

## Structural and Vibrational Analysis of Liquid Storage Tanks

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### ABSTRACT

The finite element method (FEM) is used to conduct an analysis of two sets of liquid storage tanks. The first set comprises of ten broad tanks with height-to-radius ratio generally less than 1.5. These tanks are representatives of a typical oil storage tank farm. The second set of tanks comprises of ten tall tanks, with height-to-radius ratio generally greater than 1.5. These tanks are representative of tanks which buckled or collapsed during past earthquakes. In the present study the tanks are analyzed as ideal shells, not considering possible geometric imperfections. The FEM is used to carry out a free vibration analysis, and a nonlinear collapse analysis for simulated seismic loadings. Step changes in thickness are accounted for in the first set of tanks, and the presence of a plate or conical roof or stiffening girder are considered for the second set of tanks. The procedures of the study are extensively validated by making comparisons with available results in the literature. The current report provides results for the frequencies of the tanks in empty state conditions, and for the collapse loads for geometries without imperfections. In a further study it is planned to provide results for frequencies of full tanks, and for the collapse loads of tanks with imperfections.

### INTRODUCTION

The problem of the behavior of above-ground vertical steel cylindrical liquid storage tanks subjected to seismic or wind action remains a major concern in structural engineering [1-11]. Failures of such tanks have been recorded in a number of earthquakes, including recent ones. Research is continuing to provide additional data on tank behavior, which will give a basis for the further improvement of tank design codes.

Liquid storage tanks appear in two major identifiable forms. The tanks may be broad with low L/R (height-to-radius) ratio, or tall with large L/R ratio. During seismic action a tank of either form is subjected to horizontal excitation which causes acceleration of the liquid in the tank. The acceleration leads to a large horizontal nonsymmetric pressure on the tank wall producing an overturning moment that can lead to buckling failure. For broad tanks the failure may take the form of elephant-foot bulging (EFB), while for tall tanks the failure may be in the form of diamond shaped buckling. While the response of tanks under seismic action is dynamic, a quasi-static analysis may be carried out to reduce the computational effort [12].

In this study two sets of anchored steel liquid storage tanks are analyzed. The tanks are idealized as perfect cylindrical shells. Using the FEM the natural frequencies of vibration of the tanks are found for empty state conditions. A simple formula is used to complement the FEM vibration results. Using an equivalent static load approximation, an FEM collapse analysis is conducted, that predicts the seismic acceleration coefficient necessary for failure.

### DESCRIPTION OF TANK SETS AND SEISMIC LOADING

Haroun, Izzeddine, and Mode [10] have compiled a list of steel tanks found in a tank farm in the Caribbean. The description of ten of these tanks (A1-A10) is given in Table 1. The tank radius  $R$  varies approximately from 10-31m, while the tank height  $L$  varies from 9-18m. The tanks consist of 5-8 courses of steel plates, with the bottom course having a thickness varying from 13-35mm, and the top course having a thickness varying from 6-8mm. These are mostly broad tanks with the L/R ratio varying from 0.5-1.6, and the  $R/h_m$  ratio varying from 1100-1700, where  $h_m$  represents a simple mean wall thickness. In the collapse analysis of this study the tanks of this group are assumed anchored, filled with water, and uncovered. The tanks are analyzed accounting for the step changes in the wall thickness, and also for a mean value of the wall thickness.

Hamdan [8] has compiled a list of steel tanks which buckled or collapsed during recent earthquakes. The description of ten of these tanks (B1-B10) is given in Table 2. The tank radius varies approximately from 1.4-6.4m, while the tank height varies from 5-13m. The thickness  $h$ , assumed constant for a given tank, varies from about 2-10mm. These are mostly tall tanks with the L/R ratio varying from 1.5-4.6, and the  $R/h$  ratio varying from 550-1100. Information about the covering of the tanks was not available, but for this study shell roofs with specified thickness ( $h_R$ ) are assumed for the tanks. In the collapse analysis conducted herein the tanks are assumed anchored, of constant thickness, and filled with water. Analyses are carried out for uncovered conditions, and also with plate or conical roof covers, or with a stiffening ring at the top.

