# The Influence of Weld Strength Mis-Match on Crack Opening Areas

Richard Charles[1], Alex Warren[1], John Sharples[1] and Peter Budden[2]

[1]Serco Assurance, Walton House, Birchwood Park, Warrington, Cheshire, WA3 6GA, UK.
[2]British Energy Generation Ltd., Location 94, Barnett Way, Barnwood, Gloucester, GL4 3RS, UK.

## ABSTRACT

Leak-before-break (LbB) arguments can be used to demonstrate the safety of a structure containing defects when fracture mechanics analyses fail to show adequate margins against the critical crack size. A successful LbB argument will demonstrate that leakage of fluid through a crack in the wall of a pressure component can be detected prior to the crack reaching instability, at which time a sudden catastrophic failure could occur. The calculation of crack opening areas (COA) is a fundamental part of producing the leak detection leg of an LbB argument.

This paper presents results from finite element analyses of a through thickness, straight fronted crack in a finite width plate, subjected to primary membrane loading, containing various weld mis-match conditions. The aim of this work is to observe the significance of strength mis-match on the COA, and ultimately provide a simplified approach for the treatment of cracks in a structure containing weld strength mis-match in LbB assessments. In this study, the crack has been positioned on the boundary between the two mis-matched materials. The results show that:

(i) for the geometry, and the crack size analysed, an elastic analysis produces a more conservative, lower COA result compared to that of an elastic-plastic analysis.
(ii) when comparing the COA for the homogeneous cases of the higher yield stress materials with the detailed mis-match cases, a conservative result is obtained in both instances. The homogeneous case of the lower yield stress material however, gives a non-conservative result.

A relationship between the elastic and the elastic-plastic COA has been derived, whereby an elastic-plastic value of COA can be calculated, based upon an elastic solution, i.e. there is no requirement to carry out a detailed elastic-plastic finite element analysis.

## INTRODUCTION

Leak-before-break (LbB) arguments can be used to demonstrate the safety of a structure containing defects when fracture mechanics analyses fail to show adequate margins against the critical crack size. A successful LbB argument will demonstrate that leakage of fluid through a crack in the wall of a pressure component can be detected prior to the crack reaching instability, at which time a sudden catastrophic failure could occur. The calculation of the crack opening area (COA) is a fundamental part of producing the LbB argument.

Advice on the calculation of COAs for postulated through-thickness cracks in pressure systems is given in Section III.11 of R6 [Reference 1]. This calculation depends primarily on how the crack is idealised, the crack opening mode and the material properties used. R6 includes a comprehensive list of references for a wide range of published solutions for idealised cracks in simple geometries subjected to standard basic loads such as pressure, membrane, global bending and through-wall bending. In recent years, these solutions have been extended by undertaking finite element studies on diverging cracks whereby the length of the defect on one side of the vessel or pipe wall is different to that on the other side [Reference 2].

The studies of more complex defect situations have now been extended to consider cracks present in structures, which contain strength mis-match conditions. This paper presents results from finite element analyses of a through thickness, straight fronted crack in a finite width plate, subjected to primary membrane loading and, containing various weld mis-match conditions. The aim of this work is to observe the significance of strength mis-match on the COA, and ultimately provide a simplified approach for the treatment of cracks in a structure containing weld strength mis-match in LbB assessments.

In this study, the crack has been positioned on the boundary between the two mis-matched materials.

## STRENGTH MIS-MATCH

The plastic zone at the tip of a crack increases in size as the applied load increases. Eventually the plasticity will consume the entire section of the component, and the load carrying capacity will be lost. The limit load is defined as the load at which the structure no longer has any load carrying capacity assuming perfectly plastic material behaviour.

Strength mis-match affects the evolution of the plastic zone at the crack tip under increasing load, which is due to the presence of another material nearby, either weaker or stronger. Hence, strength mis-match may affect the load carrying capacity of a structure and thus the limit load.

Section III.8 of the R6 procedure [Reference 1] provides guidance on allowance for strength mis-match effects. An 'equivalent' stress-strain curve is defined, in terms of yield strength and limit load ratios, as a weighted average of the curves of the constituent materials of a weldment.

