

# Axisymmetric Axial Crushing Analysis of Thin Walled Metallic Frusta and Tubes

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## Abstract

Axisymmetric axial crushing analysis of thin metallic frusta using straight fold model with partly inside and partly outside folding is presented. The first fold of the model ends at the mean diameter and all subsequent folds initiate and end at the mean diameter. The relations for obtaining the inside and outside fold lengths in tubes are derived taking different values of yield stress of the material in compression and tension. The variation of circumferential strain during the formation of a fold has been taken into account for the purpose of computing the variation of crushing load. The results have been compared with experiments and good agreement has been observed.

**Key words:** axial crushing, frusta, tubes, crushing, folding, straight fold

## 1. INTRODUCTION

Cylindrical tubes and frusta are the most frequently used devices that absorb energy in an accidental crash by material deformation and are installed in Aircraft, ships and road vehicles. The performance of frusta as an energy absorbing device has been compared in the past [1] with the tubes. The mechanics of axial crushing of thin metallic tubes and frusta has been extensively studied in the past experimentally [1-3]. Experiments on tubes have shown that under axial loads when these deform in axisymmetric mode, the fold formed is partly inside and partly outside the initial tube diameter [3-6]. Analytical solutions available have made several simplifying assumptions which include the deformation to be inextensional and the fold to be only outside [7] or inside [8] the initial diameter of the tube. Some studies [3,8] considered inside/outside folding in tubes and assumed that both parts are equal in length and those available for the load-deformation computation assume the fold shape. Experiments have shown that this is not true and the inner fold is smaller than the outer fold [4, 5]. Such analysis for conical frusta does not exist, and those available [6] consider the folding to be only outside.

In the present paper, a mathematical formulation is presented for axisymmetric crushing of frusta with partly inside and partly outside folding based on energy considerations. The model developed considers the variation of circumferential strain during the formation of a fold and the difference in yield strength of material in tension and compression. The existing total outside fold model of frusta [6] and the partly inside and partly outside fold models [5] have been derived from the proposed model. The mean and the variation of crushing load for frusta and tubes have been computed. The results have been compared with experiments and reasonably good agreement has been observed.

## 2. ANALYSIS OF FRUSTA

Considering a thin metallic frustum of initial thickness  $t_0$ , smaller end radius  $R_1$ , and angle of taper  $\alpha$  as shown in Fig. 1. The folding model for axisymmetric crushing of the frustum is also shown in this figure, by the formation of partly inside and partly outside straight folding. It can be seen from Fig. 1 that there are three limbs in the model instead of two taken in the earlier model [4]. After the completion of the second fold, the first limb of second fold gets in line with the third limb of the first fold thus the hinge at their junction located at the initial line of the frusta gets eliminated. The bottom portion of frustum a'-b'-c'-d'-e'-f' in undeformed state assumes the shape a-b-c-d-e-f after axisymmetric crushing. In the first fold, the length of first and second limbs are  $h_1$  and  $h_2$  respectively out of which  $mh_1$  and  $mh_2$  are inside the initial line of frustum for first and second limbs respectively, where  $m$  is the folding parameter which is the ratio of inside length to the total length of fold. The length of third limb is  $mh_1$ , which is inside the initial line of the frustum. In the second and other subsequent folds, the length of first limb is  $(1-m)h_1$ , second is  $h_2$  and the third is  $mh_1$ . The relationship between the lengths of the limbs can be obtained from the geometry in complete crushed state:

$$h_1 = Kh_2 \quad \text{where, } K = (1 + \sin \alpha) / (1 - \sin \alpha) \quad (1)$$

The total vertical crushed length of first fold is  $[(1-m)h_1 + h_2]$  and its value for other folds is  $(h_1 + h_2)$  as shown in Fig. 1. The angle of inclination of the limbs of the fold,  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  (from geometry  $\theta_1 = \theta_3$ ) have been measured from the initial line of the frustum. The yield strength of the material of the frustum in compression and

