Analyses of Soil-Structure Systems

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ABSTRACT

To simulate the dynamic behavior of soil-structure systems a numerical method is developed and implemented. Special attention is given to properly simulate the behavior of pile foundations. The verification of the computational model is achieved through the analysis of two examples and comparison with results reported in the literature.

INTRODUCTION

The analysis of structures founded on or in soil deposits and subjected to dynamic loads is addressed. The excitation can be defined as applied dynamic forces or as seismic excitations. Problems like wave propagation and structural behavior due to machinery vibration, traffic loads or earthquakes can be solved.

NUMERICAL FORMULATION

In the general case of embedded structures, the soil-structure system is divided into three sub-regions according to the Flexible Volume Method (Lysmer et al., 1988): the original soil *deposit* without the presence of the structure, the *structure*, and the soil displaced for the basement, in the following called *excavated* soil as displayed in Figure 1. The three subsystems are connected through the *interaction nodes*: nodes belonging to all three subsystems. The degrees of freedom of the *structure* are subdivided into those connected with the *interaction nodes* (located at or below the ground level) and those situated on the *superstructure* (located above the ground level).

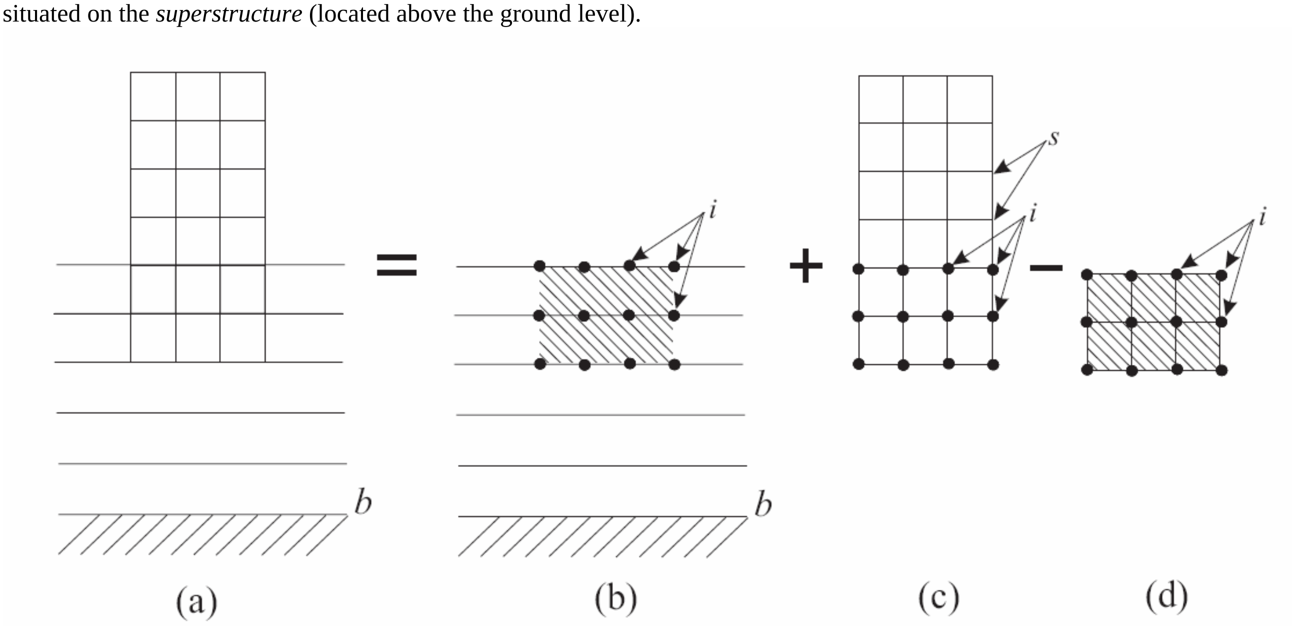

Figure 1. Substructures of the system: (a) *total* system; (b) soil *deposit*; (c) *structure*; (d) *excavated* soil (modified after Lysmer et al., 1988)

The calculation is performed in the frequency domain, where the input (loads) and output (displacements) are connected to the time domain through the Fourier Transformation. Material energy dissipation is introduced in form of a complex elasticity modulus $E = (1 + i2\beta)$, where E is the Youngs modulus, $i^2 = -1$ and β the hysteretic damping coefficient. Special consideration is given to a sufficient representation of pile foundations.

