Mathematical Modeling of Pencil-Thin Nuclear Fuel Rods

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ABSTRACT

Inside the core of a typical pressurized water reactor are pencil-thin nuclear fuel rods, each about 12 feet long, which are grouped by the hundreds in bundles called fuel assemblies. Inside each fuel rod, pellets of uranium or, more commonly, uranium oxide are stacked end to end. Heat from uranium to surroundings is transformed though the heat conducting layer separating it from the fuel rod. The correct approach to mathematical modeling of the heat conductivity and stress analysis consists in considering a change of layer thickness in the process of the thin-walled nuclear fuel rods deformation.

The problem formulated as the unilateral contact of shell through the heat-conducting layer. The approach consists in considering a change of layer thickness in a process of the shell deformation. 3-D equations of thermoelasticity and heat coduction are expanded into a polynomial Legendre series in terms of the thickness. The equations of *N*-th approximations have been obtained and the first-approximations equations have been studied in detail.

KEY WORDS: fuel rod, heat-conducting, thermoelasticity, thermo-mechanical contact.

INTRODUCTION

Nuclear energy is energy released from the nuclear fuel. Nuclear energy provides transformation of the energy released from nuclear fission and nuclear fusion processes into another kind of energy, for example into heat and electricity [1, 2]. Nuclear fuel is any material that can be consumed to derive nuclear energy. The most common fissile nuclear fuels are enriched uranium or plutonium. For use as nuclear fuel enriched uranium processed into pellet form. The pellets are then fired in a high-temperature, sintering furnace to create hard, ceramic pellets of enriched uranium. The cylindrical pellets then undergo a grinding process to achieve a uniform pellet size. The pellets are stacked, according to each nuclear core's design specifications, into pencil-thin tubes of corrosion-resistant metal alloy. The tubes are sealed to contain the fuel pellets: these tubes are called fuel rods. The finished fuel rods are grouped in special fuel assemblies that are then used to build up the nuclear fuel core of a power reactor. The metal used for the tubes depends on the design of the reactor. It can be a stainless steel or a zirconium alloy. For the most common types of reactors the tubes are assembled into bundles with the tubes spaced precise distances apart. Thus the nuclear pencil-thin fuel rods are the main elements of nuclear reactor that generate thermal energy. Their typical construction is presented on Fig. 1.

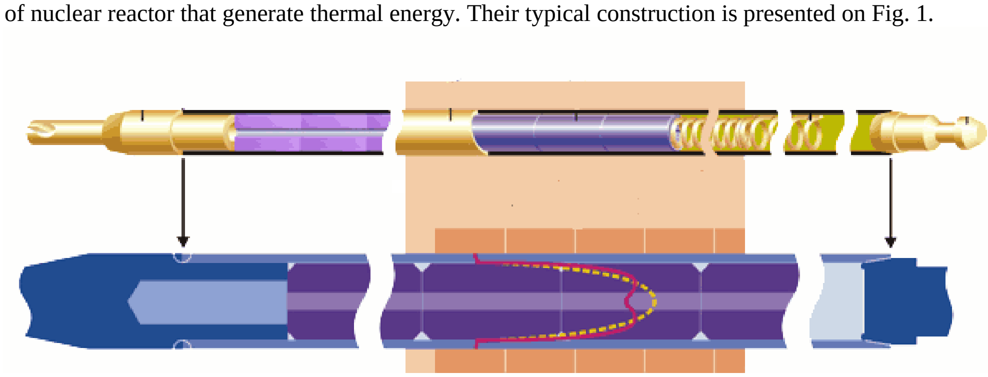

Fig.1. The nuclear fuel rod.

Heat from uranium to surroundings is transformed though the heat conducting layer separating it from the fuel rod. In our previous publications [3, 4] it was shown that for correct calculation of heat transfer from the uranium core it is necessary to take into account deformation of the pencil-thin fuel rod shell. In order to illustrate main idea let us consider very simple example of the heat transfer in the case homogeneous temperature distribution. In this case equations of the heat conductivity and pencil-thin fuel rod shell deformation have the form

$$\varepsilon_0^2\theta^0 - F_0 = 0\ ,\ \varepsilon_1^2\theta^1 - F_1 = 0\ ,\ 4\beta^4 w - \beta_0\theta^0 = 0\,. \qquad (1)$$

This system of equations is a special case of the Eqs. (24). The notations here agree with those in Eqs. (24). On the Fig 2. are presented temperature distribution and displacements calculated for different temperature of core surface 1 with and 2 without considering influence of the deformation of the pencil-thin fuel rod shell correspondently.

