Fatigue Design of Welded Joints through Inspection Planning

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ABSTRACT

A Two Phase Model (TPM) for the fatigue design was developed and calibrated. The number of cycles to crack initiation is modeled by a local strain approach using the Manson-Coffin equation, whereas the propagation phase is modeled by fracture mechanics adopting the simple version of the Paris law. The model is capable of taking into account the effect of the global geometry of the joint, the local weld toe geometry, applied stress ratio and the residual stress condition. Our study is carried out for non-load carrying fillet-welded joints made of C-Mn steel with nominal yield stress 345 MPa (50 ksi). The notch effect of the weld was characterized directly by the stress concentration factor K_t. The *S-N* curves constructed from the two-phase model will gradually change slope from 3 in the upper part (stress range higher than 150 MPa) towards 10 at stress ranges lower than 60 MPa.

At accelerated laboratory conditions (stress range of 150 MPa) the initiation phase was determined to be 30% of the entire fatigue life. When the stresses are lowered towards service stresses the initiation phase will dominate the entire fatigue life. At a stress range of 60 MPa the initiation phase is more than 90% of the entire fatigue life.

In the present paper, emphasis is put on the practical consequences of the predictions made by the TPM with respect to allowable service stresses and inspection strategies.

The first practical consequence of the present two-phase model is that it predicts longer lives at low stress ranges. With the application of the constructed *S-N* curve in the lower stress region it is possible to reduce dimensions by 30 - 40% and still achieve the same fatigue life as for the F-class *S-N* curve in the BS 7910 rules.

The second practical consequence is that in-service inspection strategy may be changed. This is owed to the fact that the crack path leading to final fracture is quite different from the path calculated by a pure fracture mechanics model. The two-phase model with its long initiation phase will give a more hidden path for the crack evolution. Hence, an inspection program with increased inspection frequency at the end of service life can be proven to be favorable.

The two practical consequences listed above will become even more pronounced for a butt joint and under variable amplitude loading with a stress distribution situated near lower stress ranges.

Keywords: Welded steel joints, Fatigue behavior, Crack initiation, Crack propagation, Life predictions, Inspection planning.

INTRODUCTION AND OBJECTIVES

Concerning the fatigue behavior of fillet-welded steel joints where shallow elliptical cracks emanate from the weld toe, the understanding and application of fracture mechanics has developed vigorously in the last decades. Nevertheless the condition under which a crack will initiate in welded joints remains somewhat elusive. This is due to a highly variable local toe geometry and metallurgical complexity at the initiation locus. As a consequence, almost all guidance given in rules and regulations is based on the simple *S-N* curve approach and applied fracture mechanics.

However, one may then argue that a Two Phase Model (TPM), including twice crack initiation and crack propagation phases is more consistent with the experimental evidence. On the other hand, it is interesting to check if the results derived from a TPM indeed provide information that will have practical consequences regarding the dimensions and inspection planning of a joint, as compared to a pure Fracture Mechanics Model (FMM). If not, one may argue that the FMM although lacking a firm footing for the early crack growth, is good enough for all practical purposes.

With this background a TPM is investigated and compared with the FMM. The crack initiation phase is based on the Manson-Coffin equation, whereas the propagation phase is based on a simple integration of Paris crack propagation law, as presented in part 2 of reference [1]. Lawrence et al. first suggested the TPM and a good overview is given in references [2] and [3]. As the method now stands its accuracy depends greatly on calibration experiments. Our objective in the present work is to study the consequence of the use of a TPM on the planning inspection procedure.

THE TWO PHASE MODEL

The Two Phase Model (TPM) for the fatigue process in welded joints is developed and calibrated in part 2 of reference [1]. The model is calibrated to fit the crack initiation data for a test series subjected axial loading with a constant stress range of 150 MPa, and the fatigue life data for similar test series subjected to stress ranges below 105 MPa. To make the model fit all test data it is crucial to select a sufficiently low transition depth between the initiation phase and the propagation phase. A transition depth of 0.1 mm was determined. Furthermore, the material parameters in the Manson-Coffin equation were determined directly from the early cracking in the weld toe for full-scale fillet welds and not from test carried out with small-scale smooth specimens. This is essential if the model is to account for the actual surface condition at the weld toe. The model is capable of taking into account the effect of the global geometry of the joint, the local weld toe geometry, applied stress ratio and the residual stress condition.

