Building Structures Under Pressurization of High Energy Line Break – Non-Linear Analysis Method

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ABSTRACT

Compartment pressurization is considered an impulse load since it is a transient (time varying) load determined by an external source and is relatively insensitive to structural response. Linear methods such as calculating the Dynamic Load Factor (DLF) may not be sufficient to qualify existing structures. The yield line/plastic hinge method is used to evaluate the components beyond the yield point. The multi-degree of freedom system is transformed into an equivalent single degree of freedom system. A numerical method using predictor-corrector algorithm is used to solve the elastic-plastic problem using spread sheet software. Ductility and hinge rotation are checked to ensure that the component can withstand the large deformation. The elastic-plastic method was used to qualify structural components that could not pass using the linear elastic methods thus avoiding costly modifications.

INTRODUCTION

Design Basis Specifications for most nuclear power plants require that rooms or compartments that are affected by the post accident pressure of a high energy line break (HELB) are designed and constructed to minimize or resist this pressure. An evaluation of the various structural components in any given room is required for the pressure transient that is exerted in that room. Recently some utilities have-re-evaluated this loading due to revised predicted failure pressure of blowout panels, increased pressure due to power uprate or new steam generator installation.

Compartment pressurization is considered an impulse load since it is a transient (time varying) load determined by an external source and is relatively insensitive to structural response. Linear methods such as calculating the DLF, as described in References [1] and [2], may not be sufficient to qualify existing structures. The yield line/plastic hinge method is used to evaluate the structural components beyond the yield point. This method uses a yield-line (collapse mechanism) approach to obtain the ultimate load of the component. Ductility and hinge rotation are checked to ensure that the component can withstand the large deformation.

METHODOLOGY

The capacity of a component can be determined by calculating the pressure that will cause a plastic collapse mechanism. For a plate (e.g., a slab), this requires finding a plastic yield line pattern that produces the minimum energy. This internal resistance energy is equated to the external work and thus the applied pressure to cause failure can be determined. Many slabs are designed (by virtue of their steel reinforcement) to be one-way slabs. Hence, these slabs can be modeled as a beam of width equals to unity. This will simplify the problem and reduces it to a plastic hinge-mechanism evaluation.

The approach used in this evaluation is based on the method described in References [3] and [4]. The method transforms the multi-degree of freedom (MDOF) system (e.g., slab or beam) into a single degree of freedom (SDOF) system and use numerical integration to solve the non-linear dynamic problem.

Converting MDOF to SDOF

To convert a MDOF system into a dynamically equivalent SDOF system, it is necessary to evaluate the parameters of the system: namely, the equivalent mass, M_E, the equivalent spring constant, K_E, and the equivalent load, F_E. The equivalent system is selected usually so that the deflection of the concentrated mass is the same as that for a significant portion of the structure. The dynamic deflected shape of the structure is taken as the shape resulting from the static application of the dynamic loads. Load, mass, and resistance transformation factors are introduced to simplify the process of conversion. This method is presented in Reference [3] and is summarized here for convenience.

With F_E being the equivalent force and F as the applied dynamic force, the load factor, K_L, is defined as:

$$K_L = F_E/F \qquad (1)$$

The load factor is derived by setting the external work done by the equivalent load, F_E, on the equivalent system equal to the external work done by the actual load, F, on the actual element deflecting to the assumed deflected shape. With M_E being the mass of the equivalent system and M as the total mass of the actual system, the mass factor, K_M, is defined as:

$$K_M = M_E/M \tag{2}$$

The mass factor is derived by setting the kinetic energy of the equivalent system equal to the kinetic energy of the actual structure as determined from the deflected shape. Values of K_L and K_M for one-way elements were derived and presented in Reference [3]. With R_E being the resistance (restoring force) of the equivalent system and R as the total resistance of the structural element, the resistance factor, K_R, is defined as:

$$K_R = R_E/R \tag{3}$$

To obtain the resistance factor, the strain energy of the equivalent system is equated to the strain energy of the structural element, as computed from the assumed deflected shape. Since the resistance of an element is the internal force tending to restore the element to its unloaded static position, it can be shown that the resistance factor, K_R, must always equal the load factor, K_L.

