General Limit Load and B_2 Stress Index Equation for Pipe Bends under In Plane Bending

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Abstract

Structural integrity assurance of piping and piping components is essential for safe operation of Nuclear Power Plants. Section III of the ASME Boiler and Pressure Vessel Code (ASME) [1] provides rules for the design of nuclear power plant piping and piping components against plastic collapse. Pipe bends or elbows among the other piping components are very flexible and exhibits highly strained regions in the piping system and their accurate limit / collapse load assessment is necessary to prevent their plastic collapse or excessive deformation. For pipe bends / elbows subjected to bending load, ASME procedure usage a B_2 index over the nominal pipe bending stress and compare with the allowable limit of straight pipe which is based on its plastic collapse. In this paper, a new equation for elbow / pipe bend B_2 index has been proposed which is based on the margin consistent definition for B_2 stress index given by Tan [2]. Here the B_2 index for elbow is defined as the ratio of the pipe collapse load to the collapse load of corresponding elbow. Rigorous Finite Element (FE) analyses of around 40 elbows covering wide range of geometry parameters such as elbow bend radius (r_b) to mean pipe radius (R) ratio and pipe radius (R) to thickness (t) ratio, have been carried out. The material is assumed as elastic perfectly plastic and the geometric nonlinearity effect is included in FE analysis. Based on FE analysis results, the collapse load (or limit load, since the material is elastic perfectly plastic) is calculated using twice-elastic slope method. A general close form equation for elbow collapse load has been derived for 90^o elbow subjected to in plane bending load. The developed equation has been rigorously validated with available literature equation as well as experimental results. The derived equation can be applied for any bend radius elbow.

Introduction

Pipe bends or elbows are commonly used components in a piping system. They are very flexible compared to the straight pipes. Pipe bend normally reduces the reaction forces and moments within the piping system under thermal loading and it becomes easier to satisfy the stress limits. Because of this increased flexibility, they often accommodate large displacements arising from the differential thermal movements during normal operation conditions or from the seismic movements. Hence they exhibit highly strained regions in the piping system and care must be taken so that the resistance to deformation does not decrease rapidly leading to the failure/collapse of the system. To prevent such failure of piping elbows, accurate collapse load assessment is necessary to prevent their excessive deformation or collapse

The Section III of ASME Boiler and Pressure Vessel Code, (ASME) [1] allows designing of piping and piping components against plastic collapse by simplified analysis using stress indices and following equation.

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M \le 1.5 S_m \qquad (1)$$

Here B_1 and B_2 are stress indices for internal pressure and bending. For elbows, the ASME has defined the B_2 stress index as $1.3/\lambda^{2/3}$. Here λ is elbow characteristic and is defines as

$$\lambda = \frac{r_b t}{R^2} = \frac{r_b/R}{R/t} \qquad (2)$$

Recently Tan [2] has given a margin consistent definition of B_2-index for elbows. As per this definition, the B_2 index for elbow is defined as the ratio of the pipe collapse load to the collapse load of corresponding elbow and is given as below:

$$B_2 = \frac{\text{Collpase Load of Pipe}}{\text{Collpase Load of Elbow}} \qquad (3)$$

The exact plastic collapse load of straight pipe under bending load has been defined as $M_{CL}^{P} = 4R^{2}t\sigma_{y}$. Here, the material constitutive behaviour is assumed as elastic perfectly plastic and σ_y is the yield stress. However for accurate assessment of B_2 stress index, accurate collapse load equation for elbow subjected to in-plane bending load is required. In past many investigators [3 to 8] have proposed equations for collapse load of elbow subjected to in-plane closing bending. Most of these equations have been derived from finite element analysis of standard elbows (where bend radius is 1.5 times of pipe diameter) or short radius elbow (where bend radius is equal to pipe diameter). These equations express the elbow collapse load, in term of pipe collapse load equation multiplied with a load reduction / weakening factor (W_f). The elbow weakening factor, W_f, basically is a measure of the loss in load carrying capacity of an elbow with respect to corresponding pipe and is defined as below which in turn depends on bend radius (r_b) and R/t.

$$W_f = \frac{\text{Collpase Load of Elbow}}{\text{Collpase Load of Pipe}} = \frac{M_{CL}^{E}}{M_{CL}^{P}} \tag{4}$$

