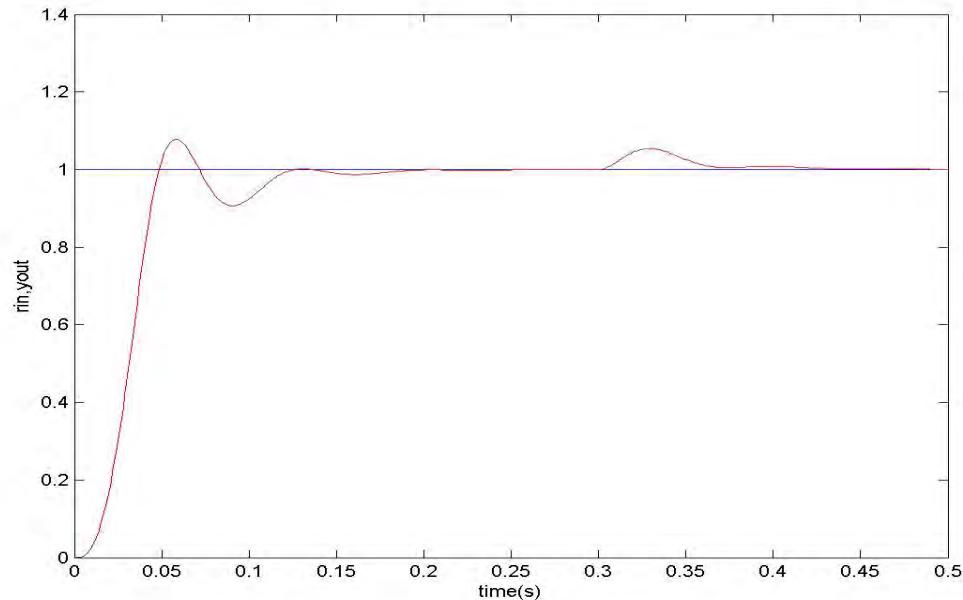
Then  $\dot{v} = S\dot{S} = -qS^2 - \varepsilon |S| < 0$ , which satisfy the Lyapunov stability condition and the system is stable.

### SIMULATION RESULTS

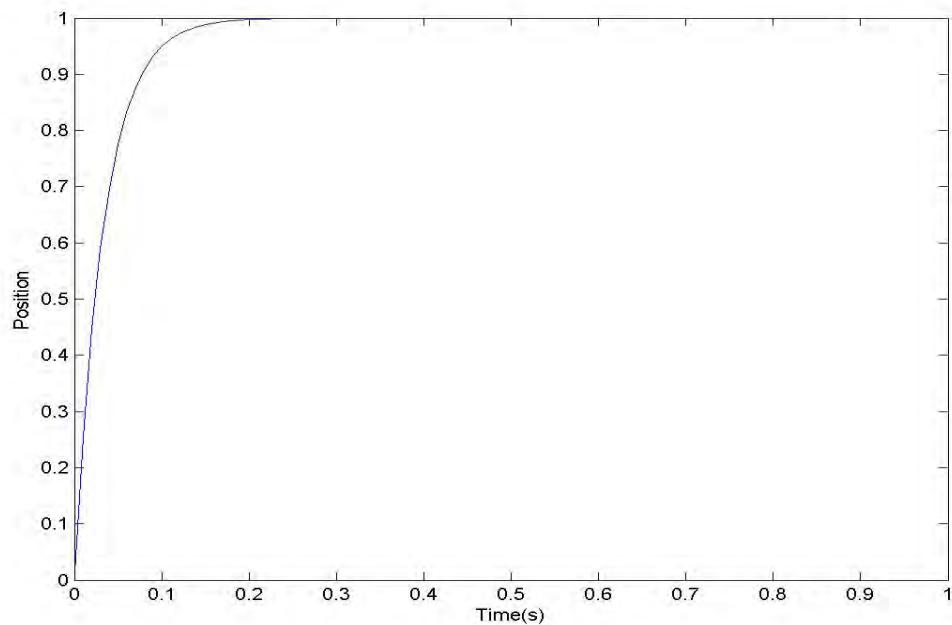
Using the software Matlab to implement the simulation works. Here are these results below.



**Fig .4:** Step Response of PD Controller



**Fig.5** Response with a impulse at 0.3s of Fuzzy-logic controller



**Fig.6 Step Response of Sliding Mode Controller**

From these results, we can find that all the performances of the three methods are acceptable. But nonlinear control methods have better performance than linear methods, especially sliding mode controller.

## CONCLUSION

In this paper we have considered three control schemes for a large air-gap magnetic bearing system. Compare to the nonlinear control methods like fuzzy-logic controller and sliding mode controller, the traditional linear PD controller performs not so well. Consider about the reality factor like leakage of magnetic flux, the nonlinear situation will be much more serious. The selection of nonlinear controller for the large air-gap magnetic bearing system should be a better choice.

## REFERENCES

1. Schweitzer, G., Bleuler, H., Traxler, A., 1994. Active magnetic bearings: basics, properties and applications of active magnetic bearings. Vdf Hochschulverlag AG an der ETH Zurich.
2. M.K.Habib, Control of Dual Acting Magnetic Bearing Actuator System Using Fuzzy Logic, Proceedings 2003 IEEE International Symposium on Computational Intelligence in Robotics and Automation. July 16-20.2003
3. V.U tkin. Varibale structure systems with sliding mode [J]. IEEE Trans.Auto. Control, 1997, AC-22: 212-222.
4. K K D Young, H G Kwatny. Variable structure servo-mechanism design and application to overspeed protection [J]. Automation. 1982,18: 385-400.