## FINITE ELEMENT ANALYSIS

### Model Overview

A flat plate of length 600mm, width 200mm and 25mm in thickness is modelled using a 2D plane strain finite element model. Only one half of the plate has been modelled due to symmetry along the length of the plate. The model represents a flat plate. The plate model is divided into two equal sections, representing a joint between two differing materials. A through thickness crack, 20mm in length, is located in the centre of the plate and is through-thickness. The crack is located on the material interface perpendicular to the line of symmetry.

The model is constructed using 4-noded, bilinear plane strain elements, ABAQUS [Reference 3 ] element type CPE4. The model contained 201,422 nodes and 100,382 elements and is shown in Figure 1. The area of the model in the vicinity of the bimaterial joint and the crack is modelled using an array of elements of size equal to 0.5mm. This provided the level of refinement required for the subsequent calculation of COA. The remainder of the plate is modelled more coarsely.

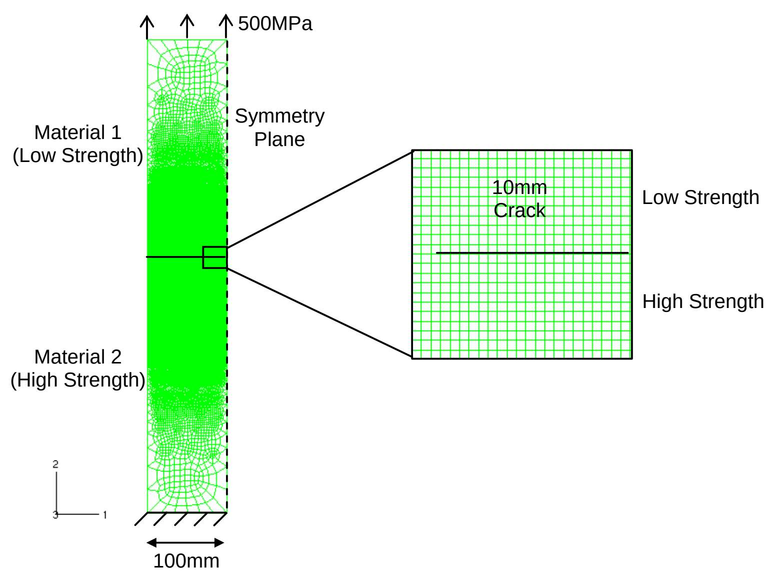


Figure 1. Finite element mesh of a through thickness crack in a flat plate.

### Material Properties

Stress-strain data were generated using the Ramberg-Osgood equation and the material representations used for all analysis cases are shown in Figure 2. The curves cover a range of 0.2% proof stress values, $\sigma_y$, that represent the different materials in the weldment. Note that the curve for a 0.2% proof stress of 272MPa is the equivalent material constructed according to R6 Section III.8.

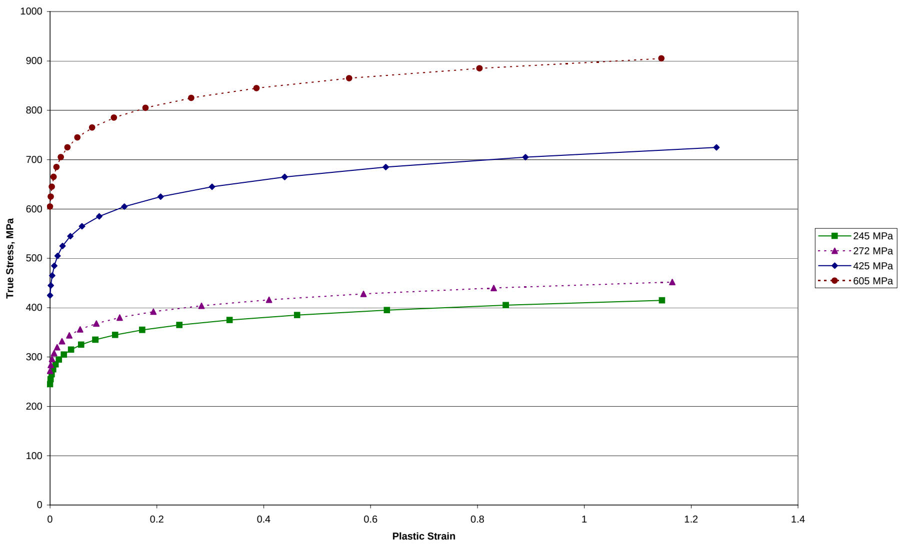


Figure 2. Stress-plastic strain data generated for $\sigma_y$ values of 245, 272, 425 and 605 MPa.