The load-mass factor is a factor formed by combining the two basic transformation factors, K_L and K_M. The equation of motion of the actual system neglecting damping is given as:

$$F - R = Ma \tag{4}$$

And for the equivalent system

$$K_L F - K_L R = K_M Ma \tag{5}$$

Which can be re-written as

$$F - R = (K_M/K_L)Ma \tag{6}$$

Or

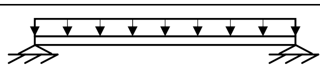
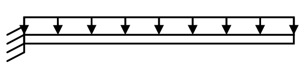
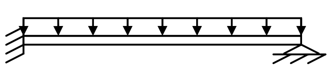
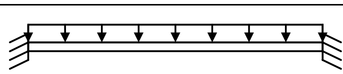
$$F - R = K_{LM}Ma = M_e a \tag{7}$$

where

K_{LM} = the load-mass factor = K_M/K_L M_e = effective total mass of the equivalent system

A sample of K_L, K_M, and K_{LM} values for one-way elements is shown in Table 1.

Table 1. Transformation Factors for One-Way Elements, [3]

Configuration	Range of behavior	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}
	Elastic	0.64	0.50	0.78
	Plastic	0.50	0.33	0.66
	Elastic	0.58	0.45	0.78
	Elasto-Plastic	0.64	0.50	0.78
	Plastic	0.50	0.33	0.66
	Elastic	0.53	0.41	0.77
	Elasto-Plastic	0.64	0.50	0.78
	Plastic	0.50	0.33	0.66
	Elastic	0.40	0.26	0.65
	Plastic	0.50	0.33	0.66

Hence, a multi-degree of freedom (MDOF) system can be transformed into a single degree of freedom (SDOF) system that can be solved with ease.

ELASTIC-PLASTIC ANALYSIS OF A SDOF SYSTEM

The equation of motion for an elastic single degree of freedom system can be written as:

$$F - kx - cv = ma \tag{8}$$

where:

F = applied force	v = velocity
k = stiffness of the system	m = mass
x = displacement	a = acceleration
c = damping	

This equation can be re-written for an elastic-plastic single degree of freedom system by including the restoring force R instead of the term kx.

$$F - R - cv = ma \tag{9}$$

Using the average acceleration method, the velocity and displacement is expressed as:

$$v_t = v_{t-1} + 1/2(a_t + a_{t-1})\Delta t \tag{10}$$
$$x_t = x_{t-1} + 1/2(v_t + v_{t-1})\Delta t \tag{11}$$

Substituting Eq. (10) into Eq. (9) and rearranging,

$$a_t = \frac{1}{m + \frac{c\Delta t}{2}}\left[F - R - c\left(v_{t-1} + \frac{a_{t-1}\Delta t}{2}\right)\right] \tag{12}$$

The term R is non-linear and depends on the displacement x at time t. To solve this problem numerically, a predictor-corrector method is utilized[3]. In this method the displacement x at time t is estimated (predicted) and then corrected. A convergence of this procedure can be obtained in a single iteration if the value of Δt is small enough. A recommended value for Δt is < T/10, where T is the period of the structure. The following step-by-step procedure is used:

1. At t = 0, compute a_t ($t = t_0$) from Eq. (9) and the initial conditions
2. Increment time: $t = t + \Delta t$
3. At $t = t + \Delta t$, set $a_t = a_{t-1}$ (t-1 = 0, initially)
4. Compute v_t and x_t from Eqs. (10) and (11)
5. Compute R
6. Compute a_t from Eq. 12

If this is the predicting pass, return to Step 4. If this is the correcting pass, set $x_{t-1} = x_t$, $v_{t-1} = v_t$, $a_t = a_{t-1}$, and return to Step 2.