Modeling the fatigue process

It is assumed that the total fatigue life consists of two major phases:

$$N=N_i + N_p \tag{1}$$

The number of cycles to crack initiation N_i is modeled by a local strain approach using the Manson-Coffin equation, whereas the propagation phase is modeled by fracture mechanics adopting the simple version of the Paris law. The model is calibrated to fit the fatigue behaviour of fillet welded joints where cracks emanate from the weld toe. The model is fitted to both experimental crack growth histories at high stress ranges and to fatigue lives at any stress level. Emphasis is laid on how to determine the variable weld toe notch factor, the transition crack depth between the two phases and the material parameters. The model is valid for high quality joints where a thorough post fabrication inspection is carried out.

The predictions for the number of cycles to crack initiation, N_i, are based on the following Manson-Coffin equation with Morrow's mean stress correction [2], [3] :

$$\frac{\Delta\varepsilon}{2} = \frac{\left(\sigma'_f - \sigma_m\right)}{E}\left(2\,N_i\right)^b + \varepsilon'_f\left(2\,N_i\right)^c \tag{2}$$

Here $\Delta\varepsilon$ is the local strain range and σ_m is the local mean stress at the weld toe. The parameters b and c are the fatigue strength and ductility exponents, and σ'_f and ε'_f are the fatigue strength and ductility coefficients respectively.

The propagation phase is modeled by linear elastic fracture mechanics adopting the simple version of the Paris law. The number of cycles to crack propagation, N_p, is given as follows:

$$N_p = \frac{1}{C}\int_{a_0}^{a_c}\frac{da}{\left(\Delta S\,\sqrt{\pi\,a}\,F(a)\right)^m} \tag{3}$$

This type of modeling has matured into the application stage and guidance on the choice of model and parameters is given in BS7910, [4]. Hence, description of this model is not emphasized in the present article.

CONSTRUCTING THE *S-N* CURVE FROM THE TWO-PHASE MODEL

One of our main goals is to establish *S-N* curves from the TPM that are consistent with the *S-N* curves in the rules and regulations. However, our model has a theoretical basis and is capable of predicting the influence of, for instance local weld toe geometry, stress ratio and stress relieving. Thus, high quality joints have long fatigue lives, whereas poor quality will be penalized. Let us begin by demonstrating that the model can predict the F-class *S-N* curve of BS 7910 under appropriate assumptions of the quality. When using the initiation model established in part 2 of reference [1], we get the fatigue lives in Table 1 at various stress levels. As can be seen from the table the time to crack initiation at test stress range of 150 MPa (21.7 ksi) is 30% of the entire fatigue life, whereas it is 88% of the fatigue life at a stress range of 80 MPa (11.5 ksi).

At stress ranges below 100 MPa (14.5 ksi), the model predicts overly optimistic results compared to the F-class *S-N* curve. As can be seen from Table 1, the total fatigue life is close to 5.5 times longer than prediction made by the F-class at 80 MPa (11.5 ksi). When comparing the figures we must bear in mind that the figures derived from the TPM model correspond to the test series in database 1, i.e. Stress Relieved (SR) and with an applied stress ratio of $R = 0.3$. This stress relieving has a strong bearing on the time to crack initiation through the Morrow mean stress effect at long lives. The vast majority of tests used to determine the F-class curve are in As Welded (AW) conditions and often tested at a stress ratio close to $R = 0.1$. Our next step is to simulate these conditions for the initiation part of our model by setting the residual stress equal to the material yield stress, i.e. 400 MPa (58 ksi). The results are shown in Table 2. As can be seen, the discrepancy between the TPM predictions and F-class predictions has been reduced, but the TPM still predicts a fatigue life of 2.5 times longer than the F-class at 80 MPa (11.5 ksi).

Table 1 – Results derived from the TPM at various stress ranges. Stress Relieved (SR), *R* = 0.3.