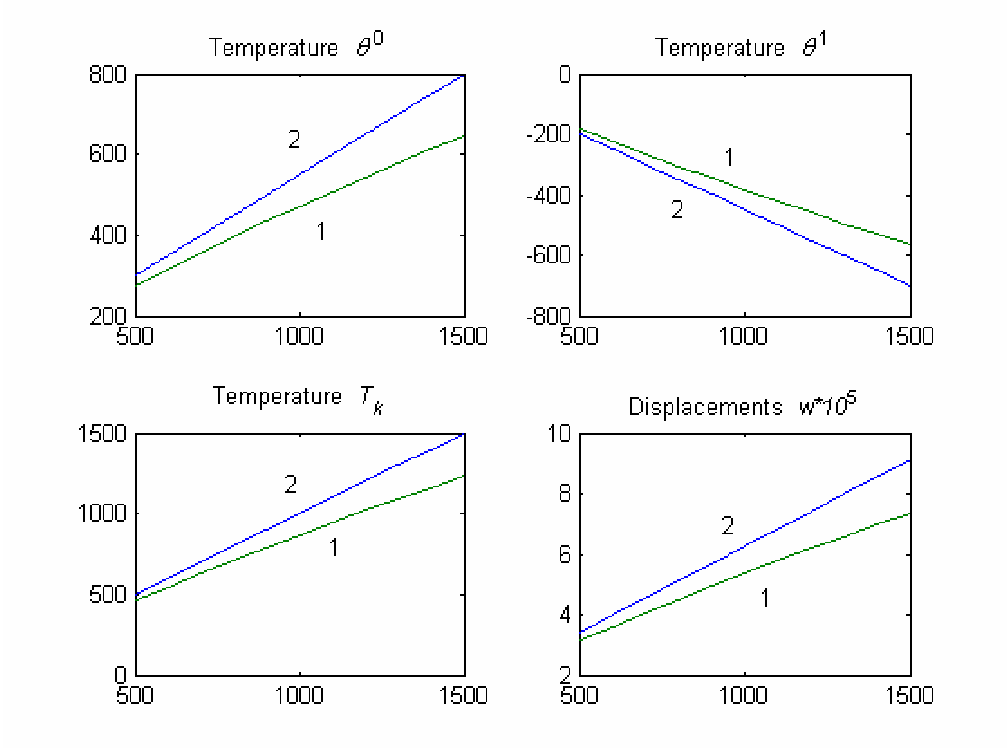

Fig.2. Temperature distribution and displacements.

Analysis of the presented data show that even in this simple case results obtained with and without considering of the deformation of the pencil-thin fuel rod shell differ in 20% and more. Further in this paper we will show that in case of inhomogeneous temperature distribution this difference is more significant. Therefore we have developed mathematical model of heat conductivity if bodies though thin heat conducting layer with considering influence of their deformation.

MATHEMATICAL MODEL

3-D formulation. We consider two elastic homogeneous isotropic bodies situated in an initial, undeformed state in a distance h_0 apart (Fig.3).

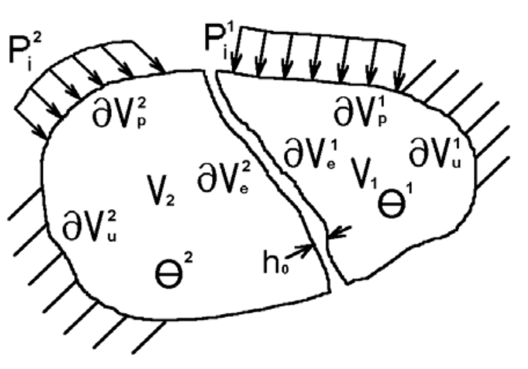

Fig.3. Two bodies in thermoelastic contact.

Here V^{α} is the volume occupied by ∂V^{α}. The body boundary may be presented in the forms $\partial V^{\alpha} = \partial V_p^{\alpha} \cup \partial V_u^{\alpha} \cup \partial V_e^{\alpha}$ and $\partial V^{\alpha} = \partial V_{\theta}^{\alpha} \cup \partial V_q^{\alpha} \cup \partial V_e^{\alpha}$. On parts ∂V_p^{α} and ∂V_u^{α} boundary conditions for displacements and traction are prescribed. On parts $\partial V_{\theta}^{\alpha}$ and ∂V_q^{α} boundary conditions for temperature and heat flux are prescribed. The ∂V_e^{α} is part of the boundary where contact of bodies take place. Different parts of the boundary do not intersect: $\partial V_p^{\alpha} \cap \partial V_u^{\alpha} \cap \partial V_e^{\alpha} = \varnothing$ and $\partial V_{\theta}^{\alpha} \cap \partial V_q^{\alpha} \cap \partial V_e^{\alpha} = \varnothing$. Here and henceforth we supply with index $\alpha = 1,2$ all parameters connected with the bodies.

We assume that h_0 is commensurable with the bodies displacements and we assume those displacements to be small. There is a heat-conducting medium in the gap between the bodies. The medium does not resist their deformation, and heat exchange between bodies is due to the thermal conductivity of the medium.