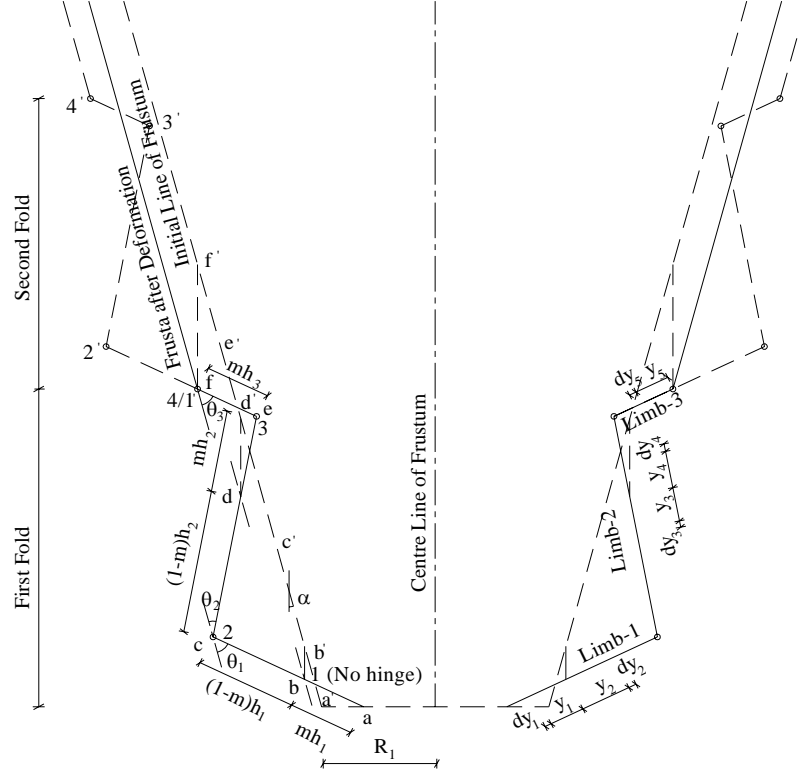


Fig. 1 Axial crushing model of a frustum

tension has been taken as  $f_{yc}$  and  $f_{yt}$  respectively, with their ratio as  $r = f_{yc}/f_{yt}$ . The plastic moment of resistance of the material of the frustum has been taken as  $M_p = (f_{yt}t_0^2)/2\sqrt{3}$  using Von-Mises criterion, where,  $t_0$  is the thickness of wall of frustum.

## 2.1. Energy Absorption in Crushing

The energy dissipated in plastic bending,  $W_{b\theta}$ , in the rotation of the lower limb upto angle  $\theta_1$  and upper limb upto angle  $\theta_2$  is given by:

$$\begin{aligned}
 W_{b\theta} &= \frac{\pi f_{yt} t_0^2}{\sqrt{3}} \left[ a_2 \int_0^{\theta_1} R_f d\theta_1 + a_2 \int_0^{\theta_1} R_e d\theta_1 + \int_0^{\theta_2} R_e d\theta_2 + \int_0^{\theta_2} R_c d\theta_2 + \int_0^{\theta_1} R_c d\theta_1 + a_1 \int_0^{\theta_1} R_b d\theta_1 \right] \\
 &\quad \text{(Hinge-4)} \quad \text{(Hinge-3)} \quad \text{(Hinge-2)} \quad \text{(Hinge-1)} \\
 &= \frac{\pi f_{yt} t_0^2}{\sqrt{3}} \left[ \int_0^{\theta_1} \{a_2(R_f + R_e) + (a_1 R_b + R_c)\} d\theta_1 + \int_0^{\theta_2} (R_c + R_e) d\theta_2 \right] \quad (2)
 \end{aligned}$$