Stress range (MPa)	N_i (cycles)	N_p (cycles)	N_t (Cycles)	N_i/N_t %	N_t F-class
150	1.4×10^5	3.3×10^5	4.7×10^5	30	5.1×10^5
120	5.6×10^5	6.5×10^5	1.2×10^6	47	1.0×10^6
100	2.1×10^6	1.1×10^6	3.2×10^6	66	1.7×10^6
80	1.6×10^7	2.2×10^6	1.8×10^7	88	3.3×10^6
60	3.7×10^8	5.1×10^6	2.9×10^8	99	8.0×10^6

Table 2 – Results derived from the TPM at various stress ranges. As Welded (AW), *R* = 0.1

Stress range (MPa)	N_i (cycles)	N_p (cycles)	N_t (Cycles)	N_i/N_t %	N_t F-class
150	1.3×10^5	3.3×10^5	4.6×10^5	28	5.1×10^5
120	4.3×10^5	6.4×10^5	1.1×10^6	40	1.0×10^6
100	1.3×10^6	1.1×10^6	2.4×10^6	54	1.7×10^6
80	6.6×10^6	2.2×10^6	8.8×10^6	75	3.4×10^6
60	8.1×10^7	5.1×10^6	8.5×10^7	94	8.0×10^6

To investigate this further, the F-class *S-N* curve is drawn in Fig. 1 along with the constructed *S-N* curves and test results in the stress region where the discrepancy is greatest, i.e. below 100 MPa (14.5 ksi). As can be seen, the basic difference between the two types of curves is that the F-class is bi-linear, whereas the TPM curves are continuously changing slope. The TPM curve is close to the F-class curve at high stress levels, and hardly any (less than 10%) discrepancy is found above a stress range level of 120 MPa (17.4 ksi). When the stresses are lowered to under 100 MPa there is a noticeable discrepancy in the way that the TPM predicts 2-9 times longer lives when we are above the endurance limit of 56 MPa (8.1 ksi). To pursue this discrepancy further we have plotted the test results pertaining to database 2 in the actual stress region. These tests are carried out on non-load carrying fillet welded joints with thickness in the range of 16 to 38 mm. They are all in As Welded condition and with positive stress ratio, i.e. there may be large residual stresses present in the specimens. As can be seen, the TPM curve for the As Welded condition is close to the mean of the experimental data between stress ranges of 60 and 100 MPa. Hence, it seems that our model is capable of correctly taking into account the effect of residual stresses and loading ratio. Furthermore, it is our judgment that the F-class curve is too conservative in the stress region under discussion. This is due to the fact that it is chosen as a straight line and based on test results that have the center of gravity for the stress ranges between 120 MPa (17.4 ksi) and 150 MPa (21.7 ksi). Hence, the curve fails to take into account the increasing fatigue life due to the importance of an initiation phase below 100 MPa. The experimental results plotted in this stress region corroborate the predictions made by the TPM.