Table 1 Description of tank set A

Tank	R(m)	L(m)	Course Thick.	h(mm)	L/R	R/h
A1	17.70	9.12	14-13-11-10-8-6	10.33	0.52	1720
A2	21.95	12.20	21-17-12-8-6	12.80	0.56	1715
A3	31.00	17.50	35-33-20-17-12-2*8	19.00	0.56	1630
A4	21.35	12.20	18-14-11-2*8	11.80	0.57	1810
A5	22.85	14.63	26-22-17-13-9-6	15.50	0.64	1470
A6	17.68	12.20	17-15-12-10-8-3*6	10.00	0.69	1770
A7	17.85	12.50	18-18-13-12-10-2*6	11.86	0.70	1500
A8	18.30	14.60	21-18-14-10-2*8	13.17	0.80	1390
A9	10.65	14.60	13-10-8-3*6	8.17	1.37	1300
A10	10.00	16.20	13-10-2*8-2*7	8.33	1.62	1135

Table 2 Description of tank set B

Tank	R(m)	L(m)	h(mm)	L/R	R/h	h <sub>R</sub> (mm)
B1	5.64	8.63	10.00	1.53	564	12.7
B2	6.41	12.20	5.85	1.90	1096	12.7
B3	6.28	13.27	6.93	2.11	906	12.7
B4	6.10	12.96	7.12	2.12	857	12.7
B5	4.58	10.68	4.77	2.33	960	12.7
B6	4.27	12.20	4.77	2.86	895	10
B7	1.98	6.10	1.98	3.08	1000	5
B8	1.45	5.34	1.98	3.68	732	5
B9	3.05	12.20	4.77	4.00	639	7
B10	2.60	12.00	4.68	4.62	556	6

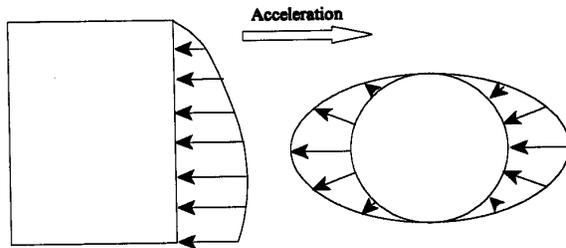


Fig. 1 Pressure distribution due to horizontal acceleration

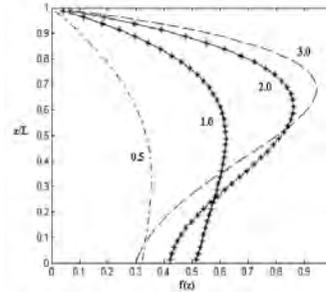


Fig. 2 Pressure profiles for various L/R ratios

The typical tank is modelled as a vertical cylindrical shell, fixed at the base and free at the top. In the filled state there is a steady axisymmetric hydrostatic pressure  $p_s$  given by

$$p_s = \rho_L g(L-z) \tag{1}$$

where  $\rho_L$  is the mass density of the liquid,  $g$  the acceleration of gravity,  $L$  the height of the tank, and  $z$  the axial coordinate measured from the base of the tank. The hydrostatic pressure clearly causes a prestress in the tank wall. Under lateral seismic action (Fig. 1) there is additionally a hydrodynamic pressure  $p_d$  which is assumed to have a circumferential variation of  $\cos \theta$ . The pressure  $p_d$  is given by

$$p_d = \rho_L R f(z) \cos \theta G g A(t) \tag{2}$$

where  $R$  is the tank radius,  $f(z)$  is a nonlinear pressure profile that gives the vertical variation of the pressure,  $G$  is the acceleration factor (as a ratio of  $g$ ), and  $A(t)$  is a dimensionless time function. In the static load approximation used in this study the time function is taken as unity, and the objective of the analysis is to determine the critical acceleration factor  $G_c$  that produces failure. The combined hydrostatic and equivalent hydrodynamic loading for tanks under seismic action

considered herein has been discussed in a number of studies including those of Wunderlich and Seiler [1], Mirfakhraei and Redekop [5], Redekop, Mirfakhraei, and Muhammad [6], and Hamdan [8].

The pressure profile  $f(z)$  stemming from the horizontal seismic action can be determined through the solution of a fluid-structure interaction problem (Wunderlich and Seiler [1], Veletsos and Tang [11]). The function is known to depend strongly on the L/R ratio and weakly on other geometric factors. For the current study it was determined by interpolating the pressure profiles given by Wunderlich and Seiler [1], which had been determined by the solution of the fluid-structure interaction problem. Sample profiles, presented in Fig. 2, agree closely with those given in their earlier paper.