It has been incorporated in the above expression by introducing a constant,  $a$ . Its value will be zero (i.e.  $a=0$ ) for first and unity (i.e.  $a=1$ ) for rest of the folds. Evaluating the integrals,  $W_{b\theta}$  is obtained as

$$W_{b\theta} = \frac{\pi f_{yt} t_0^2}{\sqrt{3}} \left[ \begin{aligned} &\{(a_1 + 2a_2 + 1)\theta_1 + 2\theta_2\}R_1 + mKh_2(a_1 + 2a_2 + 1)\theta_1 \sin \alpha \\ &- Kh_2(1 - m - ma_2)\{\cos(\theta_1 + \alpha) - \cos \alpha\} + (1 - 2m)h_2 \cos \alpha \\ &+ 2h_2\{(1 + K)(a_2\theta_1 + \theta_2) - m\theta_2\}\sin \alpha - (1 - 2m)h_2 \cos(\theta_2 - \alpha) \end{aligned} \right] \quad (3)$$

putting,  $\theta_1 = \frac{\pi}{2} - \alpha$  and  $\theta_2 = \frac{\pi}{2} + \alpha$  in the above equation gives the total energy absorbed in flexure during complete crushing of the fold:

$$W_b = \frac{\pi f_{yt} t_0^2}{\sqrt{3}} h_2 \left[ \begin{aligned} &(a_1 + 2a_2 + 1) \left( \frac{\pi}{2} - \alpha \right) \left( \frac{R_1}{h_2} + mK \sin \alpha \right) + \{1 - 2m + K(1 - m - ma_2)\} \cos \alpha \\ &+ \{(1 + K)\{a_2(\pi - 2\alpha) + (\pi + 2\alpha)\}\} \sin \alpha + (\pi + 2\alpha) \left( \frac{R_1}{h_2} - m \sin \alpha \right) \end{aligned} \right] \quad (4)$$

which for  $m = 0$  and  $a_2 = 0$  converts to the total outside fold model of frusta [6]:

$$W_b = \frac{\pi f_{yt} t_0^2}{\sqrt{3}} \left[ \left\{ (a_1 + 3) \frac{\pi}{2} - \alpha(a_1 - 1) \right\} R_1 + h_2(1 + K) \{ (\pi + 2\alpha) \sin \alpha + \cos \alpha \} \right] \quad (5)$$

and for  $\alpha = 0$ ,  $a_2 = 1$  and  $h_1 = h_2 = h$  (say), converts to the partly inside and partly outside model of cylinder of radius  $R_1$  [5]:

$$W_b = \frac{\pi f_{yt} t_0^2}{\sqrt{3}} \left[ (a_1 + 5) \frac{\pi}{2} R_1 + 2h(1 - 2m) \right] \quad (6)$$

### 2.1.1. Energy Dissipated in Circumferential Deformation

The energy dissipated in circumferential compression for the portion of the fold going inside the initial diameter of the frustum and circumferential tension for the portion of the fold going outside and rotation upto an angle  $\theta_1$  for first and third limbs and  $\theta_2$  for the second limb of the fold,  $W_{c\theta}$ , can be calculated by:

$$W_{c\theta} = \int_0^{\theta_1} \left( \frac{dW_{c1}}{d\theta_1} \right) d\theta_1 + \int_0^{\theta_2} \left( \frac{dW_{c2}}{d\theta_2} \right) d\theta_2 + \int_0^{\theta_1} \left( \frac{dW_{c3}}{d\theta_1} \right) d\theta_1 \quad (7)$$

(First Limb)    (Second Limb)    (Third Limb)

The energy dissipation in first, second and third limbs required in the above equation have been calculated in the following:

**(a) First Limb:** Considering an element of width  $dy_1$  at a distance  $y_1$  from point b in the portion of the lower limb going inside and another element of width  $dy_2$  at a distance  $y_2$  from point b in the portion of the lower limb going outside and similarly for the upper limb, the energy dissipated in circumferential deformation can be calculated as follows:

$$\frac{dW_{c1}}{d\theta_1} = ra_3 t_0 \int_0^{mh_1} f_{yt} \left| \frac{d\varepsilon_1}{d\theta_1} \right| dA_1 + t_0 \int_0^{(1-m)h_1} f_{yt} \left| \frac{d\varepsilon_2}{d\theta_1} \right| dA_2 \quad (8)$$