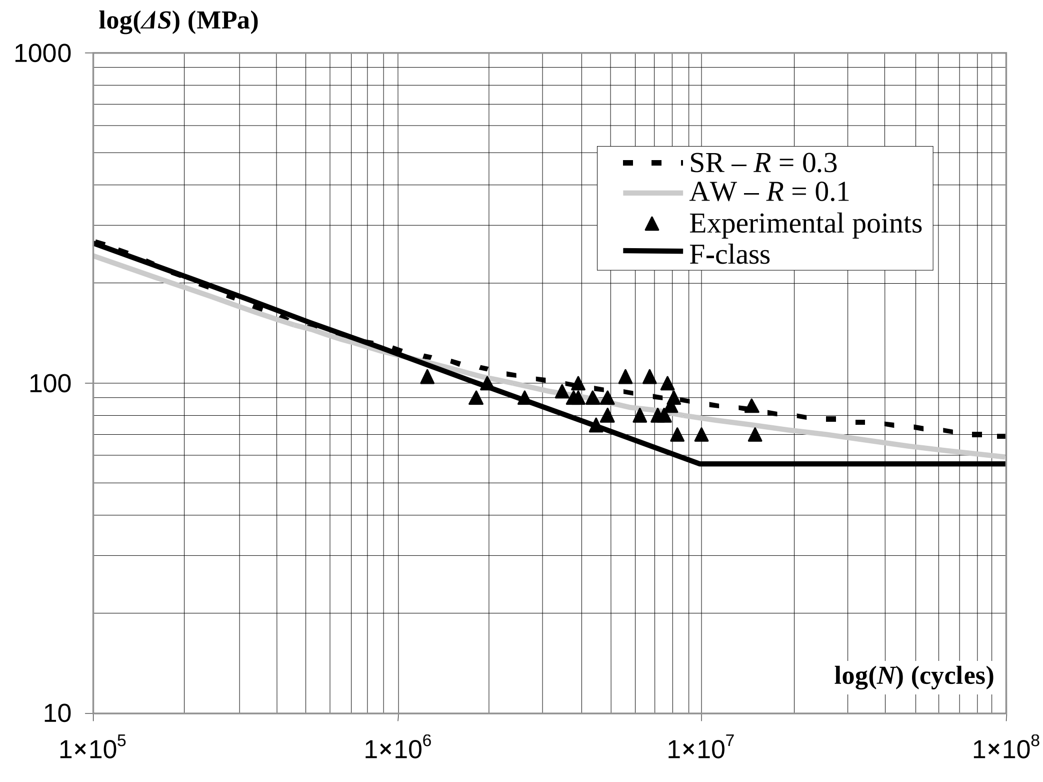


Fig. 1 – Constructed S-N curves together with the F-class S-N curve and test data.

THE PRACTICAL CONSEQUENCES OF THE TPM

General considerations

We have constructed an *S-N* curve which is nonlinear for a log-log scale and that predicts substantially longer lives at stress ranges below 100 MPa (14.5 ksi) than does the F-class linear curve. Furthermore, at these long fatigue lives (close to 5×10^6 cycles) the initiation life is at least 70% of the entire fatigue life. We will show by a simple example what are the consequences of these results with respect to:

- Predicting fatigue life for selecting dimensions for a joint
- Predicting crack growth path for inspection planning

We will compare our results with the results obtained by the F-class curve and a pure fracture mechanics model. The latter approach is traditionally used for decision making regarding inspection planning. We shall use our FMM (Fracture Mechanics Model) from reference [1], but for simplicity we decrease the growth parameter C by 10%, (cf; eq. 3). This small adjustment will make predictions made by F-class and by the FMM coincide.

Life predictions and dimensions

Our goal is to compare results obtained by:

- F-class and the FMM (which are equal when the endurance limit is neglected)
- The present TPM assuming As Welded conditions, R = 0.1

For simplicity we assume constant amplitude loading and choose a stress range in the region considered, e.g. ΔS = 80 MPa (11.5 ksi). By using the model we get the results in Table 3 for an as welded condition.

Table 3 – Life (cycles) predictions made by the TPM and the F-class.

Stress range	TPM			F-class
(MPa)	N_i	N_p	N_t	$N_t = N_p$
80	6.6×10^6	2.2×10^6	8.8×10^6	3.4×10^6
58				8.8×10^6

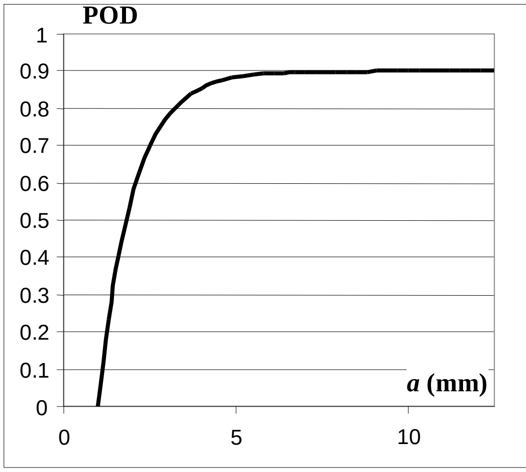
Compared with the figures in table 2 it is seen that the initiation period is reduced due to the fact that the joint is assumed to be in an As Welded condition. Hence, the residual stresses are accounted for. Details are found in reference [1] and [5]. As can be seen at 80 MPa (11.5 ksi) the TPM predicts 2.5 times longer fatigue life than the F-class, and the initiation part is close to 70% of the entire life. If dimensions are chosen according to the F-class the dimensions must be increased by 38% to give the same predicted fatigue life, i.e. 8.8×10^6 cycles. This will correspond to an allowable stress range of 58 MPa (8.4 ksi). These assessments will also be valid for a design curve if the lives predicted by the TPM have the same scatter as the F-class.