The thermodynamic state of the system, including the bodies and the heat-conducting medium, is defined by the following parameters: $\sigma_{ij}^{\alpha}(\mathbf{x})$, $\varepsilon_{ij}^{\alpha}(\mathbf{x})$ and $u_i^{\alpha}(\mathbf{x})$ are the components of the stress and strain tensors and displacement vector, and $\theta^{\alpha}(\mathbf{x})$, $\chi^{\alpha}(\mathbf{x})$, $\theta^{*}(\mathbf{x})$, $\chi^{*}(\mathbf{x})$ are the temperature and specific strength of the internal heat sources at the bodies and the medium respectively. In this case following relations take place [5]

$$\partial_j \sigma_{ij}^{(q)} + b_j^{(q)} = 0,\ \varepsilon_{ij}^{(q)} = \frac{1}{2}(\partial_i u_j^{(q)} + \partial_j u_i^{(q)}),\ \sigma_{ij}^{(q)} = c_{ijkl}^{(q)} \sigma_{kl}^{(q)} + \beta_{ij}^{(q)} \theta^{(q)} \tag{2}$$

where $\partial_i = \partial / \partial x_i$ are partial derivatives with respect to the space variables x_i, $c_{ijkl}^{(q)}$ and $\beta_{ij}^{(q)}$ are elastic modulus and the coefficients of linear thermal expansion. In the isotropic case

$$c_{ijkl}^{(q)} = \lambda^{(q)} \delta_{ij} \delta_{kl} + \mu^{(q)} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{kj}),\ \ \beta_{ij}^{(q)} = (\mu^{(q)} + 3\lambda^{(q)}) \alpha^{(q)} \delta_{ij}$$

where λ and μ are the Lame constants, $\alpha^{(q)}$ are the coefficients of linear thermal expansion. The summation convention is understood over the repeated indices here and henceforth.

The differential equations of equilibrium for the displacement vector components may be presented in the form

$$A_{ij}^{(q)} u_j^{(q)} + A_i^{(q)} \theta^{(q)} + b_i^{(q)} = 0\ , \tag{3}$$

with

$$A_{ij}^{(q)} = c_{ijkl}^{(q)} \partial_k \partial_l\ ,\ A_i^{(q)} = \beta_{ij}^{(q)} \partial_j\ , \text{ and } A_{ij}^{(q)} = \mu^2 \delta_{ij} \partial_k \partial_k + (\lambda^{(q)} + \mu^{(q)}) \partial_i \partial_j\ ,\ A_i^{(q)} = (\mu^{(q)} + 3\lambda^{(q)}) \alpha^{(q)} \partial_i$$

in anisotropic and isotropic case correspondently.

On the parts ∂V_p^{α} and ∂V_u^{α} boundary conditions for displacements and traction have the form

$$p_i^{\alpha} = \sigma_{ij}^{\alpha} n_j = \psi_i^{\alpha}, \forall \mathbf{x} \in \partial V_p^{\alpha}\ ,\ \ u_i^{\alpha} = \varphi_i^{\alpha}\ \forall \mathbf{x} \in \partial V_u^{\alpha}\ ,\ \forall\ \mathbf{x} \in V^{\alpha} \tag{4}$$

On the part ∂V_e^{α} mechanical boundary conditions have the form of inequalities [3, 4, 6]

$$\begin{gathered} \Delta u_n = u_n^1 - u_n^2 \geq h_0\ , q_n \geq 0\ , (\Delta u_n - h_0) q_n = 0 \\ |\mathbf{q}_{\tau}| < k_f q_n \to \partial_t \mathbf{u}_{\tau}^{\alpha} = 0\ ; |\mathbf{q}_{\tau}| = k_f q_n \to \partial_t \mathbf{u}_{\tau}^{\alpha} = -\lambda \mathbf{p}_{\tau}^{\alpha} \\ \mathbf{p}^1 = -\mathbf{p}^2 = \mathbf{q}\ , \mathbf{n} = \mathbf{n}_1 = -\mathbf{n}, \forall \mathbf{x} \in \partial V_e\ , \partial V_e = \partial V_e^1 \cap \partial V_e^2 \end{gathered} \tag{5}$$

where $q_n^{(q)}$, $\Delta u_n^{(q)} = u_n^{(q)} - u_n^{(q+1)}$, $\mathbf{q}_{\tau}^{(q)}$ and $\Delta \mathbf{u}_{\tau}^{(q)} = \mathbf{u}_{\tau}^{(q)} - \mathbf{u}_{\tau}^{(q+1)}$ are the normal and tangential components of the contact force vector and the displacement discontinuity vector respectively, $k_{\tau}^{(q)}$ and $\lambda_{\tau}^{(q)}$ are coefficients which depend upon the properties of the contact surfaces.