where,  $dA_1$  and  $dA_2$  are the area of elemental rings, given by

$$dA_1 = 2\pi \{R_b - y_1 \sin(\theta_1 + \alpha)\} dy_1 \quad \text{and} \quad dA_2 = 2\pi \{R_b + y_2 \sin(\theta_1 + \alpha)\} dy_2 \quad (9)$$

and  $\varepsilon_1$  and  $\varepsilon_2$  are the circumferential strains in the two elements, given by

$$\varepsilon_1 = \frac{2\pi[\{R_b - y_1 \sin(\theta_1 + \alpha)\} - (R_b - y_1 \sin \alpha)]}{2\pi(R_b - y_1 \sin \alpha)} \quad (10)$$

$$\varepsilon_2 = \frac{2\pi[\{R_b + y_2 \sin(\theta_1 + \alpha)\} - (R_b + y_2 \sin \alpha)]}{2\pi(R_b + y_2 \sin \alpha)} \quad (11)$$

Differentiating the above two equations, we get,

$$\frac{d\varepsilon_1}{d\theta_1} = -\frac{y_1 \cos(\theta_1 + \alpha)}{R_b - y_1 \sin \alpha}, \quad \frac{d\varepsilon_2}{d\theta_1} = -\frac{y_2 \cos(\theta_1 + \alpha)}{R_b + y_2 \sin \alpha} \quad (12)$$

**(b) Second Limb:** Considering an element of width  $dy_3$  at a distance  $y_3$  from point d in the portion of the second limb going inside and another element of width  $dy_4$  at a distance  $y_4$  from point d in the portion of the second limb going outside, the energy dissipated in circumferential deformation can be calculated as follows:

$$\frac{dW_{c2}}{d\theta_2} = t_0 \int_0^{(1-m)h_2} f_{yt} \left| \frac{d\varepsilon_3}{d\theta_2} \right| dA_3 + rt_0 \int_0^{mh_2} f_{yt} \left| \frac{d\varepsilon_4}{d\theta_2} \right| dA_4 \quad (13)$$

$$\text{where, } dA_3 = 2\pi\{R_d + y_3 \sin(\theta_2 - \alpha)\}dy_3 \text{ and } dA_4 = 2\pi\{R_d - y_4 \sin(\theta_2 - \alpha)\}dy_4 \quad (14)$$

$\varepsilon_1$  and  $\varepsilon_2$  are the circumferential strains in the two elements which when differentiated, as done above, gives:

$$\frac{d\varepsilon_3}{d\theta_2} = \frac{y_3 \cos(\theta_2 - \alpha)}{R_d - y_3 \sin \alpha}, \quad \frac{d\varepsilon_4}{d\theta_2} = -\frac{y_4 \cos(\theta_2 - \alpha)}{R_d + y_4 \sin \alpha} \quad (15)$$

**(c) Third Limb:** Considering an element of width  $dy_5$  at a distance  $y_5$  from point f in the third limb, the energy dissipated in circumferential compression can be calculated as follows:

$$\frac{dW_{c3}}{d\theta_1} = rt_0 \int_0^{mh_3} f_{yt} \left| \frac{d\varepsilon_5}{d\theta_1} \right| dA_5 \quad (16)$$

$$\text{where, } dA_5 = 2\pi\{R_f - y_5 \sin(\theta_1 + \alpha)\}dy_5, \quad \varepsilon_5 = \frac{2\pi[\{R_f - y_5 \sin(\theta_1 + \alpha)\} - (R_f - y_5 \sin \alpha)]}{2\pi(R_f - y_5 \sin \alpha)} \quad (17)$$

$$\text{Differentiating the above equation, gives, } \frac{d\varepsilon_5}{d\theta_1} = -\frac{y_5 \cos(\theta_1 + \alpha)}{R_f - y_5 \sin \alpha} \quad (18)$$

