Characterisation of Co-Planar Surface-Breaking Flaws under Cleavage Failure Conditions

Mark Wilkes¹, David Beardsmore¹, Chris Watson², Andrew Sherry³ and Martin Goldthorpe³

¹Serco Assurance, Birchwood Park, Warrington, Cheshire, WA3 6GA, UK
²Rolls-Royce plc, PO Box 2000, Derby, DE21 7XX, UK
³University of Manchester, Sackville Street, Manchester, M60 1QD, UK

ABSTRACT

In order to assess the effect of multiple interacting flaws using existing methodologies, it is necessary to characterise the flaws into a single, larger flaw that can then be assessed. Recent experimental studies of components with interacting surface-breaking flaws in bending have shown that while the characterisation rules are conservative for ductile tearing, they may be non-conservative in the case of cleavage fracture. It was postulated that the increase in cleavage failure probability for closely-spaced flaws was associated with elevated levels of crack driving force in the region where the defects are closest, and may be particularly prevalent if the flaws coalesce locally (the region of coalescence is termed the re-entrant feature).

To investigate this further, finite element analyses have been undertaken of a postulated test component containing two surface-breaking semi-elliptical flaws under tensile and bending loads. The flaws have been assumed to be identical, coplanar, and either in contact or closely-spaced. To assess the conservatism of the characterisation procedure, the corresponding characterised flaws have also been modelled, with the characterisation performed using the formulation in the R6 procedures. Details and results of the analytical work are presented in the paper, together with a brief overview of some related experimental work.

The cases studied to date consist of pairs of identical surface-breaking semi-elliptical flaws with aspect ratios (= crack semi-length/crack depth) of 1.0, 0.8, 0.44 and 0.2. In all cases the flaw depth is 10mm in a specimen of 25mm thickness. For each aspect ratio, flaw separation is varied from 0 to 10mm in increments of 2mm. For each of these twin flaw configurations, the corresponding characterised flaw has been modelled. This consists of a single surface-breaking semi-elliptical flaw. The flaw depth is equal to the depth of the twin flaws and length is equal to the combined length of the twin flaws plus their separation. Use has been made of planes of symmetry to reduce model size. The cases have been repeated for tensile and bending loading up to a load magnitude of one-half of the local plastic limit load in each case. The material is 50D steel and the test temperature is -196°C.

The failure probability has been estimated using three methods. The first method is based on integration of the stress intensity factor along the crack front to obtain the cleavage failure probability based on the master curve methodology. This can additionally be combined with information on crack tip constraint, estimated via the T-stress, to obtain a modified estimate of the cleavage failure probability (the second method). The third method uses the Beremin methodology based on calculation of Weibull stress in the region ahead of the crack tip to obtain the cleavage probability directly. In each case, the failure probabilities due to the actual flaw are compared with those for the corresponding characterised flaw in order to determine the level of conservatism or non-conservatism. In addition, the results for contacting flaws with aspect ratio 0.8 are compared with limited data from an experimental study of the same nominal geometry under tensile loading.

The results indicate that the characterisation procedure is conservative for flaws which are not in contact, but that non-conservatism is indicated for contacting flaws. The level of non-conservatism is greater for bending loading than for tensile loading. The detailed representation of the re-entrant feature also increases the level of non-conservatism in some cases. Reasonable agreement with the experimental results is obtained, validating the cleavage calculation methods.

AIM

In order to assess the effect of multiple interacting flaws using existing methodologies [1,2], it is necessary to characterise the flaws into a single, larger flaw that can then be assessed. Figure 1 shows a schematic of the characterisation process.
The characterisation process is conservative if:

Failure Probability for C > Failure Probability for A, or:
Cleavage Load for C < Cleavage Load for A

Recent experimental studies of components with interacting surface-breaking flaws in bending [3, 4] have shown that while the characterisation rules are conservative for ductile tearing, they may be non-conservative in the case of cleavage fracture. It was postulated that the increase in cleavage failure probability for closely-spaced flaws was associated with elevated levels of crack driving force in the region where the defects are closest, and may be particularly prevalent if the flaws coalesce locally (the region of flaw coalescence is here termed the re-entrant feature as in previous work).