Linear equations for heat conductivity have the form

$$\lambda_{ij}^{\alpha} \partial_i \partial_j \theta^{\alpha} - \chi^{\alpha} = 0\ ,\ \forall\ \mathbf{x} \in V^{\alpha} \tag{6}$$

Here $\lambda_{ij}^{\alpha} = \delta_{ij} \lambda_T^{\alpha}$ are the coefficients of thermal conductivity.

On the parts ∂V_θ^α and ∂V_q^α boundary conditions for temperature and heat flux have the form

$$\theta^\alpha = \theta_b^\alpha \,, \forall \mathbf{x} \in \partial V_\theta \,, \mathbf{q}^\alpha = \mathbf{q}_b^\alpha \,, \forall \mathbf{x} \in \partial V_q \tag{7}$$

The temperature distribution within the heat-conducting medium is described by the equations of heat conductivity

$$\lambda_{ij}^* \partial_i \partial_j \theta^* - \chi^* = 0 \;, \; \forall \mathbf{x} \in V^* \tag{8}$$

Boundary conditions on the lateral sides of the heat-conducting medium will be considered in the form

$$\lambda_{ij} \partial_n \theta^\alpha + \beta_{ij} \left(\theta^\alpha - \theta_b^\alpha \right) = 0 \tag{9}$$

Conditions of heat conductivity through the heat-conducting medium have the form

$$\theta^* = \theta^\alpha \,, \lambda_{ij}^* \partial_n \theta^* = \lambda_{ij}^\alpha \partial_n \theta^\alpha \quad , \; \forall \mathbf{x} \in \partial V_e^\alpha \tag{10}$$

In the area of close mechanical contact the thermal conditions are transformed into the form

$$q_\theta = \alpha_e \left(\theta^\alpha - \theta_b^\alpha \right) \,, \; \forall \mathbf{x} \in \partial V_e^\alpha \tag{11}$$

where q_θ is the heat flux passing across the close mechanical contact area, α_e is the contacts thermal conductivity.

2-D formulation. Now we assume that bodies 1 and 2 have one size less than two another ones and may be considered as plates or shells of constant thickness $2h_1$ and $2h_2$ as it is shown in Fig. 4.

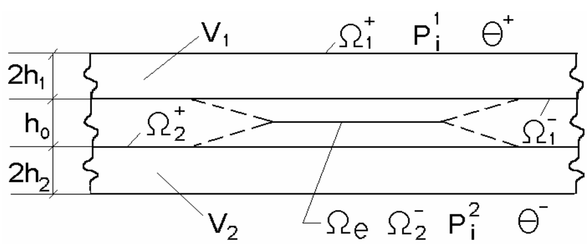

Fig. 4. Two thick bodies in thermo-elastic contact.

Let us expand the thermodynamic parameters, which describe the state of the system shown in the Fig.4 in Legendre polynomials series along the coordinate x_3 (see [7] for details). Then we will get the equations of the problem in terms of the coefficients of this expansion. As a result, we obtain a 2-D system of equations.

The Eqs. (3) of thermo-elasticity are transformed into its 2-D form

$$\begin{gathered}
\sum_{l=0}^{\infty} L_{11}^{kl} u_1^l + L_{12}^k u_2^k + \sum_{l=0}^{\infty} L_{13}^{kl} u_3^l + \frac{2k+1}{2h}\left[p_1^+ - (-1)^k p_1^- \right] - b_1^k = 0 \\
\sum_{l=0}^{\infty} L_{22}^{kl} u_2^l + L_{21}^k u_1^k + \sum_{l=0}^{\infty} L_{23}^{kl} u_3^l + \frac{2k+1}{2h}\left[p_2^+ - (-1)^k p_2^- \right] - b_2^k = 0 \\
\sum_{l=0}^{\infty} L_{31}^{kl} u_1^l + \sum_{l=0}^{\infty} L_{32}^{kl} u_2^l + \sum_{l=0}^{\infty} u_3^l + \frac{2k+1}{2h}\left[p_3^+ - (-1)^k p_3^- \right] + b_3^k = 0
\end{gathered} \tag{12}$$

The Eqs. (6) of heat-conductivity are transformed into its 2-D form

$$\Delta_0 \theta^k + \frac{2k+1}{2h}\left[Q_3^+ - (-1)^k Q_3^- \right] - \frac{2k+1}{h}\left[Q_3^{k-1} + Q_3^{k-3} \ldots \right] + (k_1 + k_2) Q_3^k + \frac{\chi_k}{\lambda_0} = 0 \tag{13}$$