This method was implemented using a spread sheet program. The results of the response of the SDOF system to a triangular forcing function is shown on Table 2 and Figure 1. The triangular forcing function is maximum at time zero and decreases linearly to zero at T=0.1 sec. Note that in Table 2, the data between time points 0.2 and 0.4 sec are hidden to save publication space. With T_N being the natural period of the structure, Ru as the ultimate resistance, P as the peak pressure, T/T_N= 0.1/0.1= 1.0, Ru/P= 750/1000= 0.75 and the resulting ductility is calculated to be Xm/X_E = 3.54 where Xm and X_E are the maximum and elastic displacement respectively. The maximum displacement occurs at tm=0.08 sec, i.e., tm/T= 0.8. These results match the results reported on Figures 3-54 and 3-55 of Reference [3].

Table 2. SDOF Elastic-Plastic Solution for Triangular Forcing Function

k=	9860		pi	3.141593		dt=	0.01		T=	0.1		
m=	2.5					P=	1000					
d=	0.001		Ru	750								
wn	62.80127		T_natural	0.100049								
wd	62.80124											
Initial Conditions												
v(initial) =	0											
x(initial)	0											
time	F	a(predict)	v(predict)	x(predict)	R(predict)	a(correct)	v(correct)	x(correct)	R(correct)	state	xp(plastic)	% change in x
0	1000	0	0	0	0	400	0	0	0	elastic	0	
0.01	900	400	4	0.02	197.2	281.1186	3.405593	0.017028	167.8957	elastic	0	17.45%
0.02	800	281.1186	6.21678	0.06514	642.2787	63.08646	5.126619	0.059689	588.5338	elastic	0	9.13%
0.03	700	63.08646	5.757483	0.11411	750	-20.00214	5.34204	0.112032	750	plastic	0.035967	1.85%
0.04	600	-20.00214	5.142019	0.164453	750	-60.00198	4.94202	0.163453	750	plastic	0.087388	0.61%
0.05	500	-60.00198	4.342	0.209873	750	-100.0017	4.142002	0.208873	750	plastic	0.132808	0.48%
0.06	400	-100.0017	3.141985	0.245293	750	-140.0012	2.941987	0.244293	750	plastic	0.168228	0.41%
0.07	300	-140.0012	1.541976	0.266712	750	-180.0005	1.341979	0.265713	750	plastic	0.189648	0.38%
0.08	200	-180.0005	-0.458027	0.270132	750	-219.9997	-0.658023	0.269132	750	plastic	0.193067	0.37%
0.09	100	-219.9997	-2.85802	0.251552	576.6591	-190.6626	-2.711334	0.252286	583.8907	elastic	0.193067	0.29%
0.1	1.11E-13	-190.6626	-4.61796	0.215639	222.5565	-89.02097	-4.109752	0.21818	247.6112	elastic	0.193067	1.16%
0.11	0	-89.02097	-4.999961	0.172632	-201.4977	80.60072	-4.151853	0.176872	-159.6859	elastic	0.193067	2.40%
0.12	0	80.60072	-3.345846	0.139384	-529.3225	211.7301	-2.690199	0.142662	-496.9991	elastic	0.193067	2.30%
0.13	0	211.7301	-0.572898	0.126346	-657.8698	263.148	-0.315809	0.127632	-645.1952	elastic	0.193067	1.01%
0.14	0	263.148	2.315672	0.137631	-546.602	218.64	2.093131	0.136518	-557.5732	elastic	0.193067	0.82%
0.15	0	218.64	4.279531	0.168382	-243.401	97.35892	3.673126	0.16535	-273.2967	elastic	0.193067	1.83%
0.16	0	97.35892	4.646715	0.206949	136.8714	-54.75012	3.88617	0.203146	99.37653	elastic	0.193067	1.87%
0.17	0	-54.75012	3.338669	0.23927	455.5611	-182.2255	2.701292	0.236083	424.1384	elastic	0.193067	1.35%
0.18	0	-182.2255	0.879037	0.253985	600.6486	-240.2597	0.588866	0.252534	586.3431	elastic	0.193067	0.57%
0.19	0	-240.2597	-1.813731	0.24641	525.9573	-210.3823	-1.664344	0.247157	533.3221	elastic	0.193067	0.30%
0.2	0	-210.3823	-3.768166	0.219994	265.4993	-106.1984	-3.247247	0.222599	291.1807	elastic	0.193067	1.17%
0.4	0	-167.4715	-2.060034	0.223028	295.4128	-118.1644	-1.813498	0.224261	307.567	elastic	0.193067	0.55%
0.41	0	-118.1644	-2.995142	0.200218	70.50105	-28.1994	-2.545317	0.202467	92.67742	elastic	0.193067	1.11%
0.42	0	-28.1994	-2.827311	0.175604	-172.1932	68.8782	-2.341923	0.178031	-148.2635	elastic	0.193067	1.36%
0.43	0	68.8782	-1.653141	0.158055	-345.2202	138.0886	-1.307089	0.159785	-328.1599	elastic	0.193067	1.08%
0.44	0	138.0886	0.073797	0.153619	-388.9612	155.5844	0.161276	0.154056	-384.6485	elastic	0.193067	0.28%
Max F	1000						X max	0.269132				
Min F	0						X min	0				
Xe	0.076065						X max/Xe	3.538192				
							|X min/Xe|	0				