We consider one spectral component with circular frequency ω.

The equation of motion of the system is than given by:

$$\tilde{\mathbf{K}}\,\tilde{\mathbf{U}} = \tilde{\mathbf{P}}\,, \tag{1}$$

where $(\tilde{\ })$ indicates complex and frequency-dependent values, $\tilde{\mathbf{U}}$ is the vector of total displacements at the nodal points, $\tilde{\mathbf{P}}$ is the load vector, and $\tilde{\mathbf{K}}$ is the dynamic stiffness matrix or impedance matrix.

The seismic excitation shall be defined as any combination of body waves. Considering an excitation at the base b and partitioning the matrices according to Figure 1, Eq. (1) can be written as:

$$\begin{bmatrix} \tilde{\mathbf{K}}^{s}_{ss} & \tilde{\mathbf{K}}^{s}_{si} \\ \tilde{\mathbf{K}}^{s}_{is} & \left(\tilde{\mathbf{K}}^{s}_{ii} + \tilde{\mathbf{K}}^{d}_{ii} - \tilde{\mathbf{K}}^{e}_{ii}\right) \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{U}}_{s} \\ \tilde{\mathbf{U}}_{i} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \left[\tilde{\mathbf{K}}^{d}_{ii}\right]\left\{\tilde{\mathbf{U}}'_{i}\right\} \end{Bmatrix} \tag{2}$$

The subscripts s and i correspond to *superstructure nodes* and *interaction* nodes, respectively. The superscripts d, s and e correspond to soil *deposit*, *structure* and *excavated* soil, respectively. $\tilde{\mathbf{U}}'_{i}$ is the vector of seismic *free field* displacements, namely without the consideration of the structure, at the *interaction* nodes.

The impedance matrices for the *structure* and the *excavated* soil are formulated with help of the Finite Element Method (Bathe 1974). The soil *deposit* impedance matrix is obtained through a semi-discrete technique called Thin Layer Method (Kausel 1999):

$$\left[\tilde{\mathbf{K}}^{d}_{ii}\right] = \left[\tilde{\mathbf{F}}^{d}_{ii}\right]^{-1}, \tag{3}$$

where $\tilde{\mathbf{F}}^{d}_{ii}$ is the soil *deposit* compliance matrix, whose elements f_{ij} are determined by successively applying unit amplitude loads at each degree of freedom j of the interaction nodes and computing the corresponding complex displacements at each degree of freedom i of the interaction nodes.

In case of pile foundations, the interaction nodes are selected along the pile axis. To simulate the load transfer from a pile with radius r in its cross section and inclined by an angle α with respect to a vertical line, as shown in Figure 1., the displacement field of ellipse-shaped distributed loads is approximated in terms of the displacement field of equivalent circular distributed loads. A disk load distribution with the same cross area as the horizontal ellipse is considered at the interaction node at the pile tip. A circle ring load distribution with the same circumference as the horizontal ellipse is considered at the remaining interaction nodes. The displacement field computed in terms of the radial, vertical and tangential components is transformed to cartesian coordinates.

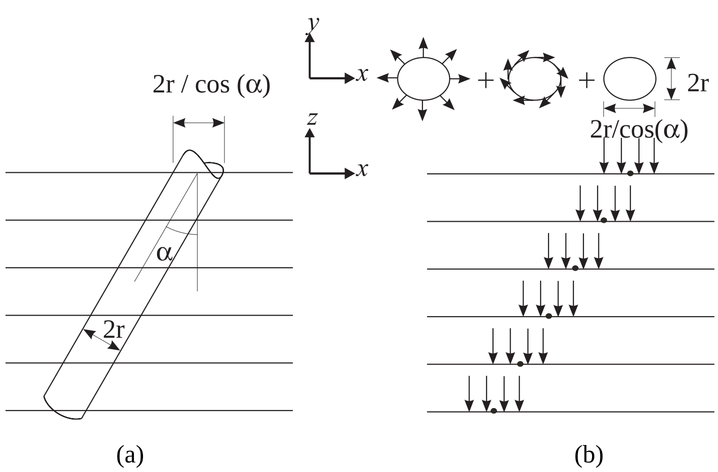

Figure 1. Inclined pile: a) physical model; b) ellipse-shaped load distributions at the interaction nodes.