The equations of heat-conductivity for the layer are transformed into 2-D form

$$\Delta_0\theta_*^k+\Delta_* hT_2^k+\Delta_* h_* T_1^k+\left(\nabla_* h\cdot\nabla T_2^k\right)+\left(\nabla_* h_*\cdot\nabla T_1^k\right)-\frac{2k+1}{2}\left[\theta_*^+\Delta_* h^+ +(-1)^k\theta_*^-\Delta_* h^-\right]+\left(\nabla_* h\cdot\mathbf{Q}_2^k\right)+\left(\nabla_* h\cdot\mathbf{Q}_1^k\right)$$
$$+\frac{2k+1}{2h}\left[Q_3^+-(-1)^k Q_3^-\right]-\frac{2k+1}{2h}\left(Q_3^{k-1}+Q_3^{k-3}+\ldots\right)+(k_1+k_2)Q_3^k+\frac{\chi^k}{\lambda_*}=0 \qquad (14)$$

where

$$\Delta_0=\frac{1}{A_1A_2}\left(\partial_1\frac{A_2}{A_1}\partial_1+\partial_2\frac{A_1}{A_2}\partial_2\right)\ ,\ \Delta_*=\frac{A_2}{A_1}\partial_1\frac{1}{h}\partial_1+\frac{A_1}{A_2}\partial_2\frac{1}{h}\partial_2\ ,\ \nabla_*=\frac{1}{h}\left(\frac{A_2}{A_1}\partial_1+\frac{A_1}{A_2}\partial_2\right),$$
$$\nabla=\partial_1+\partial_2\ ,\ \mathbf{Q}_1^k=\left(\underline{Q}_1^k,\underline{Q}_2^k\right),\mathbf{Q}_2^k=\left(\underline{Q}_{-1}^k,\underline{Q}_{-2}^k\right)$$

The equations thermo-elasticity and heat conductivity are written in coordinates connected with the principal curvatures of the shell surfaces. Here A_α are coefficients of the first quadratic form, and k_α are principal curvatures. Analytical expressions for differential operators L_{ij}^{kl} and for $\mathbf{Q}_\alpha^k$ may be found in [3, 4].

The boundary conditions Eqs. (4) and (7) are transformed into 2-D form

$$p_i^k=\psi_i^k\ ,\forall\,\mathbf{x}\in\partial\Omega_p;\ u_i^k=\varphi_i^k\ ,\forall\,\mathbf{x}\in\partial\Omega_u\ ,$$
$$\theta^k=\theta_b^k\ ,\forall\,\mathbf{x}\in\partial\Omega_\theta;\ \mathbf{q}^k=\mathbf{q}_b^k\ ,\forall\,\mathbf{x}\in\partial\Omega_q \qquad (15)$$
$$\sum_{k=0}^{\infty}p_i^k=p_i^+\ ,\forall\,\mathbf{x}\in\Omega^+;\ \sum_{k=0}^{\infty}(-1)^k p_i^k=p_i^-\ ,\forall\,\mathbf{x}\in\Omega^-$$

and the contact conditions are transformed into 2-D form considering the Eqs. (5)

$$\sum_{k=0}^{\infty}(-1)^k u_{n(1)}^k=u_n^+;\ \sum_{k=0}^{\infty}u_{n(2)}^k=u_n^-;\ q_i=\sum_{k=0}^{\infty}(-1)^k p_{i(1)}^k=-\sum_{k=0}^{\infty}(-1)^k p_{i(2)}^k \qquad (16)$$
$$\sum_{k=0}^{\infty}(-1)^k\theta_{(1)}^k=\sum_{k=0}^{\infty}\theta_*^k;\ \lambda\sum_{k=0}^{\infty}(-1)^k Q_{2(1)}^k=\lambda_*\sum_{k=0}^{\infty}Q_{3(*)}^k;\ \sum_{k=0}^{\infty}\theta_{(2)}^k=\sum_{k=0}^{\infty}(-1)^k\theta_*^k;\ \lambda\sum_{k=0}^{\infty}Q_{3(2)}^k=\lambda_*\sum_{k=0}^{\infty}(-1)^k Q_{3(*)}^k\ .$$

First approximation equations. In the first approximation, the shell theory considers only the first two terms of the Legendre polynomials series [3, 4, 7]. In this case the thermodynamic parameters, which describe the state of the system can be presented in the form

$$\sigma_{ij}(\mathbf{x})=\sigma_{ij}^0(\mathbf{x}_\nu)P_0(\omega)+\sigma_{ij}^1(\mathbf{x}_\nu)P_1(\omega)\ ,\ u_i(\mathbf{x})=u_i^0(\mathbf{x}_\nu)+u_i^1(\mathbf{x}_\nu)\frac{x_3}{h} \qquad (17)$$
$$\varepsilon_{ij}(\mathbf{x})=\varepsilon_{ij}^0(\mathbf{x}_\nu)P_0(\omega)+\varepsilon_{ij}^1(\mathbf{x}_\nu)P_1(\omega)\ ,\ \theta(\mathbf{x})=\theta^0(\mathbf{x}_\nu)+\theta^1(\mathbf{x}_\nu)\frac{x_3}{h}$$