**Boundary Conditions and Loading**

A symmetry boundary condition is applied to the symmetry plane of the model as shown in Figure 1. The bottom of the plate was fixed in the y direction (see Figure 1).

A primary membrane loading is applied as a uniform pressure (maximum of 500MPa), to the top surface of the plate in the longitudinal (2) direction such that it tended to open the crack..

**Analysis Cases**

As previously noted, the objective of this study is ultimately to propose a validated methodology whereby the effects of material strength mis-match may be evaluated in LbB calculations.

In this initial stage of the work a series of seven finite element analyses have been carried out in order to observe the effect of material strength mis-match on the COA of a through-thickness crack in a flat plate. The analyses were as follows:

- a cracked body analysis for a homogeneous flat plate, using elastic stress-strain properties (assuming the elastic properties are the same for both low and high yield stress materials)

- three cracked body analyses for a homogeneous flat plate, using elastic-plastic stress-strain properties (Figure 2), for materials:
  - $\sigma_y$ = 245MPa
  - $\sigma_y$ = 425MPa
  - $\sigma_y$ = 605MPa

- two cracked body analyses of the flat plate with two materials, i.e. including a bimaterial interface, using elastic-plastic stress-strain properties (one material being designated as "b" and the other as "w" and $\sigma_{yb}$ always being the lowest of the two values)
  - $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 425MPa
  - $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 605MPa

- one cracked body analysis for a homogeneous flat plate using elastic-plastic stress-strain properties, for an equivalent yield stress, $\sigma_{ye}$, of 272MPa, based upon values of $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 425MPa and $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 605MPa

**RESULTS AND DISCUSSION**

Figure 3 gives the results for COA versus load for all cases, and Table 1 gives the results at a specific load of 75MN, along with the load case identifier in Column 2. From the results shown in Figure 3, it can be seen that the COA for the elastic-plastic cases are always greater than that of the elastic case. The result from the elastic case are therefore conservative with respect to all of the elastic-plastic cases for a leak detection case. Figure 3 also plots the analytical solution for an infinitely wide elastic plate. This is very close to the finite element elastic results.

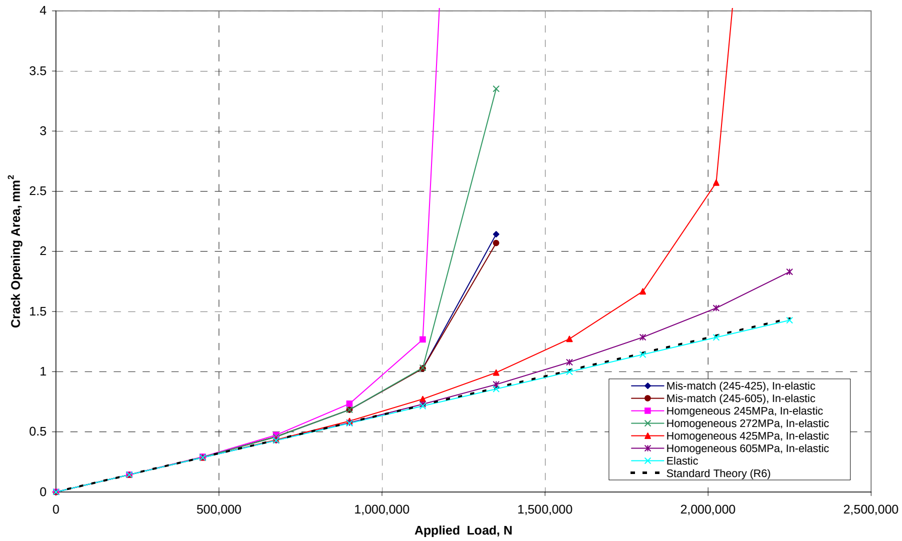


Figure 3. Calculated crack opening area versus applied load.

Figure 4 shows the COA results plotted against the R6 $L_r$ parameter, the ratio of load to limit load (e.g. the limit load used for the equivalent material is 1.2MN). The elastic-plastic COA results for the detailed mis-match cases deviate from the elastic results of COA at an $L_r$ value of approximately 0.5.