## FEM ANALYSIS

The FEM analysis of the current study was conducted using the software ADINA [13]. The tanks in all cases were assumed anchored and thus the analysis was for the structure only, excluding the foundation. This choice of shell geometry, with fixed-free boundary conditions for uncovered tanks, concurs with that used by many other analysts.

An aim in the meshing was to obtain well proportioned elements, and mesh variations were made to ensure a converged solution. The meshing for uncovered tanks was with four-node 20 degree-of-freedom plate/shell elements. For covered tanks some triangular elements were used in the roof. In the circumferential direction, elements of equal size were selected, whereas in the axial direction scaling of element size by a ratio of one to three from bottom to top was undertaken. The small elements at the bottom were desirable to better represent the expected EFB in the collapse analysis. As symmetry exists, the majority of the meshes were for half-tank models, to reduce the computational effort. For some cases of analysis full-tank models were also conducted to demonstrate the validity of the half-tank models. The mesh sizes used are indicated in the following for some sample cases.

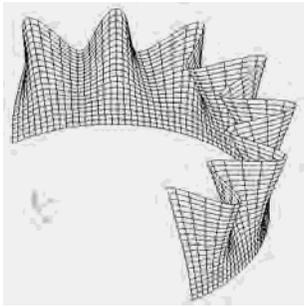


Fig. 3 First vibration mode for open tank V2

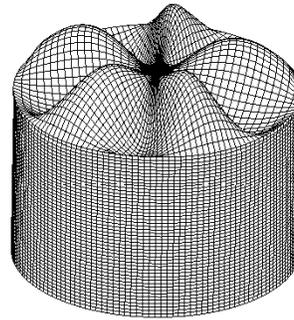


Fig. 4 First vibration mode for tank V5 with conical roof

Two separate approaches were taken to ensure accuracy of the FEM calculations. The first was to conduct the customary convergence study. The second was to validate the current FEM procedure against available vibration and buckling results. Results for frequencies and linear buckling loads for both uncovered and covered tanks were available in the literature. Meshes used in the validation served then as guides in the design of meshes used in the further analyses of the study.

### Natural Frequency Analysis

A frequency analysis was carried out first to become acquainted with the characteristics of the tanks and to obtain experience with the meshing. This analysis was conducted for empty tank conditions. The material of the shell was assumed to be steel, with Young's modulus  $E=200$  GPa, Poisson's ratio  $\nu=0.3$ , and mass density  $\rho_s=7800$  kg/m<sup>3</sup>.

### Linearized Buckling Analysis

A linearized buckling (LB) analysis (Bathe [14]) was carried out to obtain further experience with the meshing. Elastic conditions are assumed, and the bifurcation loading level is sought. This eigenvalue analysis was carried out for two different cases, one for an uncovered tank under wind pressure, and the other for covered tanks under internal pressure.

### Nonlinear Collapse Analysis

The main FEM analysis of the study was a nonlinear FEM collapse analysis. In the ADINA software a small-strain large displacement 'static' formulation with elastic-perfectly-plastic material property was selected. For the nonlinear FEM analysis the yield point of the shell material was taken as 250 MPa, with no strain hardening. This collapse analysis (CA), of course differs from the linearized buckling (LB) analysis in which an elastic material is assumed.

As discussed in the preceding the loading on the tank comprises of two effects. The first, the hydrostatic pressure  $p_s$ , given by equation (1), was input simply as a time-invariant axisymmetric loading varying linearly in the axial direction.

The second, the equivalent hydrodynamic pressure  $p_d$ , given by equation (2), was input as an array, calculated for an assumed acceleration factor  $G=1$ , and having a profile  $f(z)$  appropriate with the  $L/R$  ratio. In the collapse analysis the hydrodynamic pressure was then changed by factoring  $G$ , starting from a small value and increasing in a number of discrete steps. The increase in  $G$  was continued until the numerical process became unstable, indicating structural instability. The failure pressure was taken as the mean of the pressure at the last stable point and the pressure after instability. From this defined failure condition the theoretical critical acceleration factor  $G_c$  was identified.