The weakening factor or elbow collapse load can be evaluated by rearranging Eq.3 along with ASME B_2 stress index definition [1].

$$M_{CL}^{E} = \frac{M_{CL}^{P}}{B_2} = 0.769\lambda^{2/3}M_{CL}^{P} \quad \text{but not} > M_{CL}^{P} \tag{5}$$

Spence and Findlay [3] gave a theoretically obtained expression for in-plane lower bound limit moment of an elbow as

$$\begin{aligned} M_{CL}^{E} &= 0.8\lambda^{0.6}M_{CL}^{P} \quad \text{for } \lambda<1.45 \\ &= M_{CL}^{P} \quad \text{for } \lambda \geq 1.45 \end{aligned} \tag{6}$$

From Eq.2 and Eq.5 it is observed that the elbow collapse load based on ASME B_2 stress index are in good agreement with the lower bound limit load given by Spence and Findlay [3]. Calladine [4] has has also developed in-plane lower bound limit moment as

$$M_{CL}^{E} = 0.935\lambda^{2/3}M_{CL}^{P} \quad \text{for } \lambda<0.5 \tag{7}$$

Both the above given theoretical expressions (Eq.6 and Eq.7) were based on small displacement analyses and assume ideal plastic material behavior. Based on large displacement analysis Goodall [5] proposed the maximum load carrying capacity of the defect free elbow subjected to closing bending moment as

$$M_{CL}^{E} = \frac{1.04\lambda^{2/3}}{1+\beta}M_{CL}^{P}$$

Where (8)

$$\beta = \left(2+\frac{(3\lambda)^{2/3}}{3}\right)\left(\frac{4\sqrt{3(1-\nu^2)}}{\pi}\frac{\sigma_y}{E}\frac{R}{t}\right)$$

Based on experimental investigations, Touboul [6] has proposed different equations for collapse load under in-plane closing bending and in-plane opening bending. Equation of in-plane closing collapse load is given as below

$$M_{CL}^{E} = 0.715\lambda^{2/3}M_{CL}^{P} \tag{9}$$

Based on large number of finite element analyses, Chattopadhayay [7] has also proposed an equation for collapse moment of elbow considering pipe constraining effect and material as a elastic perfectly plastic and geometry nonlinearity in the analysis.

$$M_{CL}^{E} = 1.07\lambda^{2/3}M_{CL}^{P} \quad \text{for } \lambda<0.6 \tag{10}$$

Recently, Y.J.Kim [8] has given a equation for collapse moment of elbow under closing bending moment based on large displacement analysis and wide range of combinations of r_b/R and R/t but the resulting values of λ are limited from λ=0.1 to λ=0.5. It was shown that collpase moment not only depend on λ but also on R/t ratio.

$$M_{CL}^{E} = A_c(\lambda + k_c)^{n_c} M_{CL}^{P}$$

where (11)

$$A_c = 0.800\left(\frac{R}{t}\right)^{-0.017} \quad ; \quad k_c = 1.460\left(\frac{R}{t}\right)^{-0.911} \quad ; \quad n_c = 0.423\left(\frac{R}{t}\right)^{0.127}$$

It has been observed that these equations have limited applicability and do not cover wide range of bend radius to pipe radius (r_b/R) ratios which are used in power plant piping. Moreover, the elbow collapse load equation should approach to straight pipe collapse load as elbow bend radius increases. In other words, the weakening factor, W_f of an elbow, should approach to 1.0 as the bend radius (or λ) increases. None of above equations satisfies this asymptotic behaviour of an elbow. Available expressions, except Eq.8 and Eq.11 assumes that any two elbows with same elbow characteristic (λ) will have same $W_{f..}$ Also the present B_2 index definition is found to be very conservative when investigated with recently proposed collapse load equations (Eq.10, and Eq.11). Keeping this fact in mind, a series of finite element analyses of elbows having different geometric parameters bend radius, pipe radius and thickness have been performed. The ASME B_2 index equation for elbow has been re-looked in the context elbow collapse load equations and a margin consistent B_2 definition recently given by Tan [2]. Based on finite element analyses, a new B_2-index equation has been developed which is applicable for any bend radius and reduces undue conservatism in current ASME elbow B_2-index definition

Weakening factor (W_f) Evaluation using Finite Element Analysis

Rigorous elastic plastic finite element analyses of around 70 elbows covering wide range of geometry parameters such as elbow bend radius (r_b) to mean pipe radius (R) ratio and pipe radius (R) to thickness (t) ratio, have been carried out. The parameters selected for investigation are given in table1 and covers wide range of bend radius, radius to thickness ratio of 90^o elbows. The analysis matrix has been designed with aim to understand dependence of W_f on λ, r_b/R, R/t and its asymptotic behaviour with respect to increase in elbow bend radius.