The complex displacements, f_{ij}, at the interaction nodes are computed using closed-form solutions of three-dimensional Green's functions in a layered medium over a rigid base (Waas 1980, Tajimi 1980, Kausel & Peek 1982, Waas et al., 1985). This formulation has been implemented in the computer program SASSIG: A <u>S</u>ystem for the <u>A</u>nalysis of the

Soil-Structure Interaction using Green's functions (García 2002) as an extension of the computer program SASSI (Lysmer et al., 1988). The extension gives high accuracy in the analysis of the soil *deposit* impedance and allows the analysis of soil-pile interaction.

VERIFICATION OF THE COMPUTATIONAL MODEL

To verify the computer programs as well as the discretization criteria employed, two examples are analyzed and compared with results reported in the literature. The representation form used in the reference results is maintained. Little changes are introduced in the variable defined in the literature in order to make it consistent with that used in this investigation.

Pile foundation under harmonic force excitation

The soil and pile parameters of the analyzed problem are shown in Table 1. It corresponds to the following configuration:

- Homogeneous damped elastic half space,
- Pile foundation rigidly connected to the cap,
  - a) Single pile
  - b) 2x2 pile groups
- Pile groups with different pile spacing:
  - a) s/d=2
  - b) s/d=5
  - c) s/d=10,where s is the distance center to center between consecutive piles and d is the pile diameter.
- Rigid massless pile cap, without contact to the ground surface
- Harmonic force excitation at the pile cap:
  - a) Horizontal direction
  - b) Vertical direction.

In this particular example, *soil deposit* effects are not present because no resonance frequencies are associated with the homogeneous half space; the soil-foundation interaction is computed with no mass for the pile cap and only the mass of the piles.

Table 1. Soil and pile parameters for single pile and vertical pile group under externally applied harmonic forces.

System	E [MN/m²]	ν [-]	ρ [kg/m³]	β [-]	H [m]	L [m]	d [m]	s [m]	number of piles [-]	α [°]
Soil	42.0	0,40	1800,0	0,05	∞					
Pile	$4.2\text{x}10^4$	0,25	2500,0	0,00		9,00	0,60	1.2, 3.0, 6.0	1, 4	0,00

The reference results are reported by Kaynia (1982). The computational method used to obtain the reference results can be identified as a boundary element type formulation of the Thin Layer Method. Therefore, it displays similarities with the formulation used in this investigation. In the reference publication, the following nomenclature was employed: The complex dynamic stiffness functions K are expressed in the form as given by Kaynia (1982):

$$K= k+ia_0c \tag{4}$$

where k is the dynamic real component, c is the dynamic imaginary component, i^2= -1, and a_0 is the non-dimensional frequency defined as: $a_0=\frac{\omega d}{c_s}$ which ω is the circular frequency of harmonic vibration, d is the pile diameter and c_s is the shearwave velocity of the homogeneous half space. For pile groups subjected to horizontal excitation, the dynamic stiffness component $k^G{}_{xx}$, and the dynamic damping component $c^G{}_{xx}$ are normalized with respect to N times the static stiffness of a single pile, $k^S{}_{xx}$, where N is the number of piles in the group, the superscripts G and S identify a pile group and a single pile respectively, the first of the subscripts xx describes the direction of the stiffness and the second of the subscripts xx describes the direction of the excitation. Similar nomenclature and normalization is followed in the vertical direction. The dynamic stiffness functions are calculated and compared with the reference results as it is displayed in Figure 3.