### FORMULA FOR NATURAL FREQUENCY ANALYSIS

An approximate formula for the natural frequency of vibration  $\omega$ , of a thin-walled cylindrical shell is given by Soedel [15]. The formula is based on the Donnell-Mushtari-Vlasov thin shell theory which is of the Kirchoff-Love type. In deriving the formula, use is made of the Galerkin method in which beam functions are used as the trial functions. For a cylindrical shell with fixed-free support conditions the formula is

$$\omega_{mn}^2 = \{ [ (Eh\lambda_m^4) / [R^2 \{ (n/R)^2 + \lambda_m^2 \}^2] + D [ (n/R)^2 + \lambda_m^2 ]^2 \} / (\rho_s h) \quad (3)$$

where the  $\lambda$  set is given by  $\lambda_1=1.875104$ ,  $\lambda_2=4.694091$ ,  $\lambda_3=7.854757$ , ... (Nachtigall, Gebbeken, and Urrutia-Galicia [2]). The  $m$  and  $n$  are the wave numbers in the axial and circumferential directions, and  $D=Eh^3/12(1-\nu^2)$ . In the formula, the thickness measure  $h$  is taken as the simple mean thickness  $h_m$  of the tank wall.

Table 3 Description of validation tank set V

Tank	Geometry		Wall thickness		Ratios		Roof		Study
	R(m)	L(m)	Course thickness (mm)	$h_m$ (mm)	L/R	$R/h_m$	$v/r$	$h_R$ (mm)	
V1	15.24	19.34	12.7-9.5-6*7.9	8.7	1.27	1750	-	-	VGS [3]
V2	15.24	12.19	12.7-9.5-3*7.9	9.18	0.80	1660	-	-	
V3	15.24	7.26	12.7-9.5-7.9	10.00	0.48	1520	-	-	
V4	15.24	19.34	12.7-9.5-6*7.9	8.7	1.27	1750	0	12.7	
V5	15.24	19.34	12.7-9.5-6*7.9	8.7	1.27	1750	3/16	12.7	
V6	15.24	7.26	12.7-9.5-7.9	10.00	0.48	1520	0	12.7	
V7	15.24	7.26	12.7-9.5-7.9	10.00	0.48	1520	3/16	12.7	
V8	19.00	7.60	10	10.0	0.40	1900	-	-	FG [9]
V9	5.00	5.0	4	4	1.00	1250	1/6	4	HM [4]
V10	5.00	5.0	8	8	1.00	625	1/6	8	
V11	5.00	5.0	12	12	1.00	417	1/6	12	
V12	10.00	5.0	4	4	0.50	2500	1/6	4	
V13	10.00	5.0	8	8	0.50	1250	1/6	8	
V14	10.00	5.0	12	12	0.50	833	1/6	12	
V15	15.00	5.0	4	4	0.33	3750	1/6	4	
V16	15.00	5.0	8	8	0.33	1875	1/6	8	
V17	15.00	5.0	12	12	0.33	1250	1/6	12	

### VALIDATION

To validate the procedure of the current study results for natural frequencies and buckling loads were computed for a number of storage tanks reported in the literature. The geometric details of the 17 tanks considered in the validation are given in Table 3. Tanks V1-V7 are large broad tanks with step-variation in wall thickness. The first three of these are uncovered while the last four have either plate or cone shell roofs. The natural periods of vibration of these tanks were determined earlier by Virella, Godoy, and Suarez [3], using the FEM software ABAQUS. Tank V8 is a broad tank that was studied by Flores and Godoy [9], from the point of view of buckling under hurricane wind loading. Tanks V9-V17 are small tanks with conical roofs, studied earlier by Hagihara and Miyazaki [4]. In the tables, and subsequent work herein, the results from these three studies will be referred to using the symbols VGS, FG, and HM respectively. Buckling pressures were determined under uniform internal pressure loading. In the table  $h_m$  indicates the simple mean wall thickness,  $v/r$  indicates the rise over run of the roof, and  $h_R$  the thickness of the roof.