Then the Eqs. (12) of the 2-D thermo-elasticity have the form

$$L_{ij}^{00}u_j^0+L_{ij}^{01}u_j^1+L_i^0\left(\theta^0-\theta_0^0\right)+b_i^0=0$$
$$L_{ij}^{10}u_j^0+L_{ij}^{11}u_j^1+L_i^1\left(\theta^1-\theta_0^1\right)+b_i^1=0 \qquad (18)$$

and the Eqs. (13) of the 2-D heat-conductivity have the form

$$\Delta_0\theta^0+\frac{1}{2h}\left(Q_3^+-Q_3^-\right)+(k_1+k_2)Q_3^0+\frac{\chi^0}{\lambda_0}=0 \qquad (19)$$
$$\Delta_0\theta^1+\frac{3}{2h}\left(Q_3^+-Q_3^-\right)+(k_1+k_2)Q_3^1+\frac{\chi^1}{\lambda_0}=0$$

We will consider only one term in the Legendre polynomials series for θ_*. Then the Eq. (13) of the 2-D heat conductivity for the layer have the form

$$\Delta_0\theta_*^0+\Delta h\theta_*^0+\left(\nabla^* h\cdot\nabla\theta_*^0\right)-\frac{1}{2}\left(\theta_*^+\Delta_* h^+ +\theta_*^-\Delta_* h_*\right)+\left(\nabla^* h\cdot\mathbf{Q}_2^0\right)+\frac{1}{2h}(Q^{*+}_3-Q^{*-}_3)+\left(k_1+k_2\right)Q^{*0}_3+\frac{\chi^0}{\lambda_*}=0 \quad (20)$$

The unknown parameters in the Eqs (19) have the form

$$Q_3^+-Q_3^-=\frac{3}{4h}\left(\theta^++T_k\right)+\frac{3\theta^0}{2h}\ ,\ Q_3^0=\frac{1}{2h}\left(\theta^+-T_k\right) \quad (21)$$
$$Q_3^+-Q_3^-=\frac{3}{2h}\left(\theta^+-T_k\right)-\frac{5\theta^1}{2h}\ ,\ Q_3^1=\frac{3}{2h}\left(\theta^++T_k\right)-\frac{3\theta^1}{2h}$$

where

$$T_k^1=\frac{\left[9\lambda_0^2\left(h_0-u_3^1-u_3^2\right)+\lambda_0\lambda_* h_2\right]\left(3\theta^++6\theta_1^0-10\theta_1^1\right)+\lambda_0\lambda_* h_1\left(3\theta^-+6\theta_2^0-10\theta_2^1\right)}{9\left[9\lambda_0^2\left(h_0-u_3^1-u_3^2\right)+\lambda_0\lambda_*\left(h_1+h_2\right)\right]} \quad (22)$$
$$T_k^2=\frac{\left[9\lambda_0^2\left(h_0-u_3^1-u_3^2\right)+\lambda_0\lambda_* h_1\right]\left(3\theta^-+6\theta_2^0-10\theta_2^1\right)+\lambda_0\lambda_* h_2\left(3\theta^++6\theta_1^0-10\theta_1^1\right)}{9\left[9\lambda_0^2\left(h_0-u_3^1-u_3^2\right)+\lambda_0\lambda_*\left(h_1+h_2\right)\right]}$$

in the case of two shells in thermo-mechanical contact, and

$$T_k=\frac{\lambda_0\left(h_0-u_3\right)\left(3\theta^++6\theta^0-10\theta^1\right)+\lambda_* h\theta^-}{9\lambda_0\left(h_0-u_3\right)+\lambda_* h} \quad (23)$$

in the case of the shell contact with foundation.

NUMERICAL EXAMPLE

We consider heat transfer from the rigid body though heat conducting layer and axisymmetrical cylindrical shell with possibility for unilateral contact, as it is shown in the Fig 5. In contrast to Eqs. 1 this model can be applied to inhomogeneous temperature distribution in the pencil-thin fuel rods.

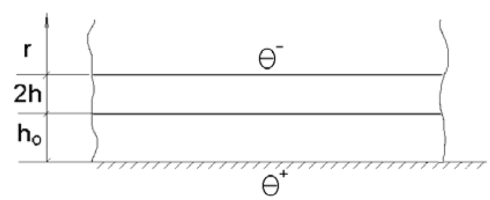

Fig.5. Heat transfer from the rigid body though heat conducting layer.