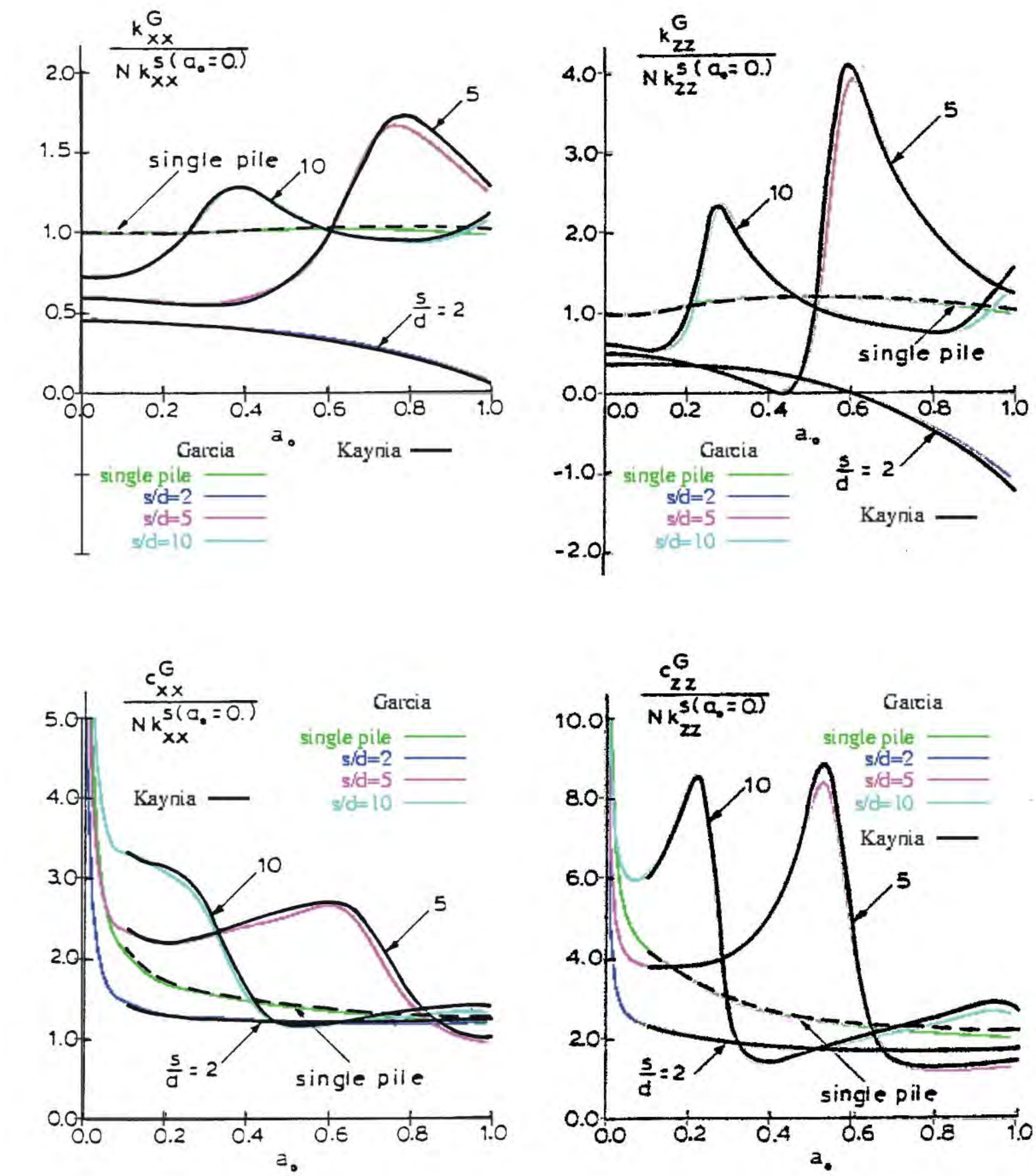

Figure 3. Dynamic stiffnesses of single pile and pile group under externally applied harmonic forces. Top left: horizontal excitation, real component; top right: vertical excitation, real component; bottom left: horizontal excitation, imaginary component; bottom right: vertical excitation, imaginary component.

A very good agreement can be seen for the dynamic stiffness functions calculated. Kaynia (1982) has discussed these results in detail. Nevertheless, some general comments will be stated here. The dynamic stiffness and damping components for a single pile show relatively light frequency dependence in the frequency range computed. These behavior is also partially displayed for a closely spaced 2x2 pile group (*s*/*d*=2). The pile spacing for a 2x2 pile group (*s*/*d*=5 and *s*/*d*=10) has very strong frequency dependence; depending of the pile spacing, resonance is observed at specific frequencies in the dynamic stiffness components as well as in the dynamic damping components. A very different vibration behavior of the pile group is therefore expected for every pile spacing.