Table 4 Validation for vibration of empty open tanks (V1, V2, V3) - first 19 periods (sec)

Mode	V1			V2			V3		
	VGS	Formula	FEM	VGS	Formula	FEM	VGS	Formula	FEM
1	0.696	0.687 (11)	0.697	0.424	0.422(13)	0.426	0.244	0.242(16)	0.246
3	0.677	0.687 (10)	0.678	0.417	0.413(14)	0.419	0.243	0.240(17)	0.245
5	0.672	0.649 (12)	0.674	0.416	0.412(12)	0.418	0.240	0.237(15)	0.242
7	0.631	0.636 (09)	0.631	0.397	0.392(15)	0.399	0.237	0.233(18)	0.239
9	0.606	0.591 (13)	0.607	0.393	0.382(11)	0.395	0.232	0.225(14)	0.234
11	0.572	0.545 (08)	0.573	0.371	0.364(16)	0.373	0.226	0.222(19)	0.227
13	0.514	0.529 (14)	0.514	0.355	0.338(10)	0.357	0.221	0.210(20)	0.223
15	0.512	0.471 (15)	0.513	0.343	0.334(17)	0.345	0.210	0.207(13)	0.211
17	0.459	0.437 (07)	0.459	0.315	0.305(18)	0.317	0.209	0.196(21)	0.211
19	0.411	0.419 (16)	0.411	0.310	0.287(09)	0.311	0.197	0.186(12)	0.199

Table 5 Validation for vibration of empty covered tanks - first 19 periods (sec)

Mode	V4		V5		V6		V7	
	VGS(b)	FEM(b)	VGS	FEM	VGS(b)	FEM(b)	VGS(b)	FEM(b)
1	7.873	7.873	0.377b	0.377b	7.873	7.890	0.371	0.371
3	3.773	3.773	0.374b	0.374b	3.773	3.781	0.371	0.370
5	2.294	2.294	0.358b	0.358b	2.293	2.298	0.356	0.355
7	1.564	1.564	0.357b	0.357b	1.564	1.567	0.344	0.344
9	1.311	1.311	0.338	0.336	1.311	1.313	0.333	0.331
11	1.144	1.144	0.338	0.335	1.143	1.145	0.307	0.305
13	0.941	0.941	0.333b	0.333b	0.941	0.942	0.295	0.289
15	0.892	0.892	0.328	0.327	0.892	0.894	0.280	0.278
17	0.876	0.876	0.328	0.326	0.876	0.878	0.275	0.275
19	0.715	0.715	0.313	0.310	0.715	0.716	0.255	0.253

A comparison of results for the lowest 19 periods of natural vibration for the open tanks V1-V3, under empty conditions, is given in Table 4. These periods appear in pairs, and only the odd-numbered values are cited. For each tank the values given by VGS, the approximate formula (3), and the current FEM analysis are given. The circumferential wave number as predicted by the approximate formula is given in the brackets. The current FEM values show agreement within 1% in comparison with the VGS values for all three tanks and all 19 modes. The formula values agree within 1-2% for the first period. It is noted that the L/R ratio of the tanks V1 to V3 decreases from 1.27 to 0.48. The wave numbers of the fundamental frequency increase as the L/R ratio decreases, indicating the need for a finer mesh in the circumferential direction for the shorter tanks.

A comparison of results for the lowest 19 periods of natural vibration for the covered tanks V4-V7, under empty conditions, is given in Table 5. Tanks V4 and V6 had flat roofs, while tanks V5 and V7 had conical roofs. The thickness of all the tank covers was taken as 12.7mm. The periods again appear in pairs, and only the odd-numbered ones are recorded. For each tank the values given by VGS and the current FEM analysis are given. The (b) symbol indicates that the vibration mode occurred in the tank roof. There is again excellent agreement in the two sets of results. The tanks with the flat roof have higher natural periods than the comparable tanks with the conical roof. The tank V5 with the conical roof has lower natural periods than the comparable uncovered tank V1 (Table 4), while tank V7 has higher periods than tank V3.

Plots as determined in the current study are given in Figs. 3-4 for the fundamental mode shapes for the open tank V2 and the covered tank V5. The mesh in the current study for the open tank was made for the half-model, as opposed to the full model in the paper by Virella, Godoy, and Suarez [3]. Comparison of the current Fig. 3 with Fig. 5 of their paper shows a close similarity in mode shape, with fourteen circumferential waves indicated in both cases. Comparison of Fig. 4 with Fig. 7a of their paper again shows a close similarity, with a large displacement pattern involving five waves in the roof in each case.