To investigate this further, finite element (FE) analyses have been undertaken of a postulated test component with two surface-breaking semi-elliptical flaws under tensile and bending loads [5, 6]. The flaws have been assumed to be identical, coplanar, and either in contact or closely-spaced. The flaw geometries included are summarised below. To assess the conservatism of the characterisation procedure, the corresponding characterised flaws have also been modelled, with the characterisation performed using the formulation shown in Figure 1. In this formulation, the characterised flaw is of the same form as the twin flaws, with depth equal to the twin flaw depth and length equal to the sum of the twin flaw lengths, plus the flaw separation. Figure 2 shows the arrangement of the flaws.
In the FE analyses, use has been made of planes of symmetry, as shown in Figure 2, in order to reduce model size. Some example FE models are shown in Figure 3.

**Figure 3 : Example FE Models**

**SCOPE**

The cases studied to date consist of identical coplanar twin surface-breaking semi-elliptical flaws of aspect ratios 1.0, 0.8, 0.44 and 0.2. Here the aspect ratio is defined as the ratio of the flaw depth (a in Figure 1) and the flaw semi-length (c in Figure 1). In all of the cases studied here a is 10mm. The flaw separation is varied from 0 to 10mm. For each configuration, the corresponding characterised flaw has been modelled. The analyses have been undertaken for tensile and bending loading up to a load magnitude of one-half of the local plastic limit load in each case. The material is 50D steel and the test temperature is -196°C, at this temperature cleavage fracture will be expected to dominate behaviour. The material yield stress is taken as 640MPa and a power-law approximation of stress-strain behaviour has been used.

The cleavage failure probability has been estimated using three methods. The first method (the uncorrected master curve method) is based on integration of the stress intensity factor along the crack front to obtain the cleavage probability via a method based on weakest link statistics. The second method (the constraint-corrected master curve method), is based on the first, but here the information on stress intensity factor is combined with information on crack tip constraint, estimated via the T-stress, to obtain a constraint-corrected cleavage failure probability. The third (Beremin) method uses the Beremin cleavage assessment methodology which is based on a calculation of Weibull stress in the region ahead of the crack tip to obtain the cleavage probability directly. The three methods are described in more detail below.

**Uncorrected Master Curve Method**

To undertake a calculation of cleavage failure probability by this method, it is first necessary to obtain values of $K_I$ along the flaw front from the FE model. We can then calculate the failure probability by weakest link statistics as the product of the survivor functions for all elemental lengths along the flaw front, integrated and scaled for length:

$$P_F = 1 - \exp \left\{ -\frac{1}{B_0} \sum_{i=1}^n \left( \frac{K_{I(i)} - K_{min}}{K_0 - K_{min}} \right)^4 \Delta l_i \right\}$$

(1)

Here $P_F$ is the cleavage failure probability, $\Delta l_i$ is the length of the $i$th element of the crack front and $K_{I(i)}$ is the stress intensity factor acting over that length (taken as the mean of the nodal $K_I$ values
associated with the element). \( B_0 \) is a characterizing length taken as 25mm and \( K_{\text{min}} \) is a threshold value of toughness which we take as 20MPa\( \cdot \)m. \( K_0 \) is defined in terms of the reference temperature \( T_0 \) as:

\[
K_0 = 31 + 77 * \exp(0.019 * (T - T_0))
\]  
(2)

In this study, \( T_0 \) is taken as -100°C, based on experimental data; \( T \) is the test temperature.