Table 1: Geometry details of the elbows analyzed (Mean Pipe Radius, R=250mm)

r_b/r →	*2*	*3*	*6*	*9*	*12*	*18*	**pipe**
R/t ↓	↓ **Parameter : λ (Elbow Characteristics) = (r_b/r) / (R/t)**						
5	0.4	0.6	1.2	1.8	2.4	3.6	∞
7.5		0.4	0.8	1.2	1.6	2.4	∞
10	0.2	0.3	0.6	0.9	1.2	1.8	∞
15		0.2	0.4	0.6	0.8	1.2	∞
20	0.1	0.15	0.3	0.45	0.6	0.9	∞
30		0.1	0.2	0.3	0.4	0.6	∞
r_b/r →	*2*	*3*	*6*	*9*	*12*	*18*	**pipe**

Taking advantage of the double symmetry, one quarter of the elbow was modeled using three dimensional 20 noded brick elements in commercial finite element code. To account for the effect of constraint posed by connected piping, the elbow is modeled with connected pipes with length equal to three times pipe diameter. Figure 1 shows meshes for two typical cases, small bend radius thin elbow (r_b/R=2 and R/t=30) and other extra large bend radius thick elbow (rb/R=9 and R/t=5). Pure in plane bending moment has been applied to the end plane of connected straight pipe. The material constitutive behaviour is assumed as elastic perfectly plastic and finite element analysis was performed including the geometric nonlinearity effects. The material's Young's Modulus, yield stress and Poisson's ratio is taken equal to 203GPa, 250 MPa and 0.3 respectively. The analyses have been carried out for both in-plane closing and opening bending moment A typical moment versus end rotation (M-ϕ) curve (for r_b/R=3, R/t=20) is shown in figure 2. Here, it is observed that under closing bending the M-ϕ curve becomes almost flat at certain load while under opening bending it continues to rise as the applied moment increases. This is due to the difference in the deformation (ovalization) behaviour of elbow cross-section when subjected to closing and opening bending as shown in figure 2. The collapse moments (or limit load, since the material is elastic perfectly plastic) have been evaluated from M-ϕ curve by twice elastic slope (TES) method. The plastic part of rotation (ϕ_p) and elastic part (ϕ_e) becomes equal at the collapse load point. As shown in figure 2, it is well known that the collapse load for in plane closing bending is lowest when compared with in plane opening or out of plane bending collapse load. Hence the in plane closing collapse load forms the basis for ensuring the integrity of an elbow against plastic collapse. In view of this the present investigation will consider the in plane closing collapse load for derivation of B_2 stress index. Further the weakening factor W_f , has been evaluated for all the cases of table 1, using Eq.4 where the elbow collapse load is evaluated from FE results using TES method and the pipe collapse load is taken equal to $M_{CL}^{P} = 4R^2 t\sigma_y$. The W_f factors (that is inverse of B_2 stress index) have been given in table 2 and has been plotted in figure 3.