**(d) Total Circumferential Energy:** Using of Eqs. (8), (13) and (16), Eq. (7) gives the energy absorbed in circumferential deformation during the rotation of first and third limb upto  $\theta_1$  and second limb upto  $\theta_2$  :

$$W_{c\theta} = 2\pi f_{yt} t_0 \left( \frac{R_b}{\sin \alpha} \right)^2 \left[ \begin{array}{l} \left\{ -\ln(A^{ra_3} B) + (1-m-rma_3) \frac{h_1 \sin \alpha}{R_b} \right\} \left\{ \sin(\theta_1 + \alpha) - \sin \alpha \right\} \\ + \left\{ 2\ln(A^{ra_3} B) + ra_3 A^2 + B^2 - (1+ra_3) \right\} \left\{ \frac{\cos 2\alpha - \cos 2(\theta_1 + \alpha)}{8 \sin \alpha} \right\} \\ \left\{ -4(1-m-rma_3) \frac{h_1 \sin \alpha}{R_b} \right\} \end{array} \right]$$

$$\begin{aligned}
& -2\pi f_{yt_0} \left( \frac{R_d}{\sin \alpha} \right)^2 \left[ \begin{aligned} & \left\{ \ln(C^r D) + (1-m-rm) \frac{h_2 \sin \alpha}{R_d} \right\} \{ \sin(\theta_2 - \alpha) - \sin \alpha \} \\ & + \left\{ 2 \ln(C^r D) + rC^2 + D^2 - (1+r) \right. \\ & \left. + 4(1-m-rm) \frac{h_2 \sin \alpha}{R_d} \right\} \left\{ \frac{\cos 2\alpha - \cos 2(\theta_2 - \alpha)}{8 \sin \alpha} \right\} \end{aligned} \right] \\
& + 2\pi f_{yt_0} \left( \frac{R_f}{\sin \alpha} \right)^2 \left[ \begin{aligned} & \left\{ \ln(E^r) + rm \frac{h_1 \sin \alpha}{R_f} \right\} \{ \sin(\theta_1 + \alpha) - \sin \alpha \} \\ & + \left\{ 2 \ln(E^r) + rE^2 - r \right. \\ & \left. + 4rm \frac{h_1 \sin \alpha}{R_f} \right\} \left\{ \frac{\cos 2\alpha - \cos 2(\theta_1 + \alpha)}{8 \sin \alpha} \right\} \end{aligned} \right] \quad (19)
\end{aligned}$$

$$\text{where, } A = 1 - \frac{mh_1 \sin \alpha}{R_b}, \quad B = 1 + \frac{(1-m)h_1 \sin \alpha}{R_b}, \quad C = 1 - \frac{(1-m)h_2 \sin \alpha}{R_d} \quad (20)$$

$$D = 1 + \frac{mh_2 \sin \alpha}{R_d}, \quad E = 1 - \frac{mh_1 \sin \alpha}{R_f} \quad (21)$$

putting,  $\theta_1 = \frac{\pi}{2} - \alpha$  and  $\theta_2 = \frac{\pi}{2} + \alpha$  in Eq. (19) gives the total energy absorbed in circumferential deformation during complete crushing of the fold:

$$W_c = 2\pi f_{yt_0} \frac{1 - \sin \alpha}{\sin^2 \alpha} \left[ R_b^2 B_1 - R_d^2 B_2 + R_f^2 B_3 + \frac{1 + \sin \alpha}{4 \sin \alpha} \{ R_b^2 C_1 - R_d^2 C_2 + R_f^2 C_3 \} \right] \quad (22)$$

$$\text{where, } B_1 = -\ln(A^{ra_3} B) + (1-m-mra_3)Kh_2 \frac{\sin \alpha}{R_b} \quad (23)$$

$$B_2 = \ln(C^r D) + (1-m-mr)h_2 \frac{\sin \alpha}{R_d}, \quad B_3 = \ln(E^r) + rmKh_2 \frac{\sin \alpha}{R_f} \quad (24)$$

$$C_1 = -(1+ra_3) + B^2 + ra_3 A^2 + 2 \ln(A^{ra_3} B) - 4(1-m-mra_3)Kh_2 \frac{\sin \alpha}{R_b} \quad (25)$$

$$C_2 = -(1+r) + D^2 + rC^2 + 2 \ln(C^r D) + 4(1-m-mr)h_2 \frac{\sin \alpha}{R_d} \quad (26)$$