Predicted crack evolution and inspection planning

Let us compare the two alternatives above with respect to inspection planning. The first alternative is the TPM based design with an allowable stress of 80 MPa (11.5 ksi), the second alternative is the F-class design (the same as the FMM) with allowable stress range of 58 MPa (8.4 ksi). Our task is now to compare the crack evolutions before and up to final fracture based on the TPM and the FMM in the two cases. We shall more precisely consider the effect of a scheduled inspection program for the two growth histories. The purpose of such a program is of course to detect cracks so that they can be repaired before reaching the final critical crack size. We introduce the concept of a Probability of Detection (POD) curve to characterize the performance of the inspection technique. The POD is a function of joint type, environment and crack size and is established based on blind tests with inspectors. The POD curve for Magnetic Particle Inspection under poor conditions reads [6] :

$$POD(a) = 0.9\,[1 - e^{-(a-1)}] \qquad a > 1\,\text{mm} \tag{4}$$

The curve is shown to the left in Fig. 2. The derived crack histories for the FMM and the TPM are shown to the right. As can be seen the curve derived from the TPM has a more hidden path that makes the crack more difficult to detect at an early stage.



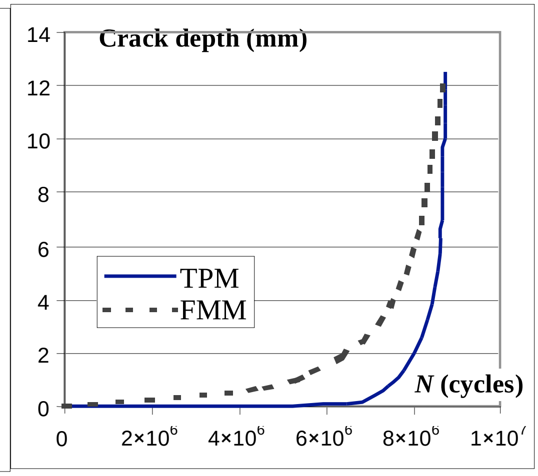


Fig. 2 – Left: POD curve Magnetic Particle Inspection; Right: Predicted crack evolution for FMM, ΔS = 58 MPa and TPM, ΔS = 80 MPa.

The reliability calculations for an inspection program are carried out using a simple quasi-stochastic approach often used in the aircraft industry. We call the method quasi-stochastic because it does not take into account the randomness of the crack evolution, but applies the mean curves as given to the right in Fig. 2. A more fully stochastic analysis is found in reference [6]. If the strategy is to implement k inspections during the planned service life, the event that all of them fail can be estimated by:

$$P_F = \prod_{i=1}^{i=k} [1 - POD(a_i)] \tag{5}$$

The expression is based on the assumption that each inspection is independent. The reliability of the inspection program is $R = 1 - P_F$.

If we are planning k = 7 inspections at a constant time interval corresponding to 5×10^5 cycles, we will for the two curves in Fig. 2 get the results given in Table 4. As can be seen, there is an important difference in achieved reliability for the given inspection program for the two different crack evolutions. The FMM predicts a reliability of 0.999, whereas the TPM predicts only 0.935. When comparing the probability of failure the difference becomes more striking, the probability of failure pertaining to the FMM is normally in the acceptable range, whereas the one pertaining to the TPM is not.

Table 4 – Reliability calculations for a given inspection program.