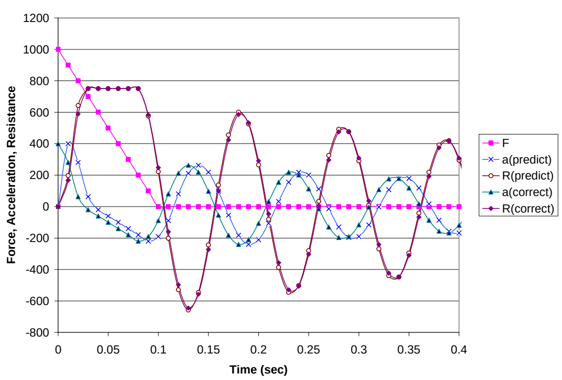


Fig. 1 Acceleration and Resistance Due to a Triangular Forcing Function

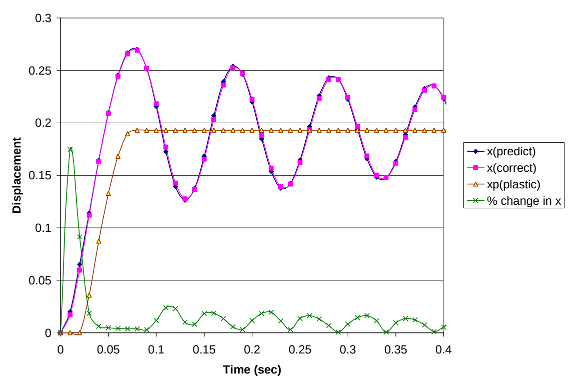


Fig. 2 Displacement and Accumulated Plastic Deformation Due to a Triangular Forcing Function

APPLICATION

A spread sheet similar to that shown on Table 2 was created for the analysis of structural components under pressurization of HELB, the following notes and rules were used:

1. The effective moment of inertia is the average of the gross and cracked moments of inertia, References [4] and [6]. The compression steel may be neglected when calculating the cracked moment of inertia.
2. The moment capacity is calculated on a step by step basis (i.e., calculating the moment that will form the first hinge, then the second hinge, etc.) until a collapse mechanism occurs. Attention is given to the sequence, the positive and negative reinforcement, and the corresponding moment capacity values.
3. For a two-step elasto-plastic system (i.e., formation of two hinges in sequence), the three slope curve is transformed to a two slope curve using the method described in Sections 3.13, 3.14 and 3.17 of Reference [3]. This is done by equating the area under the curve (resistance energy) for the two systems.
4. The load-mass factor is taken from Table 1. For small plastic deformations ($x_{max}/x_e < 5$), K_{LM} is taken as the average of the elastic and plastic values. Whereas, for larger plastic deformation ($x_{max}/x_e > 5$), the equivalent K_{LM} is taken as the plastic value, where x_{max}/x_e = ductility[3].
5. For convenience, the equation of motion used is expressed in terms of the unit area of the element (i.e. the force becomes pressure, etc.).
6. The dead load (DL) includes the equipment loads and is in the same direction as the pressure for a floor or in opposite direction to the pressure for a ceiling. The load multiplier for the DL is reduced to 1.0 for the ceiling, in order to minimize the opposing force to the pressure.
7. The pressure-time history for the applicable volume/compartment is used in the analysis.
8. The initial conditions for the predictor-corrector numerical method are zero velocity and displacement.
9. The damping value corresponds to the structural component under consideration. The critical damping is

   $$Cc = 2\,m\,\omega_n, \text{ where } \omega_n \text{ is the natural frequency} \tag{13}$$