Differential equations of thermo-elasticity and heat-conductivity for the classic Kirchhoff-Love's theory for the axisymmetrical cylindrical shell have the form

$$\frac{d^4w}{dx^4}+4\beta^4 w-\beta_0\theta^0-\beta_1\frac{d^2\theta^1}{dx^2}=\frac{1}{D}\left(p-q\right), \quad (24)$$
$$\frac{d^2\theta^0}{dx^2}-\varepsilon_0^2\theta^0+F_0=0\ ,\ \frac{d^2\theta^1}{dx^2}-\varepsilon_1^2\theta^1+F_1=0\ ,$$

where

$$\varepsilon_0=\frac{3}{h^2}\ ,\ \varepsilon_1=\frac{15}{h^2}\ ,\ \beta^4=\frac{3\left(1-\nu^2\right)}{4h^2r^2}\ ,\ \beta_0=\frac{3\left(1-\nu\right)\alpha_\tau}{h^2r}\ ,\ \beta_1=\frac{\left(1+\nu\right)\alpha_\tau}{h}\ ,\ D=\frac{2Eh^3}{3\left(1-\nu^2\right)}$$

and

$$F_0=0.5\varepsilon_0\left(\theta^-+T_k\right)+\frac{1}{2hr}\left(T_k-\theta^-\right)\ ,\ F_1=0.5\varepsilon_1\left(T_k-\theta^-\right)+\frac{3}{2hr}\left(T_k+\theta^-\right)-\frac{3}{hr}\theta^1$$

In [8-10] it was shown that the Eqs. (24) can be transformed into the integral equations of Hammerstein's type

$$\int_l G_\alpha(x,y)F_\alpha(y)dy = \theta^\alpha \ , \ \int_l W(x,y)\left\{\frac{1}{D}\left[p(y)-q(y)\right]-\beta_0 F_3(y)\right\}dy = w \qquad (25)$$

The kernels in these integral equations are fundamental solutions for corresponding differential operators of the form

$$G_i(x,y) = \exp\left(-\varepsilon_i|x-y|\right)/2\varepsilon_i \ , \ i = 0,1 \ , \ W(x,y) = \frac{1}{8\beta^3 D}\exp\left(-\beta|x-y|\right)\left[\cos\left(\beta|x-y|\right)+\sin\left(\beta|x-y|\right)\right] \qquad (26)$$

and

$$F_3 = \beta_1\left(F_1 + \varepsilon_1^2\theta^1\right) - \beta_0\theta^0$$

Stresses in the axisymmetrical cylindrical shell are calculated by formulas

$$\sigma_x = \frac{E}{1-\nu^2}\left[\frac{d^2w}{dx^2}z-(1+\nu)\alpha_t t_1\frac{z}{2h}\right] \ , \ \sigma_\theta = \frac{Ew}{r}-\alpha_t t_0 E + \frac{E}{1-\nu^2}\left[\nu\frac{d^2w}{dx^2}z-(1+\nu)\alpha_t t_1\frac{z}{2h}\right] \qquad (27)$$

Second derivative of the $w(x)$ may be represented in the form

$$\frac{d^2w(x)}{dx^2} = \beta_0\int_0^l\theta_0(\xi)\frac{d^2G(\xi,x)}{dx^2}d\xi + 4\beta^4\beta_1\int_0^l G(\xi,x)\theta_1(\xi)d\xi - \beta_1\theta_1(x) + \frac{1}{D}\int_0^l\left[p(\xi)-q(\xi)\right]\frac{d^2G(\xi,x)}{dx^2}d\xi \qquad (28)$$

Substituting expression for the second derivative of the shell displacements into Eqs. (27), finally we have the following equations for the stresses calculation

$$\sigma_x(x) = b_0\int_0^l T_0(\xi)\frac{d^2G(\xi,x)}{dx^2}d\xi + b_1\int_0^l G(\xi,x)T_1(\xi)d\xi - b_2T_1(x) + b_3\int_0^l\left[p(\xi)-q(\xi)\right]\frac{d^2G(\xi,x)}{dx^2}d\xi \qquad (28)$$

$$\sigma_\theta(x) = \nu\sigma_x(x) + \frac{Ew(x)}{r} - \alpha_t E\left[T_0(x)+T_1(x)z\right]$$

where

$$b_0 = \frac{3E\alpha_t zr^5}{(1+\nu)h^2h_0} \ , \ b_1 = \frac{3(1+\nu)E\alpha_\tau r^6 z}{4h^3h^2{}_0} \quad b_2 = \frac{E\alpha_\tau r^4 z}{(1-\nu)hh_0} \ , \ b_3 = \frac{3r^6 z}{2h^3h_0}$$

$$\beta_0 = \frac{3\alpha_t Erz}{h^2} \ , \ \beta_1 = \frac{3(1-\nu)\alpha_t Er^2 z}{h^2} \ , \ \beta_3 = \frac{r^2 z}{8h^3} .$$

Mizes stresses are calculated by the equations

$$\sigma_i^- = \sqrt{(\sigma_x^-)^2 + (\sigma_x^-)(\sigma_0^-) + (\sigma_0^-)^2} \ , \ \sigma_i^+ = 1\Big/\sqrt{2}\ \sqrt{(\sigma_x^+ - q)^2 + (\sigma_x^+ - \sigma_x^+)^2 + (\sigma_0^+ + q)^2} \qquad (29)$$

Algorithm for the problem solution consists in iterative process of the integral equations of Hammerstein's type solution and in the case if unilateral contact take place additional iterative algorithm is used. Algorithm was elaborated in [3, 4]. In the problems under consideration algorithm is convergent and convergence enough fast.