A comparison of results for linearized buckling analysis of empty tanks is given in Table 6. The results of the current study were all obtained for half-tank models. Tank V8 is a large uncovered broad tank analyzed for a wind pressure that varies in two directions, strongly in the circumferential, and mildly in the axial (Flores and Godoy [9]). The tank experiences a large overturning moment, and thus is subject to a stress state that is somewhat similar to that experienced by tanks under lateral seismic action. The non-dimensional bifurcation buckling pressure given by Flores and Godoy [9] was 2.207, determined by a modal analysis employing ring elements. In the current study the problem was solved using a mesh of

240X30 elements, where the first integer gives the number of elements circumferentially and the second axially. The result from the current CA analysis was 2.205.

Table 6 Validation for buckling and collapse pressure (kPa)

Tank	Pressure	LB (previous)	LB (current)	CA	Wall mesh	Roof mesh
V8	Wind	2.207	-	2.205	240X30	
V9	Internal	3.416	2.961	3.85	150X50	150x50
V10		18.23	18.59	20.75	150X50	
V11		41.26	58.01	51.75	150X50	
V12	Internal	0.509	0.503	1.35	280X40	248x80
V13		3.419	2.867	3.55	280X40	
V14		9.500	8.296	10.25	280X40	
V15	Internal	0.171	0.150	0.15	285X30	285x90
V16		1.117	1.113	1.15	285X30	
V17		3.422	2.845	3.75	285X30	

Results for the failure pressures under internal pressure loading of the covered tanks V9-V17 are given in Table 6. For each tank the results from a LB analysis as determined by Hagihara and Miyazaki [4] and the current study are given. Presented as well are results for a CA analysis of the current study. In the descriptions of the FEM meshes of the current study the first integer for the wall and roof meshes indicates the number of elements in the circumferential direction. The second integer, for the wall gives the number of elements in the axial direction, and for the roof the nominal number of elements in the meridional direction. The LB results of the current study show agreement with the results of the previous study, although not as close as those of the vibration analysis. The CA results for some tanks are lower than the LB results, indicating that the critical plastic stress state arises prior to the occurrence of elastic buckling.

## RESULTS FOR TANK SETS A AND B

New results are discussed in this section for the frequency and collapse analysis of the tank sets A and B (Tables 1-2). Natural frequency values cited in Table 7 for broad tank set A are the fundamental frequencies in Hz. The results given are those obtained using the Soedel formula (2), and the FEM. Two sets of FEM values are given, values based on an assumed constant mean wall thickness (FEM1), and values accounting for the step change in wall thickness (FEM2). The bracketed quantities given with the formula values are the circumferential mode numbers. Two sets of FEM values are also given for the collapse, values based on an assumed constant mean thickness (CA1 values), and values accounting for step change in thickness (CA2 values). These latter two values represent the theoretical critical acceleration factor  $G_c$  for failure.

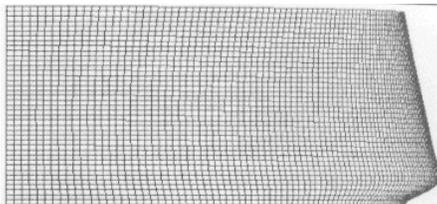


Fig. 5 Typical EFB in broad tank

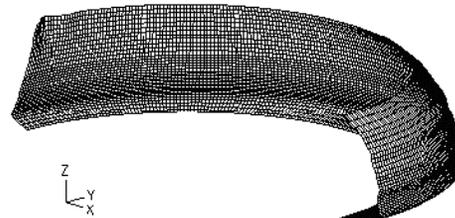


Fig. 6 Deformation - broad tank A1

It is noted that the three sets of frequencies show close agreement, with an approximate range in fundamental frequency from 1.6 to 3.1 Hz. The collapse values CA1 are significantly lower than the CA2 values, indicating the importance of using the correct values for the local wall thickness in the analysis. Numerically the CA1 values range approximately from 0.1 to 0.5, while the CA2 values from 0.5 to 0.6. In evaluation of these results it is to be recalled that the tanks have been idealized as perfect cylindrical shells.