   For damping value corresponding to 5% of critical damping, $C = 0.05Cc$
10. The equation of motion is solved for the origin at the static equilibrium as opposed to the origin at the unstretched position. The total displacement is the sum of the displacement caused by gravity and the dynamic displacement from the static equilibrium condition. The calculation of the resistance function, R, accounts for the initial stressed condition of DL and equipment load (i.e., the initial stress, whether positive or negative, is algebraically added to the stress caused by dynamic load).
11. The stress state (i.e., elastic or plastic) is monitored and updated at each time step. The plastic displacement is accumulated accordingly.
12. The plastic displacement is equal to the shift in the origin of the stress-strain curve in the subsequent unloading from the plastic state and reloading of the system. Hence, the equation for the resistance, R, is defined as follows:

   If ($x_i + DL/k - xp_{i-1}) > 0.0$ (loading)

   $$R = \text{Minimum of } [\{ k(x_i - xp_{i-1}) + DL\} \text{ or } Ru] \tag{14}$$

   Else (unloading)

   $$R = \text{Maximum of } [\{ k(x_i - xp_{i-1}) + DL\} \text{ or } -Ru] \tag{15}$$

   where:

   x_i = the displacement at time step i
   xp_{i-1} = the accumulated plastic displacement (deformation) at time step i-1
   = the shift of the origin of the resistance curve
   DL = dead load + equipment load including any load factors and accounting for sign, e.g. for ceiling this term is negative because it opposes the pressure

13. Equation 12 is adjusted to account for the initial prestress due to DL (including equipment load and any load factor) as follows:

   $$a_t = \frac{1}{m + \frac{c\Delta t}{2}}\left[1.25 * P - (R + DL) - c\left(v_{t-1} + \frac{a_{t-1}\Delta t}{2}\right)\right] \tag{16}$$

   where:

   1.25 P = pressure multiplied by 1.25 load factor, [6]
   DL = factored dead load + equipment load

14. The convergence of the numerical method is checked by monitoring the percent change of the displacement of the corrected pass and the predicted pass as follows:

$$\text{Change} = \frac{x_{correct} - x_{predict}}{x_{correct}} \tag{17}$$

This term is undefined when the displacement, $x_{correct}$, is zero. It is also very high in the first load step since the $x_{predict}$ is zero initially.

15. The ductility is calculated as:

$$\text{Ductility} = \text{Max}\left[\left(\left|\frac{x_{min} + x_{DL}}{x_e}\right|\right), \left(\left|\frac{x_{max} + x_{DL}}{x_e}\right|\right)\right] \tag{18}$$

where:

x_{min}	=	minimum dynamic displacement
x_{max}	=	maximum dynamic displacement
x_{DL}	=	initial static equilibrium displacement due to DL and equipment load
x_e	=	elastic displacement

16. The ductility is checked against the allowable ductility as defined in References [5] and [6].
17. The plastic hinge rotation is calculated based on the maximum total displacement $x_{max} + x_{DL}$ and is checked against the allowable as defined in Reference [5]. Experience has shown that the results were controlled by x_{max} and not x_{min}.