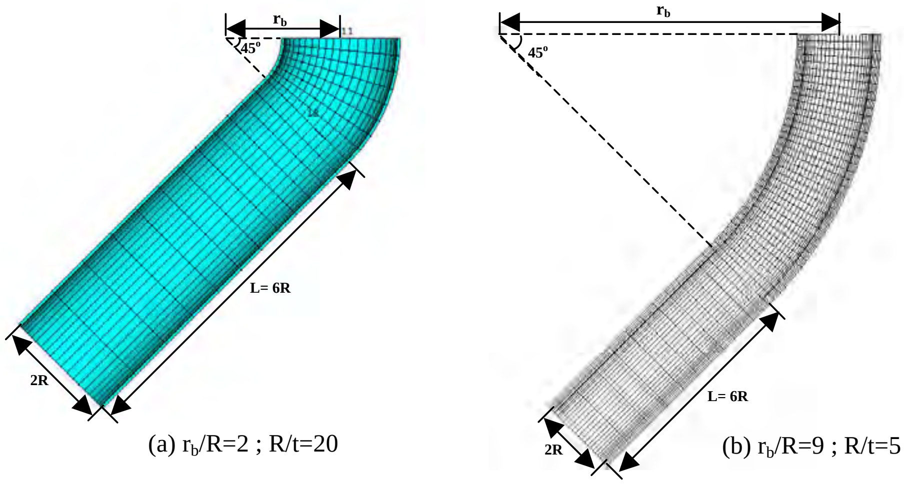

Figure 1: Typical finite element mesh for two sample cases used in analyses, (a) small bend radius thin elbow, r_b/R=2 and R/t=30 ; (b) extra large bend radius thick elbow, r_b/R=9 and R/t=5

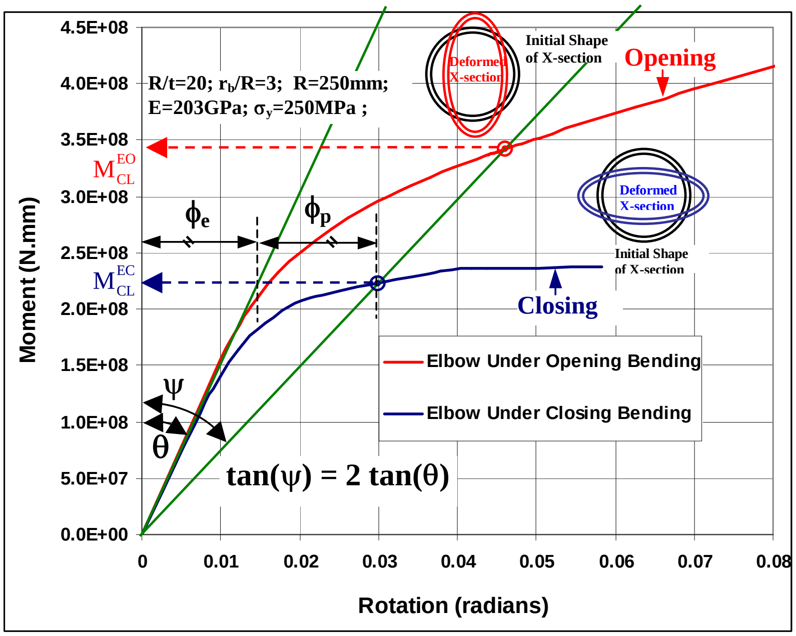

Figure 2: Typical moment versus end rotation curve for in-plane closing and opening and collapse load evaluation using TES method

Table 2: Weakening factor W_f that is Normalized Limit Moment (M^E_{CL} /4$R^2t\sigma_y$) for different elbows under closing bending moment

r_b/r →	2	3	6	9	12	18	pipe
R/t ↓	↓ Parameter : Weakening factor=W_f (FEM) Closing Bending = 1/B_2						
5	0.63	0.73	0.89	0.94	0.95	0.96	1
7.5		0.59	0.78	0.88	0.92	0.94	1
10	0.42	0.49	0.67	0.80	0.88	0.92	1
15		0.37	0.51	0.62	0.73	0.86	1
20	0.24	0.29	0.41	0.50	0.59	0.74	1
30		0.21	0.30	0.37	0.44	0.55	1
r_b/r →	2	3	6	9	12	18	pipe

The figure 3 show that for constant R/t, the weakening factor W_f approaches to unity as r_b/R increases. It also shows that for same rb/R, the W_f for thin elbows (small R/t) is significant less then thick elbow. From table 2, it is clearly noted that the W_f depends not only on elbow characteristic but also on r_b/R and R/t ratios. For example in the analysis matrix (refer table 1) there are five pairs of r_b/R and R/t [(r_b/R =3, R/t=5); (6,10); (9,15); (12, 20) and (18, 30)] , give λ equal to 0.6 and table 2 shows corresponding FEM evaluated W_f for these elbows. This clearly demonstrates that most of the available equations except Kim[8] are not adequate or applicable to address this observation. The table 2 also shows that for some constant R/t, if the bend radius of elbow is increased, the W_f approaches to 1. This observation has not been addressed by available expressions.