$$C_3 = -r + rE^2 + 2 \ln(E^r) + 4rmKh_2 \frac{\sin \alpha}{R_f} \quad (27)$$

which for total outside fold model (i.e.  $m=0$ ) converts to the expression given in Ref. [6]. Equation (22) is not valid for  $\alpha = 0$ , for which, we get,

$$W_c = \pi f_{yt_0} h^2 \left[ (a_3 + 2)rm^2 \left( 1 - \frac{mh}{3R_1} \right) + 2(1-m)^2 \left\{ 1 + \frac{(1-m)h}{3R_1} \right\} \right] \quad (28)$$

which is same as that given in Ref. [5] for cylinder of radius  $R_1$  and size of fold,  $h$ .

## 2.2. Average crushing load

Assuming that the energy dissipation in the axisymmetric axial crushing of frusta takes place in the form of flexural and circumferential deformations, therefore, the external work done can be equated to the energy absorbed in bending and circumferential stretching. The average crushing load,  $P_m$ , can, therefore, be calculated:

$$P_m = \frac{W_b + W_c}{\{1 + K(1 + a_4 m)\} h_2 \cos \alpha} \quad (29)$$

where,  $W_b$  is given by Eq. (4) for frusta and Eq. (6) for tubes, and  $W_c$  is given by Eq. (22) for frusta and Eq. (28) for tubes.

### 2.3. Size of Fold and folding parameter, $m$

Determination of the size of fold,  $h_1$  and  $h_2$ , and the folding parameter,  $m$ , requires the minimization of external work done for crushing unit length of frusta during the fold formation or the minimization of average crushing load of the fold i.e.

$$\frac{\partial P_m}{\partial h_2} = 0 \quad \text{and} \quad \frac{\partial P_m}{\partial m} = 0 \quad (30)$$

where,  $P_m$  is given by Eq. (29).

### 2.4. Variation of Crushing Load

The variation of crushing load can now be found from the following relation

$$P_\theta = \frac{d(W_{b\theta} + W_{c\theta})}{dz} = \frac{d}{d\theta} (W_{b\theta} + W_{c\theta}) \left/ \frac{dz}{d\theta} \right. \quad (31)$$

where,  $W_{b\theta}$  and  $W_{c\theta}$  are the work done in bending and circumferential stretching in rotation of lower limb of fold upto  $\theta_1$  and upper limb upto  $\theta_2$  given by Eqs. (3) and (19); and  $z$  is the crushing distance in the direction of the load which is given by:

$$z = h_2 \left[ \{1 + K(1 + a_4 m)\} \cos \alpha - K(1 + a_4 m) \cos(\theta_1 + \alpha) - \cos(\theta_2 - \alpha) \right] \quad (32)$$

## 3. COMPARISON WITH EXPERIMENTAL OBSERVATIONS

A steel cylindrical tube [5] of 43.0 mm diameter and 1.8 mm thick and an Aluminium frustum [6], 1.8 mm thick, 132.3 mm long, and with end diameters of 44.8 and 69.3 mm (Angle of taper = 5.29°) tested in axial compression have been used in the validation of the analysis presented in earlier sections. The value of yield strength of the material of steel tube and Aluminium frustum taken in the analysis is 400.0 and 92.0 MPa respectively.

The crushing load variation obtained experimentally for first fold of the Aluminium frustum has been plotted in Fig. 2. The value of size of fold was first determined numerically by minimizing the mean crushing load and this value was then used for finding out the variation of crushing load  $P_\theta / P_0$ , where,  $P_0 = \pi(R_1 + R_2) f_{yt}$ ,  $R_1$  and  $R_2$  are the end radii of frustum. The variation of crushing load for first fold taking  $r = 1.0$  and other fold  $r = 2.0$  have also been plotted in Fig. 2. The analytical load-deformation curves do not start from zero load level due to the neglect of the elastic deformation in the beginning. The post peak experimental curve for first fold is found to be close to the analytical curve. The crushing load variation obtained experimentally and analytically for the first fold of the steel tube taking  $r = 1.0$  and other folds by taking  $r = 2.0$  have been plotted in Fig. 3. The values of size of fold and folding parameter were first determined numerically by minimizing the mean crushing load and these values were then used for finding out the variation of crushing load. The post peak experimental curve is found to be close to the analytical curve.