The above methodology was used to evaluate structural components that could not be evaluated using linear analysis methods. A sample of these evaluations is summarized in the following showing the changes in the results as input parameters are varied. In these evaluations, a concrete slab which acted as a one way fixed-pin beam is converted to a SDOF system. Using the unit area of the element to solve the equations of motion, the inputs used are the following: R= 34.012 kPa/mm^2 (4.933 psi/in^2), Me = 0.001617 lb.sec^2/in^3, K_E= 5.741 psi/in, and the pressure time history is multiplied by the pressure load factor of 1.25. Figure 3 shows the results of this analysis with the DL opposing the pressure and is equal to 0.868 lb/in^2 and neglecting damping (0.1%). The resistance curve show that it reached the maximum at about 0.5 sec and continues until after 0.1 sec. The accumulated plastic deformation increases during the same time frame reaching its maximum of 27.15 mm (1.069 in). The maximum displacement occurs at 0.1 sec and is equal to 52.82 mm (2.08 in). The corresponding ductility is 2.24.

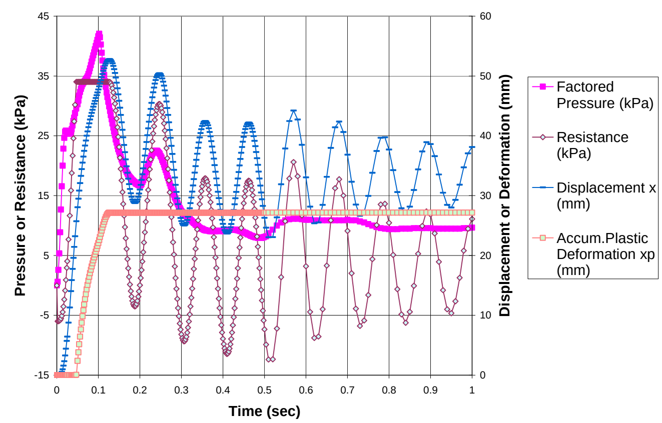


Fig. 3 Results of Pressure Transient on a Ceiling Slab Including DL Using 0.1 % Damping

The same problem was solved using 5% damping. The results, shown on Fig. 4, exhibit a reduction of the displacement and plastic deformation. The corresponding ductility is reduced to 1.74.

Figure 5 show the solution when the maximum 1.05 (DL+LL) is used as the initial static load (prestressed condition). The effective mass in this case is 0.002264 lb.sec^2/in^3. The results show, as expected, a reduction in the dynamic response and the ductility is calculated to be 1.622.

Figure 6 show the response if the DL+ LL are neglected as in the case of a reinforced concrete wall. The effective mass of the SDOF system is 0.001617 lb.sec^2/in^3. The response exhibits plastic behavior at two time regions with unloading and reloading of the structure between these regions. The ductility is calculated to be 4.27.

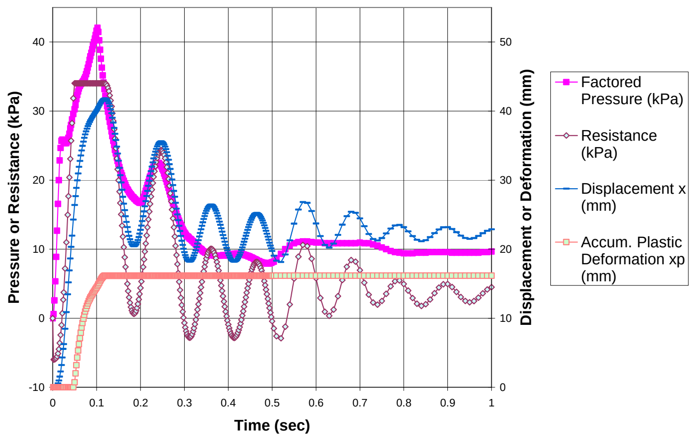


Fig. 4 Results of Pressure Transient on a Ceiling Slab
Including DL Using 5.0 % Damping