**Constraint-corrected Master Curve Method**

This method proceeds as for uncorrected master curve method above, except that the influence of crack tip constraint is represented by a \( T \)-stress-based correction factor \( \theta \), which is introduced to the weakest link relation, viz:

\[
P_F = 1 - \exp \left\{ - \frac{1}{B_0} \sum_{i=1}^{n} \left( \frac{K_j(i) / \theta - K_{\text{min}}}{K_0 - K_{\text{min}}} \right)^4 \Delta i \right\}
\]  
(3)

The factor \( \theta \) is defined in terms of the crack tip constraint, represented by the normalized \( T \)-stress as follows:

\[
\theta = \left\{ \begin{array}{ll}
1 + \alpha \left( \frac{T^* (i)}{\sigma_y} \right)^k, & T^* (i) < 0 \\
1, & T^* (i) \geq 0
\end{array} \right.
\]  
(4)

where \( T^* \) is the elastic \( T \)-stress for the \( i \)th element and \( \sigma_y \) is the material yield stress. The parameter set \( (\alpha, k) \) characterises the constraint dependence for this material. The values of these parameters are obtained by a method of data fitting which uses information from the Beremin calculations (see below).

**Beremin Method**

In this method, the stress state in the region ahead of the crack in the FE model is recorded. This information is then used to calculate the Weibull stress \( \sigma_W \) as follows:

\[
\sigma_W = \left[ \frac{1}{V_0} \sum_j \sigma_{j,\Delta V_j}^m \right]^{1/m}
\]  
(5)

Here \( \sigma_{j,\Delta V_j} \) is the maximum principal stress in the \( j \)th element, \( \Delta V_j \) is the volume of the \( j \)th element and \( V_0 \) is a normalising volume taken as 1mm\(^3\). Only those \( j \) elements which have undergone plastic yield in tension in the region ahead of the crack tip are included in the Weibull sum. From the Weibull stress, we can calculate \( P_F \) as:

\[
P_F = 1 - \exp \left\{ - \left( \frac{\sigma_W}{\sigma_y} \right)^m \right\}
\]  
(6)

Here \( (m, \sigma_y) \) are the Beremin parameters. These are material-dependent, and are not, in general, available from reference data. They can be obtained from specimen test data, but such data are unavailable in this case. In order to evaluate these parameters in this case, along with the master curve parameters \( \alpha \) and \( k \) described above, a method of data fitting has been developed. This method involves the use of results from the master curve and Beremin methods. The probability vs load curves for the constraint-corrected master curve and Beremin methods are compared with each other. If the parameters are correct the two methods will give rise to equivalent failure probabilities. Comparing Eqs. 3 and 6, we can see that when the failure probabilities are equivalent, the terms in the brackets are equivalent:
The parameter set \((\alpha, k, m, \sigma_u)\) required to enforce this equation can be determined by iteration. For best results, a number of different flaw geometries should be used to obtain a parameter set which is generally applicable for this material. Performing this for the models in this study (using widely-spaced flaws), the values of the parameter set were determined to be:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.452</td>
</tr>
<tr>
<td>(k)</td>
<td>0.772</td>
</tr>
<tr>
<td>(m)</td>
<td>15.65</td>
</tr>
<tr>
<td>(\sigma_u)</td>
<td>1710 MPa</td>
</tr>
</tbody>
</table>

As a check of these values, they can be compared with available reference data. These are unavailable for \(\alpha\) and \(k\) but values of \(m\) and \(\sigma_u\) for steel are available; these are plotted alongside the tuned parameter values in Figure 4.

![Figure 4: Tuned Values of Beremin Parameters Compared with Reference Data](image-url)

The plot shows that the tuned values lie in the centre of the range, near to the mean values, giving confidence in the tuning method.

**RESULTS**

In each case, the failure probabilities due to the actual flaw are compared with those for the corresponding characterised flaw in order to determine the level of conservatism. Example plots of failure probability for \(a/c = 1\) under tensile loading for the twin and characterised flaws are shown in Figures 5a and 5b, respectively.
Two main observations arise from these plots. Firstly, as the separation of the twin flaws is reduced, there is an increase in the predicted cleavage failure probability. This increase is associated with the interaction between the flaws and is modest as separation reduces from 10mm to 4mm, but then becomes more marked, with a very large increase as separation reduces from 2mm to zero. Secondly, as the flaw separation decreases, the predicted failure probability for the characterised flaw reduces. This second effect is smaller than the first, but is opposite in value. The two effects act in concert such that while for widely separated flaws the failure probability for the characterised flaw is greater than that for the twin flaws (i.e. the method is conservative), when the separation reduces to zero the characterised flaw underpredicts the failure probability associated with the actual system of twin flaws. In this case, the characterisation process is non-conservative.