The influence of folding parameter,  $m$ , on the non-dimensional mean crushing load has been studied by plotting its variation in Fig. 4 for the first fold of a steel tube, 24.05 mm in diameter and 1.01 mm thick with  $f_{yt} = 400.0$  MPa.

The curves in this figure have been plotted for the model of tube with  $r = 1$  (i.e. first fold). Each point on this curve corresponds to an optimal size of fold whose variation is also plotted in this figure. It is observed from this figure that the mean crushing load is maximum for total outside folding (i.e.  $m = 0$ ) which reduces with increase in the

value of  $m$  and after reaching an optimal value, it increases but the mean crushing load for the total outside folding (i.e.  $m = 1$ ) is less than that of the total outside.

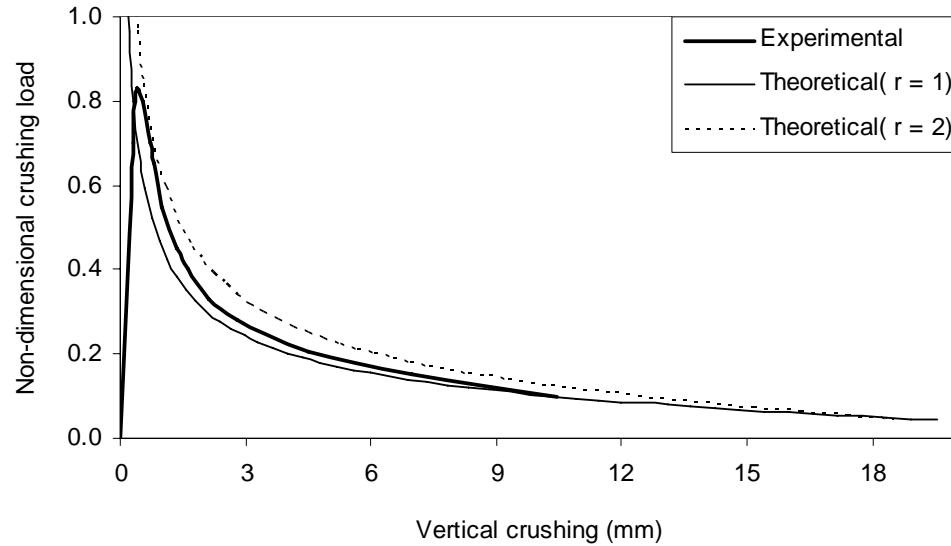


Fig. 2 Comparison of theoretical and experimental compression curves for Aluminium frusta

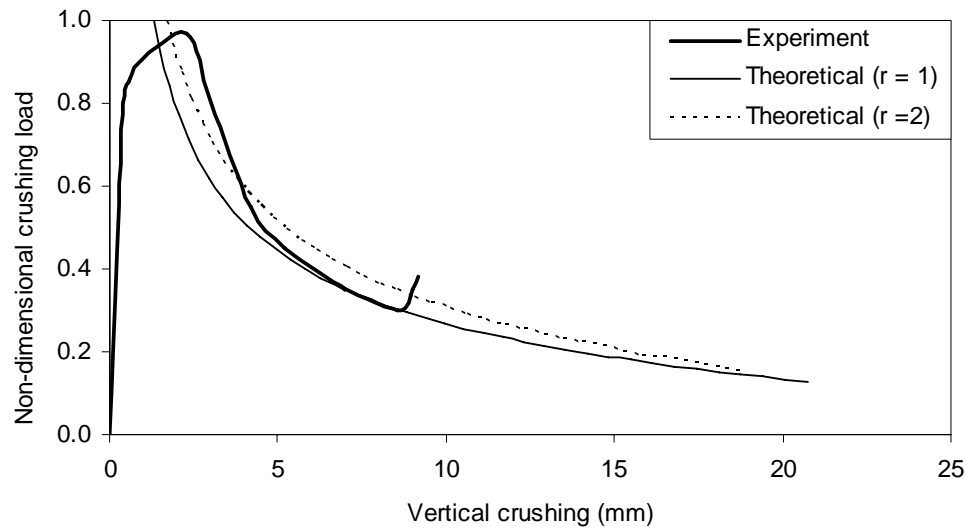


Fig. 3 Comparison of theoretical and experimental compression curves for steel tube