Pile foundation under harmonic wave propagation

The soil and pile parameters are shown in Tables 2. and 3. It corresponds to the following configuration:

- Two different soil profiles considered:
  - a) Stratified soft soil over rigid base: *soil 1*
  - b) Stratified stiff soil over rigid base: *soil 2*
- Pile foundation rigidly connected to the cap:
  - a) Single pile
  - b) 4x4 pile group
- For the pile group: pile spacing s/d=2.5
- Rigid massless pile cap, without contact with the ground surface
- Harmonic wave propagation from the top of the rigid base:
  - a) vertically propagating SV-wave
  - b) vertically propagating P-wave.

Table 2. *Soil 1* and pile parameters for pile foundation under harmonic seismic excitation.

System	Layer	E [MN/m^2]	ν [-]	ρ [kg/m^3]	β [-]	h [m]	L [m]	d [m]	s [m]	number of piles [-]
Soil 1	1	6	0,49	1500,0	0,10	12,4				
Soil 1	2	149,6	0,49	1800,0	0,10	12,0				
Soil 1	3	268,6	0,48	2000,0	0,10	26,0				
Pile		3.0x10^4	0,20	2500,0	0,00		26,40	1,30	3,25	1, 16

Table 3. *Soil 2* and pile parameters for vertical pile foundation under harmonic seismic excitation.

System	Layer	E [MN/m^2]	ν [-]	ρ [kg/m^3]	β [-]	h [m]	L [m]	d [m]	s [m]	number of piles [-]
Soil 2	1	30	0,49	1500,0	0,06	12,4				
Soil 2	2	343,3	0,49	1800,0	0,04	12,0				
Soil 2	3	652,5	0,48	2000,0	0,05	26,0				
Pile		3.0x10^4	0,20	2500,0	0,00		26,40	1,30	3,25	1, 16

The reference results are reported by Hartmann (1986). The computational method used to obtain the reference results consists of a flexibility formulation of the thin layer method. As expected, it displays similarities with the formulation used in this investigation. In the reference publication, following nomenclature was employed.

The transfer function from rock to the free surface $\ddot{u}_{1x}/\ddot{u}_{2x}$ is defined as the ratio of the acceleration amplitude at the free surface $\ddot{u}_{1x}$ to the acceleration amplitude at the rock base $\ddot{u}_{2x}$. The transfer function from rock to the pile cap $\ddot{u}_{5x}/\ddot{u}_{2x}$ is defined as the ratio of the acceleration amplitude at the pile cap $\ddot{u}_{5x}$ to the acceleration amplitude at the rock base $\ddot{u}_{2x}$. The transfer functions are computed and displayed as a function of the excitation frequency. The results are compared with the reference results in Figure 4.