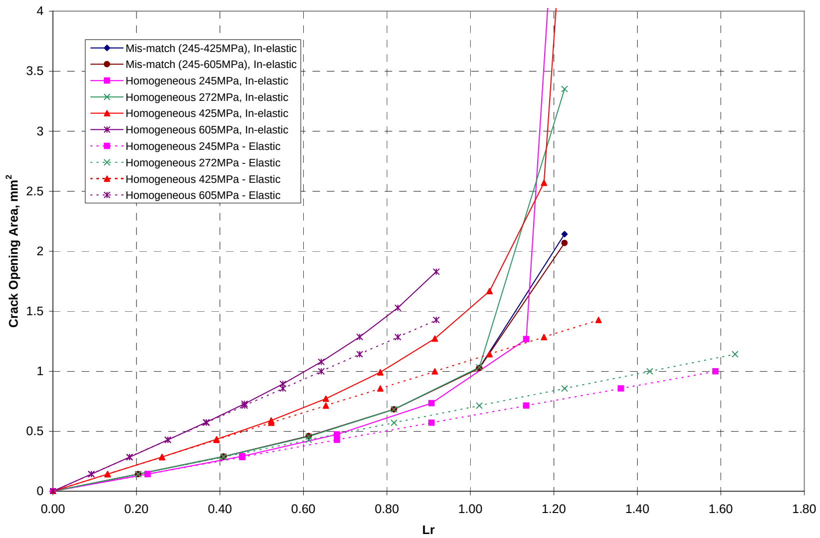


Figure 4. Calculated crack opening area versus $L_r$ (the different elastic lines displayed correspond to elastic values of $L_r$ which have been calculated for each of the specific $\sigma_y$ values used in the analyses).

The 272MPa elastic-plastic equivalent yield stress analysis gives the same values of COA as those from the detailed mis-match cases, up to $L_r$ values of approximately 1, that is at a load equal to the limit load. Such behaviour would be expected of the elastic-plastic crack driving force.

Table 1 compares the values of COA, at an applied load of 75MN from the homogeneous 245MPa case, the homogeneous 425MPa case and the detailed mis-match case of $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 425MPa (as detailed in Table 1) it is evident that the homogeneous 425MPa case gives a lower value than the detailed mis-match case, and the homogeneous 245MPa case gives a higher value than the detailed mismatch case. It is worth noting here that the centre crack opening displacement (COD) for the Material 2 side of the detailed mis-match case gives a lower value than the homogeneous 425MPa case, even though the resulting total COD and the COA are higher.

| Analysis Case | Load Case Identifier | Centre Crack Opening, mm (Material 1) | Centre Crack Opening, mm (Material 2) | Total Centre Crack Opening, mm | Crack Opening Area, $mm^2$ |
|---|---|---|---|---|---|
| **Elastic** | **(e)** | 0.0138 | 0.0138 | 0.0276 | 0.4286 |
| **Mis-Match (245 – 425)*** | **(mm1)** | 0.0164 | 0.0122 | 0.0286 | 0.4598 |
| **Mis-Match (245 – 605)*** | **(mm2)** | 0.0165 | 0.0121 | 0.0286 | 0.4597 |
| **Homogeneous 245MPa*** | **(h245)** | 0.0146 | 0.0145 | 0.0291 | 0.4732 |
| **Homogeneous 272MPa*** | **(h272)** | 0.0143 | 0.0143 | 0.0286 | 0.4569 |
| **Homogeneous 425MPa*** | **(h425)** | 0.0138 | 0.0138 | 0.0276 | 0.4334 |
| **Homogeneous 605MPa*** | **(h605)** | 00138 | 0.0138 | 0.0276 | 0.4290 |

Table 1. Comparison of crack opening and crack opening at an applied load of 75MN.

The significance of these findings is that the 425MPa homogeneous COA solution is conservative for assessing the detailed mis-matched case, but the 245MPa homogeneous solution will give a non-conservative assessment.

This is also clearly visible in Figure 5, in that the crack opening profile of the Material 1 side of the detailed mis-match case (mm1) is greater than that of the homogeneous Material 1 (h245) case, and the Material 2 detailed mis-match case (mm1) is less than that of the homogeneous Material 2 (h425) case. However when the crack opening area is observed they both look very similar in shape and size.