Development of B_2 Stress Index Equation

Based on the results as shown in table 2, a relational study was performed for weakening factor (W_f) dependence on the R/t, r_b/R and elbow characteristic 'λ' and simple expression for W_f or B_2 index has been proposed which satisfies the above discussed asymptotic behavior of elbow collapse load.

$$B_2 = \frac{1}{W_f} = 1 + \frac{C_1}{\lambda^{C_2}} \tag{12}$$

A straight line is fitted for the logarithm of the above equation for different r_b/R as shown in figure 4 then from the different equations, and coefficients C_1 and C_2 are obtained. It is found that C_1 is nearly equal to 0.2 and constant C_2 can be represented as linearly varying with respect to r_b/R as C_2 =1.028 + 0.095*r_b/R.

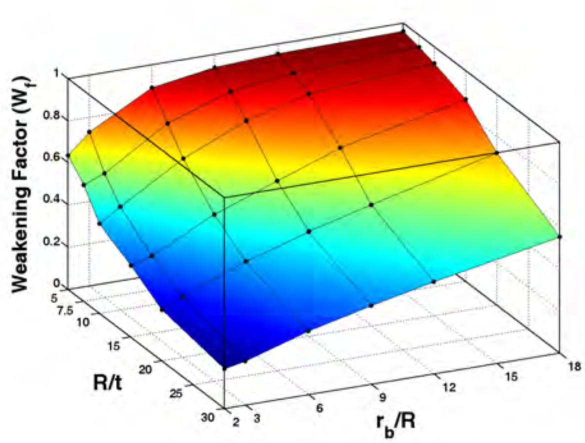

Figure 3: Surafce plot of FE evaluated weakening factor (W_f) with R/t and r_b/R

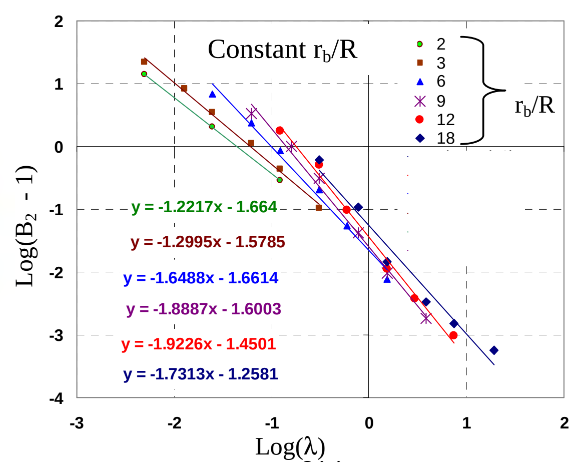

Figure 4: Evaluation of C_1 and C_2 coefficients for Eq.11 using the finite element results

Finally a simple expression for B_2 stress index (Eq.13) or a general equation (Eq.14) for elbow in plane closing collapse load is proposed which covers the entire domain of the parameters and gives good prediction of collapse load for non standard elbows also. The Proposed equation is given as below

$$B_2 = \frac{1}{W_f} = 1 + \frac{0.2}{\lambda^{1.028+0.095 r_b/R}} \tag{13}$$

and $$M_{CL}^{E} = \left(1 + \frac{0.2}{\lambda^{1.028+0.095 r_b/R}}\right)^{-1} M_{CL}^{P} \tag{14}$$

Rigorous validation has been made with respect to the available collapse load equations available in literature [3 to 8]. The error estimates of the present proposed equation (Eq.14) as well as literature equations (Eq.5 to Eq.11) with respect to the finite element evaluated collapse load has been summarized in table 3 and figure 5. The figure 5 and table 3 clearly show that the proposed equation (Eq.13) is in good agreement with the FE results for all combinations of r_b/R and R/t analyzed, and also satisfies the desired asymptotic behaviour.