To assess the level of conservatism (or non-conservatism), it is convenient to introduce a parameter $\Delta P_F = P_F(C) - P_F(A)$ where $P_F(C)$ and $P_F(A)$ are the failure probabilities due to the characterised and actual twin flaws, respectively, at a given load. Where the characterisation method remains conservative, $P_F(C)$ will be greater than $P_F(A)$ and $\Delta P_F$ will be positive. Negative values of $\Delta P_F$ indicate non-conservatism. Taking the case shown above ($a/c = 1$ under tensile loading), $\Delta P_F$ for the uncorrected master curve method (data from Figures 5a&b), the constraint-corrected master curve method and the Beremin method is shown in Figures 6a, 6b and 6c, respectively.
From examination of these results, it can be seen that for flaws not in contact, the Beremin method gives results in close agreement with the constraint-corrected master curve method. When the flaws are in contact, the Beremin method agrees more closely with the uncorrected master curve results. All of the methods indicate that there is non-conservatism in the case where the flaws are in contact, although the predicted magnitude of the non-conservatism is variable. However, this non-conservatism is removed when the flaws are separated. Repeating these calculations for the other flaw geometries obtains a similar result, however, it is found that the predicted non-conservatism increases with increasing \( \frac{a}{c} \) (i.e. as flaw size decreases) and is larger for bending than for tensile loading.

Examination of the results from the FE analyses reveals the reason for the non-conservatism. In the region of flaw coalescence there is a large augmentation of the stress intensity factor \( K_J \) (Figure 7).

![Figure 7: Stress Intensity Factor Along the Flaw Front for Twin and Characterised Flaws](image)

This augmentation is not observed at the outer end of the flaw, or for the characterised flaw. We have seen that the calculation of the cleavage failure probability is based on the 4\(^{\text{th}}\) power of \( K_J \) (Eq. 1). This region, though small, exhibits sufficiently high values of \( K_J \) to cause an increase in calculated cleavage probability over that which is observed for the characterised flaw, despite \( K_J \) for the characterised flaw being higher over most of the flaw length. This is the source of the predicted non-conservatism in the uncorrected master curve calculations. However, examination of the crack tip constraint via T-stress reveals that there is a large loss of constraint in the same region, and this constraint loss goes some way toward offsetting the enhancement due to \( K_J \), leading to a reduction in predicted failure probability for the constraint-corrected calculation. The Beremin calculation, by contrast, is based on the extent of the plastic zone ahead of the crack tip. This is plotted for the twin and characterised flaws in Figure 8.

![Figure 8: Extent of Plastic Zone for Twin Flaws (Above) and Characterised Flaw (Below)](image)
It can be seen that there is a large plastic zone associated with the re-entrant feature which is not seen for the characterised flaw. This zone provides the contribution that gives rise to increased cleavage probability for the twin flaws compared to the characterised flaw, hence producing non-conservatism.

Comparison with experimental results for contacting flaws under tensile load suggests that reasonable agreement is obtained between the Beremin and uncorrected master curve methods, as shown in Figure 9. The results of these methods appear also to be in fair agreement with the experimentally-measured cleavage failure loads (shown as data points on the probability curves).

**CONCLUSIONS**

The results indicate that the three calculation methods give rise to differing predictions of the cleavage failure probability for actual and characterised twin flaws. All of the methods do confirm that the characterisation procedure is conservative for flaws which are not in contact. However, non-conservatism is indicated for contacting flaws, using all of the methods. Reasonable agreement with the experimental results is obtained, providing some validation for the cleavage calculation methods.

**REFERENCES**