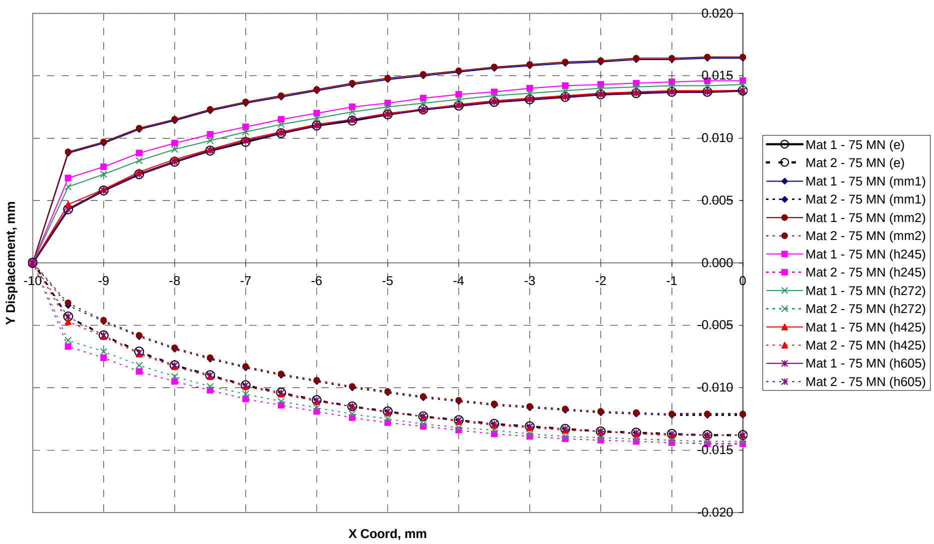


Figure 5. Comparison of crack opening profile results at an applied load of 75MN ('Mat 1' refers to profile in upper, low strength material and 'Mat 2' to profile in lower, high strength material).

The same pattern can be observed for mis-match case of $\sigma_{yb}$ = 245MPa, $\sigma_{yw}$ = 605MPa.

Again, this is clearly visible in Figure 5, in that the crack profile area of the Material 1 side of the detailed mis-match case (mm2) is greater than that of the homogeneous Material 1 (h245) case, and the Material 2 detailed mis-match case (mm2) is less than that of the Material 2 (h605) case. However when the COAs are examined they both look very similar in shape and size. Regarding the behaviour of the equivalent mis-match case, (homogenous 272MPa), the COA again produces a conservative result when compared to both detailed mis-match cases.

The failure assessment curve in R6 is effectively a crack driving force curve turned on its head, $K_{elastic}/K_{elastic\text{-}plastic}$. A similar normalized COA is then defined as $COA_{elastic}/COA_{elastic\text{-}plastic}$. This 'effective $K_r$' is plotted as a function of $L_r$ as in the conventional R6 diagram. In Figure 6, this 'effective $K_r$' value has been plotted with corresponding values of $L_r$, the measure of proximity to plastic collapse, for the two detailed mis-match cases and the 245MPa, 425MPa and 605MPa homogeneous cases in order to compare their behaviour. Figure 6 also conatins the R6 Option 1 failure assessement curve, an explicit function of $L_r$. It is encouraging to observe that all of the results compare closely to a single curve as shown in Figure 7. From this behaviour it is evident that an elastic-plastic value of COA could be obtained, for a mis-match case of a particular crack geometry and loading, by means of obtaining an elastic value of COA and then using an assessment diagram of the form shown in Figure 7. This is of course is only applicable to the material mis-match, geometry and loading cases investigated to date. If such a method can be shown to be universally applicable, this could lead the way for elastic-plastic values of crack opening area to be obtained for the same mis-match case of a particular crack geometry and loading, without carrying out any detailed finite element analysis. It is noted that the use of analytical expressions for the ratio of elastic-plastic to elastic centre crack opening displacement and hence, with an assumption on the crack shape, the COA has also been suggested by Kim and co-workers [e.g., Reference 4]