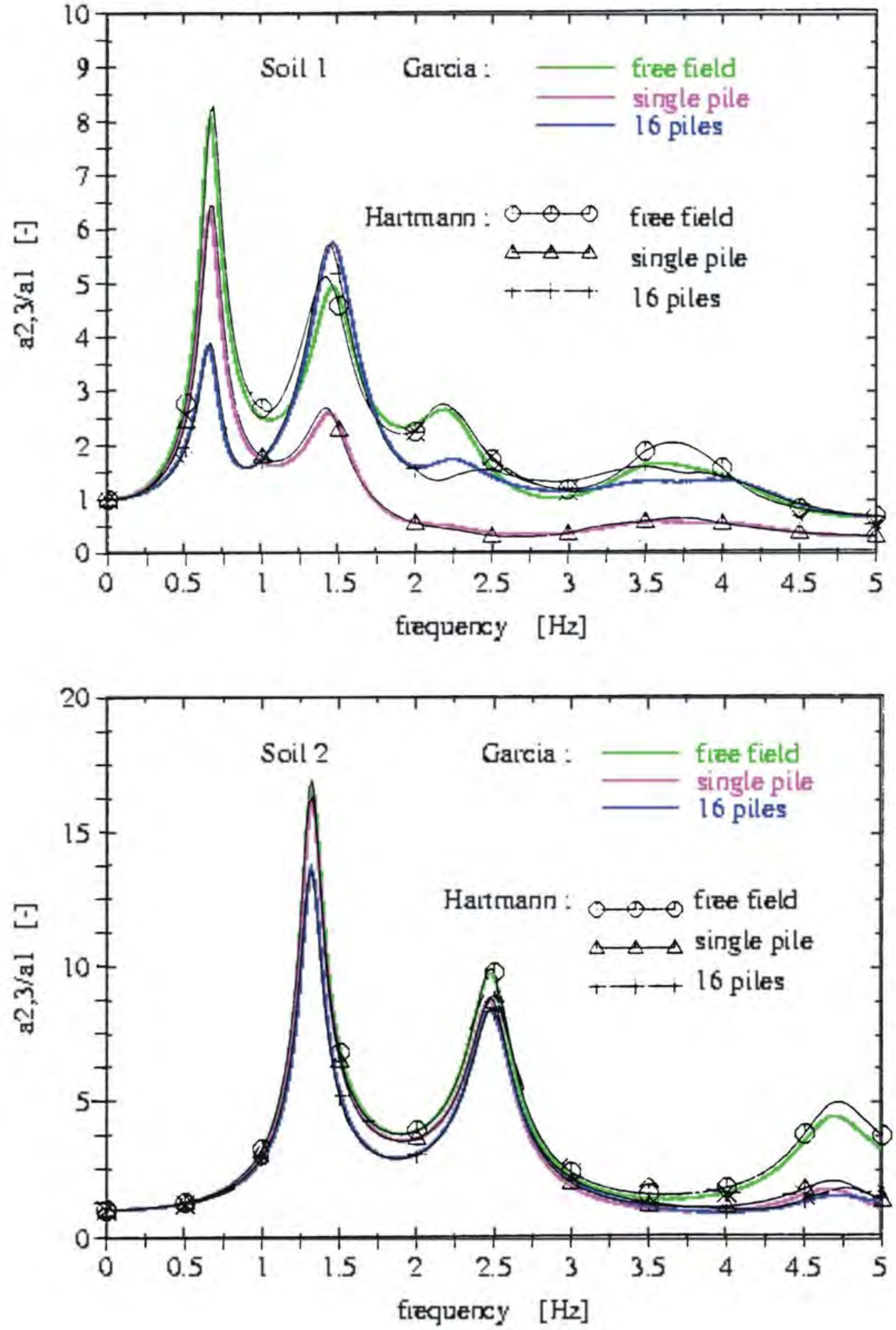

Figure 4. Horizontal harmonic foundation top response functions under harmonic seismic excitation: top: *soil 1*; bottom: *soil 2*.

The results obtained from this example match quite good with the reference results. Although the inertial effects are small because only the mass of the piles is considered, the *soil deposit* effects represent a strong component in the dynamic stiffness of the system, as can be seen in the response functions of the free field. The influence of the piles on the amplitudes of the transfer functions is higher for the soft *soil 1*, than for the stiff *soil 2*. In the latter, little difference can be detected between the transfer function for a single pile and that for the free surface.

The validation of the two different examples, namely dynamic stiffness of vertical pile foundations under harmonic force excitation, and transfer functions of vertical pile foundations under harmonic wave propagation shows that the numerical model with the discretization criteria employed are able to reproduce the dynamic behavior of single piles and pile groups, with vertical and/or inclined configurations, in homogeneous half spaces and in horizontally stratified media.

SUMMARY AND CONCLUSIONS

To simulate the dynamic behavior of soil-structure systems a numerical method is developed and implemented. The numerical method is formulated in the frequency domain, and the connection to the time domain is given by Fourier transformation techniques. The structure is described with the Finite Element Method and the unbounded soil with the Thin Layer Method, coupling the vibration of the structure and the wave propagation in the soil. The response is supposed to vary linearly with the load. Material energy dissipation is introduced in form of a linear hysteretic damping formulation. Special consideration is given to a sufficient representation of pile foundations.

The verification of the computational model shows that the numerical model with the discretization criteria employed are able to reproduce the dynamic behavior of single piles and pile groups, in homogeneous half spaces and in horizontally stratified media.

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