The natural frequency values cited in Table 8 for the tall tank set B are again the fundamental frequencies in Hz. Results for open tanks were obtained using the Soedel formula (2), and the FEM. The bracketed quantities given with the formula values are again the circumferential mode numbers. Results for covered tanks, obtained using the FEM, are given for both plate and cone roofs. The symbol  $b$  again signifies a vibration mode in the roof. The collapse values were obtained using the FEM, and are given for open tanks, for tanks with plate or cone roofs, and for tanks with a stiffening ring at the top. These latter three values represent the theoretical critical acceleration factors  $G_c$  for failure.

Table 7 Results for natural frequencies and collapse for tank set A

Tank	Fundamental frequency (Hz)			Collapse $G_c$	
	Formula	FEM1	FEM2	CA1	CA2
A1	3.12 (16)	3.04	3.13	0.491	0.604
A2	2.33 (16)	2.28	2.36	0.479	0.568
A3	1.67 (15)	1.62	1.66	0.103	0.438
A4	2.27 (16)	2.22	2.24	0.460	0.610
A5	2.09 (14)	2.05	2.16	0.477	0.539
A6	2.29 (14)	2.25	2.26	0.433	0.552
A7	2.44 (13)	2.38	2.47	0.461	0.542
A8	2.17 (12)	2.12	2.13	0.426	0.501
A9	2.24 (09)	2.20	2.21	0.426	0.531
A10	2.17 (18)	2.12	2.13	0.456	0.572

Table 8 Results for natural frequencies and collapse for tank set B

Tank	Fundamental frequency (Hz)				Collapse $G_c$		
	Open		Covered - FEM		FEM		
	Formula	FEM	Plate(b)	Cone	Open	Plate/cone	Ring
B1	5.75 (7)	5.64	0.937	11.5b	1.218	1.503	1.734
B2	2.94 (8)	2.95	0.649	5.99	0.692	0.802	0.838
B3	2.94 (7)	2.94	0.709	6.05	0.762	0.899	0.943
B4	3.11 (7)	3.11	0.756	6.39	0.793	0.800	0.859
B5	3.58 (7)	3.61	1.157	7.39	0.802	0.951	1.007
B6	3.22 (6)	3.25	1.160	6.71	0.870	1.024	1.091
B7	6.10 (6)	6.17	2.557	12.80	0.970	1.187	1.279
B8	8.14 (5)	8.14	4.662	16.89	1.327	1.747	1.999
B9	3.90 (5)	3.90	1.740	7.92	1.082	1.420	1.548
B10	4.18 (4)	4.07	2.096	8.41	1.335	1.791	1.977

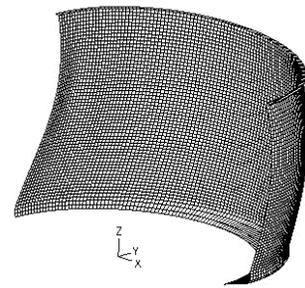
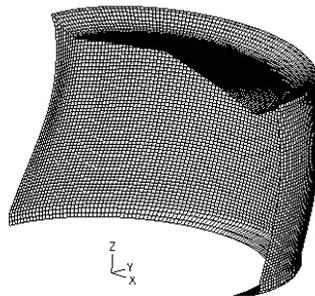
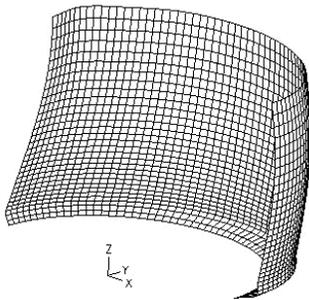


Fig. 7 Deformation - open tank B1 Fig. 8 Deformation - cone-covered tank B1 Fig. 9 Deformation - ring-stiffened tank B1

It is noted that the two sets of frequencies for open tanks show close agreement, with an approximate range from 2.9 to 8.1 Hz. The frequencies for the covered tanks differ considerably from those of the open tanks, and from each other. The collapse values for the open tanks are significantly lower than the values for the covered tanks, indicating the importance of the roof in stiffening the tanks. The collapse values for the two types of roof were identical to the figures cited, and thus are given in a single column. The collapse values for the ring stiffened tanks are larger than those for the roof-supported tanks, as expected. Numerically the approximate range for the collapse values for the open tanks is from about 0.7 to 1.3, for the covered tanks from 0.8 to 1.8, and for the ring-stiffened tanks from 0.85 to 2.0.