Table 3: Verification of proposed collapse load equation with respect to FEM results and literature equation

Sr. No	r_b/R	R/t	λ	Weak-ening Factor W_f (FEM)	$\%Error = 100 * (W_f^{FEM} - W_f^{Equation})/W_f^{FEM}$							
					Proposed Equation	Spence & Findley [3]	ASME [1], B_2 Index & Tan [2]	Calla-dine [4]	Goodal [5]	Touboul [6]	Chattop-adhaya [7]	Y.J. Kim [8]
1	*2*	*20*	0.10	0.241	3.77	16.77	31.36	16.57	16.50	36.20	4.07	-14.61
2	*2*	*10*	0.20	0.423	2.29	27.97	37.78	24.38	20.49	42.17	13.05	-5.02
3	*2*	*5*	0.40	0.634	2.05	27.17	34.12	19.92	13.59	38.76	7.93	-4.81
4	*3*	*30*	0.10	0.207	5.49	2.92	19.94	2.69	7.25	25.58	-11.88	-13.15
5	*3*	*20*	0.15	0.286	-2.50	10.30	23.99	7.62	7.74	29.35	-6.22	-11.52
6	*3*	*15*	0.20	0.369	-2.19	17.37	28.63	13.25	11.22	33.66	0.26	-5.78
7	*3*	*10*	0.30	0.486	-4.39	20.04	29.05	13.76	9.49	34.05	0.84	-4.36
8	*3*	*7.5*	0.40	0.590	-1.66	21.80	29.27	14.03	8.59	34.25	1.15	-1.98
9	*3*	*5*	0.60	0.729	1.45	19.27	24.98	NA	1.74	30.27	-4.84	-3.17
10	*6*	*30*	0.20	0.300	7.87	-1.53	12.31	-6.59	-1.00	18.49	NA	-6.17
11	*6*	*20*	0.30	0.408	-3.38	4.83	15.55	-2.65	-1.97	21.50	NA	-4.87
12	*6*	*15*	0.40	0.515	-4.15	10.34	18.89	1.41	-0.39	24.61	NA	-0.89
13	*6*	*10*	0.60	0.667	-3.21	11.74	17.97	NA	-4.17	23.76	NA	-0.11
14	*6*	*7.5*	0.80	0.779	0.15	10.17	14.90	NA	-9.53	20.90	NA	-1.01
15	*6*	*5*	1.20	0.892	2.46	-0.07	2.60	NA	-27.10	9.47	NA	-9.09
16	*9*	*30*	0.30	0.372	8.21	-4.48	7.28	-12.7	-6.27	13.82	NA	-5.48
17	*9*	*20*	0.45	0.498	-5.63	0.59	9.37	-10.2	-8.95	15.76	NA	-4.81
18	*9*	*15*	0.60	0.624	-5.20	5.64	12.31	NA	-8.08	18.49	NA	-0.96
19	*9*	*10*	0.90	0.798	-0.69	5.94	10.19	NA	-13.63	16.52	NA	-0.61
20	*9*	*7.5*	1.20	0.882	0.75	-1.15	1.55	NA	-26.28	8.49	NA	-6.64
21	*9*	*5*	1.80	0.940	0.17	-6.43	-6.43	NA	-57.61	-12.61	NA	-22.86
22	*12*	*30*	0.40	0.437	6.93	-5.61	4.46	-16.1	-9.03	11.20	NA	-5.01
23	*12*	*20*	0.60	0.593	-5.11	0.64	7.66	NA	-10.56	14.17	NA	-2.45
24	*12*	*15*	0.80	0.734	-2.88	4.66	9.68	NA	-10.92	16.04	NA	0.70
25	*12*	*10*	1.20	0.876	-0.59	-1.86	0.86	NA	-25.03	7.85	NA	-5.35
26	*12*	*7.5*	1.60	0.919	-1.51	-8.84	-8.84	NA	-46.46	-6.46	NA	-17.16
27	*12*	*5*	2.40	0.954	-1.81	-4.87	-4.87	NA	-87.62	-34.40	NA	-37.64
28	*18*	*30*	0.60	0.552	-0.09	-6.67	0.87	NA	-12.27	7.86	NA	-4.95
29	*18*	*20*	0.90	0.743	-6.27	-1.11	3.46	NA	-14.78	10.27	NA	-2.06
30	*18*	*15*	1.20	0.863	-3.30	-3.39	-0.63	NA	-22.77	6.47	NA	-4.63
31	*18*	*10*	1.80	0.922	-4.26	-8.43	-8.43	NA	-54.73	-14.72	NA	-22.81
32	*18*	*7.5*	2.40	0.944	-4.09	-5.98	-5.98	NA	-85.87	-35.83	NA	-39.04
33	*18*	*5*	3.60	0.962	-3.29	-3.91	-3.91	NA	-142.44	-74.51	NA	-64.70
34	*∞*	*any*	∞	1	0	0	0	NA	∞	∞	NA	∞