Model for prediction	First effective inspection	Last effective inspection	Number of effective inspections	Reliability	Probability of failure
FMM	5.5×10^6	8.5×10^6	7	0.9999	1.0×10^{-4}
TPM	8.0×10^6	8.5×10^6	2	0.935	6.5×10^{-2}

If the predictions made by the TPM are accepted as true, one would obtain acceptable reliability if the inspection efforts were concentrated in the last part of the service life with a decreased inspection interval of $2{\times}10^5$ cycles. For a more fully stochastic approach one can introduce the probability of a preexisting crack in the TPM. The difference in reliability will then be less, but the revealed tendency will be the same.

Other configurations, loading conditions and environment

The study presented in this paper has been limited to fillet-welded joint subjected to constant amplitude loading. Future work should carry out an investigation on other joint configurations such as butt joints and variable amplitude loading. As for a butt joint, which is the most common configuration for high load transfer, we already know that the initiation phase will play an even more important role than has been shown for the fillet-welded joint in the present work. This is due to the significantly lower stress concentration factor at the weld toe. When it comes to variable amplitude loading, the importance and consequences of an initiation phase are not obvious and an investigation is necessary before drawing conclusions. For load spectra with the center of gravity in the low stress range area (e.g. exponential distributed stress ranges) the effects revealed for constant amplitude loading will probably prevail. It is also interesting to determine the role the initiation part plays for welded joints in severe environments such as seawater in the case of offshore structures in which inspection planning is today planned entirely on applied fracture mechanics.

CONCLUSIONS

Our study is carried out for non-load carrying fillet-welded joints made of C-Mn steel with nominal yield stress 345 MPa (50 ksi). A two-phase model was used to predict the fatigue life. The initiation life was modeled by Manson-Coffin equation and the propagation phase by the simple version of Paris law. The model was validated and calibrated with the use of a large database.

The transition depth between the two phases was determined to be 0.1 mm or even smaller. The main criterion for selecting such a shallow depth was that the model should be able to construct *S-N* curves consistent with experimental data at low stress ranges.

The material parameters in the Manson-Coffin equation were determined directly from the measured time to reach 0.1 mm crack depth for joints subjected to a stress range of 150 MPa (21.7 ksi). The advantage is that the parameters then will reflect the actual surface condition at the weld toe. This will not be the case when the parameters are derived from tests with smooth specimens

At accelerated laboratory conditions (stress range of 150 MPa) the initiation phase was determined to be 30% of the entire fatigue life. When the stresses are lowered towards service stresses the initiation phase will dominate the entire fatigue life. At a stress range of 60 MPa the initiation phase is more than 90% of the entire fatigue life.

The *S-N* curves constructed from the two-phase model will gradually change slope from 3 in the upper part (stress range higher than 150 MPa) towards 10 at stress ranges lower than 60 MPa (8.7 ksi). The endurance limit will be introduced gradually as the slope changes. This non-linear shape of an *S-N* curve for a log-log scale is probably more correct than the bi-linear shape found in rules and regulation. We claim so because our model has more theoretical footing than the rather arbitrary assumption of bi-linearity.

The F-class *S-N* curve and the constructed *S-N* curve will for all practical purposes give the same results for stress ranges above 100 MPa (14.5 ksi). It is particularly between 100 to 60 MPa that the most interesting discrepancy between the two curves is found. Towards a stress range of 60 MPa the constructed model will predicted up to 9 times longer life than the bi-linear rule based curve.

The first practical consequences of the present two-phase model is that it predicts longer lives at low stress ranges. With the application of the constructed *S-N* curve in the lower stress region it is possible to reduce dimensions by 30 - 40% and still achieve the same fatigue life as for the F-class *S-N* curve.

The second practical consequence is that in-service inspection strategy may be changed. This is owed to the fact that the crack path leading to final fracture is quite different from the path calculated by a pure fracture mechanics model.

The two-phase model with its long initiation phase will give a more hidden path for the crack evolution. Hence, an inspection program with increased inspection frequency at the end of service life can be proven to be favorable.

The two practical consequences listed above will become even more pronounced for a butt joint and under variable amplitude loading with a stress distribution with a center of gravity towards lower stress ranges.

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