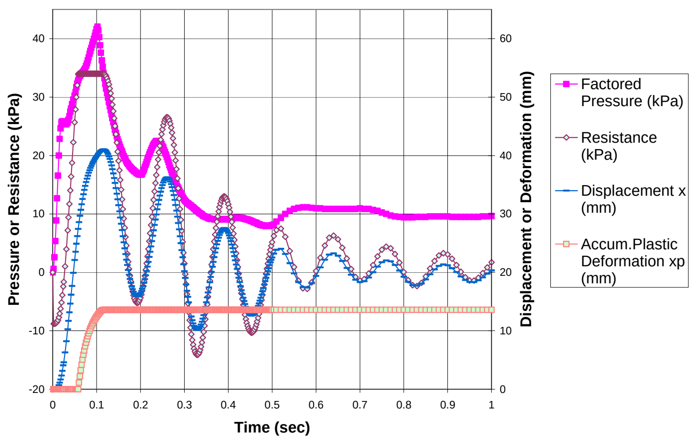


Fig. 5 Results of Pressure Transient on a Ceiling Slab
Including 1.05(DL+LL) Using 5.0 % Damping

For a floor slab with 1.05(DL), the initial static load acts in the same direction as the pressure. The effective mass is Me = 0.001483 lb.sec^2/in^3 and 1.05DL = 0.9114 lb/in^2. Figure 7 shows high displacements in which the two plastic regions of the previous figure are combined to form a wide plastic region. The ductility for this case is calculated to be 9.57. This example only serves as an illustration on how the initial conditions influence the results of the ductility considerably.

Hinge rotation is calculated using the geometric configuration of the collapse mechanism and the maximum displacement obtained from above. Using the non-linear analysis method makes it possible to qualify many existing structural components that cannot be qualified using the linear elastic method.

SUMMARY AND CONCLUSION

A numerical method using a predictor-corrector algorithm is used to solve the idealized SDPF elastic-plastic problem subjected to HELB pressure time history. The method is implemented using spread sheet software. Ductility and hinge rotation are checked to ensure that the component can withstand the large deformation. The elastic-plastic method was used to qualify structural components that could not pass using the linear elastic methods thus avoiding costly modifications.

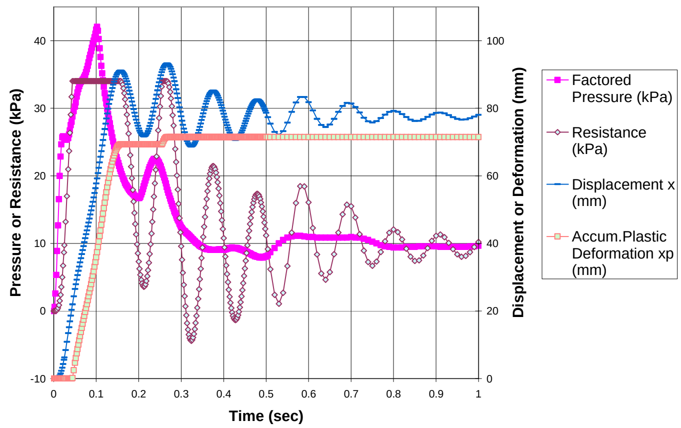


Fig. 6 Results of Pressure Transient on a Wall
Neglecting DL & LL Using 5.0 % Damping

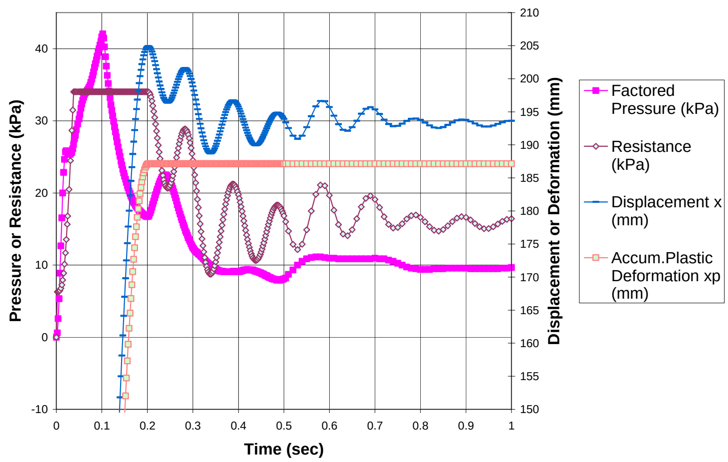


Fig. 7 Results of Pressure Transient on a Floor Slab
Including 1.05DL Using 5.0 % Damping

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