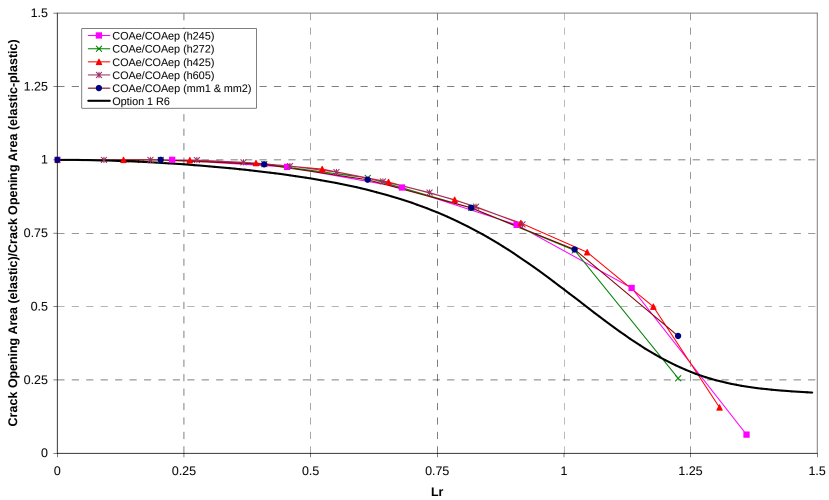


Figure 6. ‘Kr’ ($COA_{(elastic)}/COA_{(elastic-plastic)}$) versus Lr.

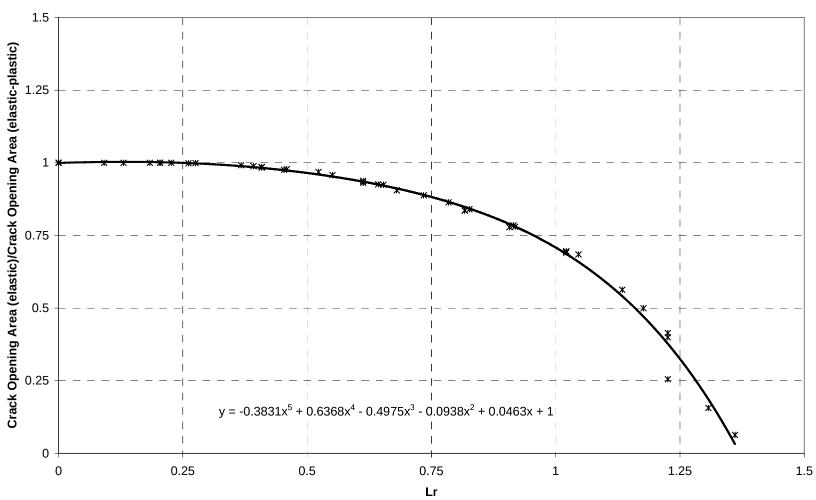


Figure 7. Curve fit to finite element data shown in Figure 6.

**CONCLUSIONS**

This paper has considered the effect of weld mis-match on a through wall crack situated along the boundary of two differing materials in a flat plate and, subjected to primary membrane loading.

Homogeneous analyses have also been carried out for comparison. The following conclusions have been obtained.

(i) For the geometry and crack size analysed, an elastic analysis produces a more conservative, lower COA compared to that of an elastic-plastic analysis.

(ii) When comparing the COA from the homogeneous cases of the higher yield stress materials with the detailed mis-match cases, a conservative result is obtained in both instances. However, the homogeneous case of the lower yield stress material gives a non-conservative, higher result.

(iii) A relationship between the elastic and the elastic-plastic COA has been obtained, whereby an elastic-plastic value of COA can be calculated, based upon an elastic solution, i.e. there is no requirement to carry out a detailed elastic-plastic finite element analysis.

## ACKNOWLEDGEMENTS

This paper is published by permission of Serco Assurance and British Energy Generation Ltd.

## REFERENCES

1 BEGL Procedure R6 Rev. 4, April 2001 with Revisions to May 2005. Assessment of the Integrity of Structures Containing Defects.

2 Yellowlees, S. F., Lynch, K. M., Warren, A., Sharples, J. K. and Budden, P. J., "Development of Guidance for Leak-Before-Break Analysis of Complex Shaped Cracks," ASME PVP Conference, 2005.

3 ABAQUS Version 6.4 Linear and Non-linear FE Analysis Package, ABAQUS Inc., U.S.A., 2003.