The mean crushing load of the tube has two components, one due to bending and the other due to circumferential deformation. The variation of these two components has also been plotted in this figure. It is observed from their variation that for smaller values of folding parameter ( $m < 0.47$ ), the energy absorption in bending is more than that in circumferential deformation but for larger values of folding parameter ( $m > 0.47$ ), it is the vice versa. At the optimal point where folding parameter is slightly more than 0.5 ( $m = 0.62$  for the tube considered here), the energy absorption in bending is slightly more than the circumferential deformation which for practical purposes may be taken to be almost equal. The value of the size of fold was first determined numerically by minimizing the mean crushing load, which was then used for finding out the mean crushing load. The curves for one value  $r$  in this figure start from the same mean crushing load at  $m = 0$  because for this case of total outside folding, there is only

extensional circumferential deformation. For the values of  $m > 0$ , the increase in the parameter  $r$  results in increase in the mean crushing load and decrease in the optimal value of folding parameter which is because of the material becoming stronger in compression thus the fold goes less inside the initial line.

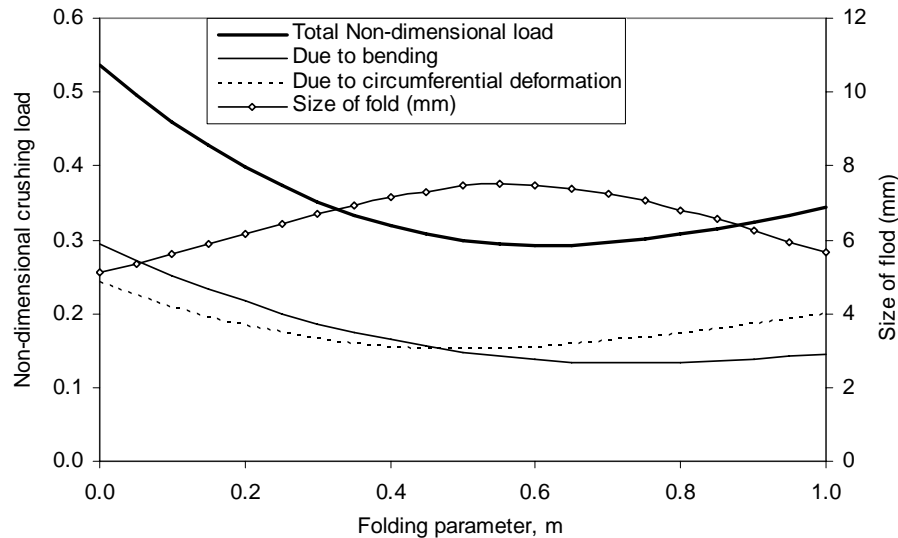


Fig. 4 Variation of non-dimensional crushing load with folding parameter for steel tube with  $r = 1$

#### 4. CONCLUSIONS

A mathematical model has been developed for axisymmetric axial crushing analysis of thin walled frusta considering straight folds that are partly inside and partly outside the initial line of the frusta. Variation of circumferential strain during the formation of folds has been considered. The existing total outside fold model of frusta and partly inside and partly outside fold model of tube can be derived from this model. The variation in crushing load and mean collapse load have been computed.

The results have been compared with experiments and reasonably good agreement has been observed. For smaller values of folding parameter, the energy absorption in bending is more than that in circumferential deformation but for larger values of folding parameter, it is the vice versa. For optimal value of folding parameter, the energy absorption in bending is slightly more than the circumferential deformation, which for practical purposes may be taken to be almost equal.

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