Calculation have been done for the data: temperature: $\theta_0^+ = 100^o C$, $\theta^* = 700^o C$, $\theta^- = 0^o C$, geometrical parameters $r = 0.01\,m$, h=0.0014m , $h_0 = 0.0007m$, material properties: $E = 100$ GPa $\nu = 0.25$ $\alpha_\tau = 2.5\cdot 10^{-5}\, 1/{}^oC$, $\lambda_1 = 20V/m^oC$, $\lambda_* = 10V/m^oC$. At the area of length $l = 0.5\,m$ there is perturbation of the temperature field, where temperature has the form $\theta^+ = \theta_0^+ + \theta^* \sin \pi x / l$.

Results of the calculations are presented on Fig. 6, where curves 1 and 2 have been obtained with and without considering influence of the deformation of the pencil-thin fuel rod shell correspondently. On the upper graphs are presented temperature distribution along area of temperature perturbation. Form these graphs follows that in some points temperature calculated with and without considering influence of the deformation of the shell differ twice. On the lower graphs are presented distribution of the shell displacements and the Mizes stresses. Comparison of the curves for shell displacements show that in the case of considering influence of the shell deformations on the process of heat transfer occur area of close mechanical contact between shell and rigid body. Contrary in the case of neglecting influence of the shell deformations on the process of heat transfer there is no mechanical contact occurs. The thermomechanical processes in these two cases differ not only quantitative but also quantitatively. As the result the temperature differ twice and Mizes stresses even more.

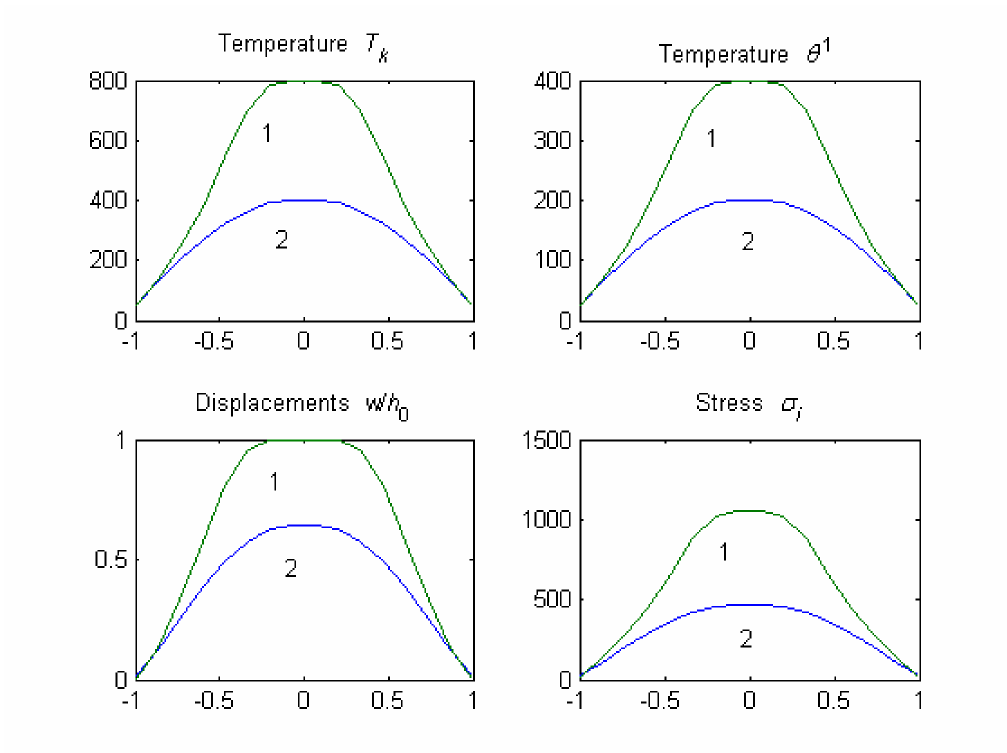

Fig.6. Contact of a cylindrical shell with foundation

CONCLUSIONS

The results presented here confirm the significance of taking into account deformation of shell during heat transform though thin layer and possibility for the mechanical contact interaction. Hence, additional extensive investigations are necessary to ensure safety of nuclear reactor and specifically the pencil-thin fuel rod.

In [9-11] presented here approach has been applied in composite material science for mathematical modeling of debonding between laminas in laminated shell subjected intensive temperature field.

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