The figure 5a compares the proposed Eq.13 with different literature equations (Eq.5 to Eq.11) for r_b/R=3. It shows that the FE results compare well with the Eq.10, (Chattopadhaya [7] and Eq.11, Kim [8]. This validates the present FE results and the new general equation since Eq.10 is based on detailed finite element investigation on standard bend radius (r_b/R=3) 90° elbow. The Eq.11, Kim [8] compares well up to λ value of 1.0 for all r_b/R and R/t combinations, however it fails to match the finite element results or new Eq.13 for higher λ values and also does not follow asymptotic behaviour. This is due to the fact the Eq.11 was developed by Kim [8] from the FE results for elbows having λ range from λ=0.1 to λ=0.5, but several combinations of r_b/R and R/t were considered. The Eq.5 (ASME [1], Tan [2]) and Eq.6 (Spence and Findley [3]) are very conservative for small to large bend radius elbows (r_b/R=2, 3 and 6 cases), however they are reasonable for extra large bend radius cases (r_b/R= 9, 12 and 18 cases). From the figure 6, it is observed that under closing bending moment, for the constant value of λ, B_2 stress index increases with increase in r_b/R (or R/t since λ is kept constant). This fact is predominant for the lower value of elbow characteristics, λ.

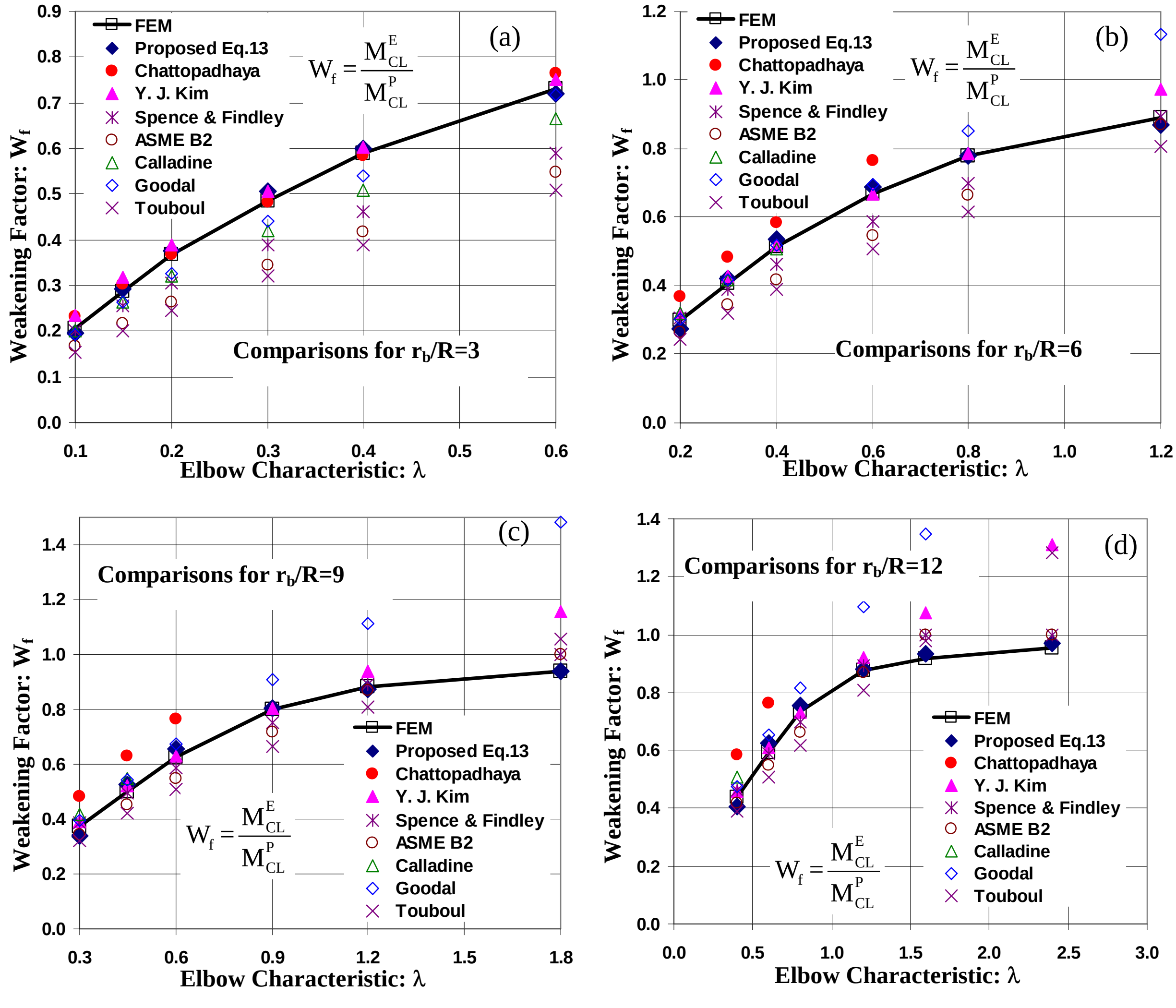

Figure 5: Comparisons of normalized elbow collapse load (W_f) evaluated using FEM, proposed Eq.13 and various literature equations for different bend radius elbows