Plots were determined in the current study for the deformed collapse shapes. For broad tanks (Figs. 5-6) EFB is generally quite prominent at the bottom, in the compression area zone. Figs. 7-9 respectively show the deformed shape for the tall tank B1 when uncovered, when closed with a conical-shell roof, and when reinforced with a stiffening ring at the top. The CA values for the three situations, given in Table 8, respectively are 1.218, 1.503, and 1.734. There is noticeable bulging at the tank bottom in the compression zone for all three cases. At the tank top there is a noticeable wave in the uncovered tank, which is absent in the tank with roof or stiffening ring.

## CONCLUSIONS

In this study several contributions are made towards the seismic analysis of liquid storage tanks. A meticulous validation for free vibration and buckling analysis is carried out, which serves to confirm previous work and establish the groundwork for conducting the current analysis. The present work indicates the importance of accounting for step-variations in tank wall thickness in carrying out buckling or collapse analysis. For tall tanks quantitative data is presented showing that collapse loads are strongly dependent on the boundary conditions at the top end of the tank, specifically that the presence of a roof or stiffening ring significantly increases the collapse load. The current report provides results for the frequencies of the tanks in empty state conditions, and for the collapse loads of tanks without imperfections. In a later report it is planned to provide results for frequencies of full tanks, and for the collapse loads of tanks with imperfections.

## REFERENCES

- 1- Wunderlich, W., and Seiler, C., "Nonlinear Treatment of Liquid-Filled Storage Tanks Under Earthquake Excitation by a Quasistatic Approach," *Comput. Struct.*, Vol. 78, 2000, pp. 385-395.
- 2- Nachtigall, I., Gebbeken, N., and Urrutia-Galicia, J. L., "On the Analysis of Vertical Circular Cylindrical Tanks Under Earthquake Excitation at its Base," *Eng. Struct.*, Vol. 25, 2003, pp. 201-213.
- 3- Virella, J. C., Godoy, L. A., and Suarez, L.E., "Influence of the Roof on the Natural Periods of Empty Steel Tanks," *Eng. Struct.*, Vol. 25, 2003, pp. 877-887.
- 4- Hagihara, S., and Miyazaki, N., "Bifurcation Buckling Analysis of Conical Roof Shell Subjected to Dynamic Internal Pressure by the Finite Element Method," *J. Press. Ves. Techn.*, Vol. 125, 2003, pp. 78-84.
- 5- Mirfakhraei, P., and Redekop, D., "Comparison of Elastic Buckling Loads for Liquid Storage Tanks," *Steel Comp. Struct.*, Vol. 2, 2002, pp. 161-170.
- 6- Redekop, D., Mirfakhraei, P., and Muhammad, T., "Nonlinear Analysis of Anchored Tanks Subjected to Equivalent Static Loading," *ASME PVP*, Vol. 442, 2002, pp. 157-163.
- 7- Malhotra, P. K., Wenk, T., and Wieland, M., "Simple Procedure for Seismic Analysis of Liquid-Storage Tanks," *Struct. Eng. Int.*, Vol. 3, 2000, pp. 97-201.
- 8- Hamdan, F. H., "Seismic Behavior of Cylindrical Steel Liquid Storage Tanks," *J. Construct. Steel Res.*, Vol. 53, 2000, pp. 307-333.
- 9- Flores, F.G., and Godoy, L. A., "Buckling of Short Tanks Due to Hurricanes," *Eng. Struct.*, Vol. 20, 1998, pp. 752-760.
- 10- Haroun, M. A., Izzeddine, W. A., and Mode, N., "Comparative Evaluation of Seismic Design Standards of Storage Tanks," *ASME PVP*, Vol. 272, 1994, pp. 51-57.
- 11- Veletsos, A. S., and Tang, Y., "Soil-Structure Interaction Effects for Laterally Excited Liquid Storage Tanks," *Earthq. Eng. Struct. Dyn.*, Vol. 19, 1990, pp. 473-496.
- 12- "API650 - Welded Steel Tanks for Oil Storage," American Petroleum Institute, Washington, 1998.
- 13- "ADINA User's Guide," ADINA R & D Inc, Worcester, 2003.
- 14- Bathe, K. J., "Finite Element Procedures," Englewood Cliffs, Prentice Hall, 1996.
- 15- Soedel, W., "Vibrations of Shells and Plates," 3rd Ed., New York, Marcel Dekker, 2004.