Conclusion

The following conclusions are drawn from the above investigation of collapse moments of subjected to in-plane bending moment.

1. It has been shown that when the bend radius is increased keeping the ratio between mean radius to thickness constant, the weakening factor W_f approaches to unity that is, the elbow collapse load approaches to that of pipe. The proposed equation (Eq.13) shows this asymptotic behavior, however literature equations fail to satisfy this.
2. It has been shown that weakening factor is not only the function of λ but also depends on r_b/R and R/t under closing bending moment. The proposed equation is valid for all values of λ and r_b/R ratio. This makes the equation applicability general for any 90^o elbow or pipe bend. The equation has rigorously validated against the recent development in elbow collapse load [7, 8].
3. The collapse moment equation given by Spence & Findlay and ASME B_2 stress index are generally very conservative for thick elbows with small elbow characteristic, λ. The new equation (Eq.13) eliminates this conservatism and gives more realistic value of B_2 which is based on the margin consistent definition recently given by Tan [2].

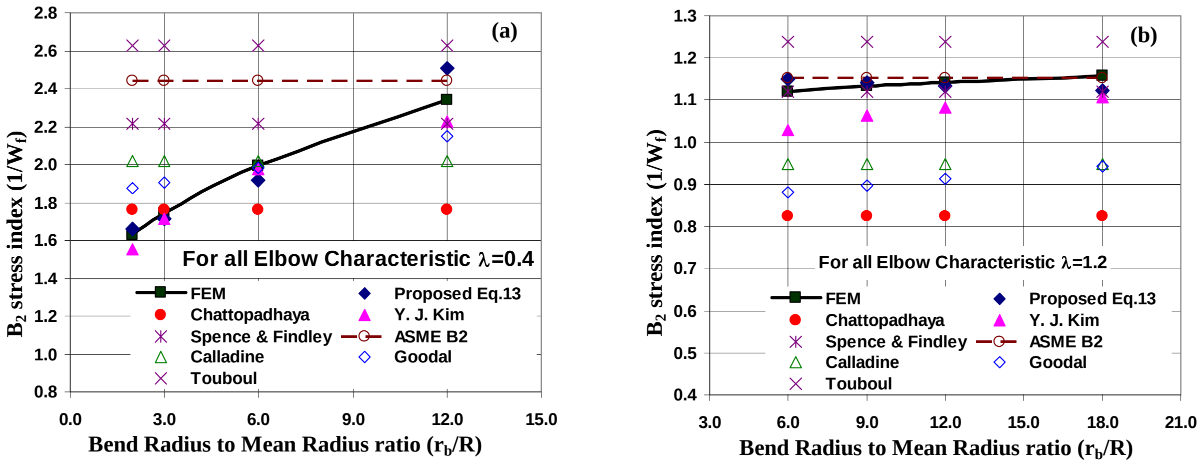

Figure 5: Comparison of B_2 stress index evaluated by different equations with FEM results.

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