# Detonation of Hydrogen and Oxygen in a Stainless Steel Pipe

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## ABSTRACT

At the closed steam line high end of a boiling water reactor plant (BWR) hydrogen and oxygen may accumulate. For the analytical investigation a mixture of hydrogen, oxygen, and steam in a stainless steel pipe is considered. The deflagration pressure, the detonation pressure, pipe inflation under the action of detonation pressure, subsequent pipe burst, the velocity of the outward flying debris, and impact resistance of a target are calculated.

## HYDROGEN AND OXYGEN ACCUMULATION IN STEAM

In a water cooled reactor hydrogen and oxygen are produced by radiation. If a steam pipe of a boiling water reactor is closed at its upper end, hydrogen and oxygen accumulate there. The mixture of steam, hydrogen and oxygen has the temperature of saturated steam at the partial steam pressure. This mixture assumes a steam concentration at minimum density. At 7.06 MPa total pressure, minimum density is 14.9 % mole fraction steam at 182 °C, see figure 1. The temperature of the gas mixture will not fall significantly below minimum density temperature.

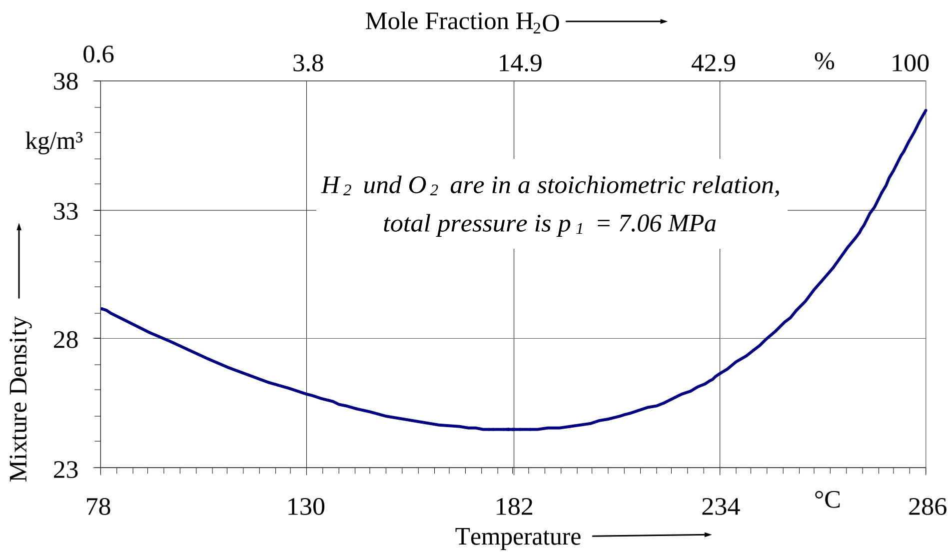


**Figure 1: Density for mixture of steam, hydrogen and oxygen**

## DEFLAGRATION

### Gas Composition Before Explosion

A mixture of hydrogen, oxygen, and steam at $p_1$ = 7.06 MPa and $\vartheta_1$ = 182 °C, or $T_1$ = 455.15 K, is analyzed with hydrogen and oxygen being in a stoichiometric relation. The mixture has the steam concentration and the corresponding temperature at minimum density. Table 1 reveals the parameters of the three gas components. The steam density was taken from a table for saturated steam properties. Hydrogen and oxygen densities are as for ideal gases.

**Table 1: Radiolysis Gas Before Explosion, at $p_1$ = 7.06 MPa and at $\vartheta_1$ =182 °C**

| Gas | Mole Fraction | Molar Mass kg/kmol | Specific Gas Constant J/(kg*K) | Mass Fraction | Density kg/m³ | Enthalpy $H^0$ for Single Gas J/mol | Enthalpy $h^0$ for Single Gas J/kg | Enthalpy $h^0$ for Mixture J/kg |
|---|---|---|---|---|---|---|---|---|
| $H_2O$ | 14.90 % | 18.015 | 461.5 | 22.0 % | 5.391 | 15271 | 847688 | 186843 |
| $H_2$ | 56.73 % | 2.016 | 4124.4 | 8.7 % | 2.134 | 13038 | 6467377 | 564182 |
| $O_2$ | 28.37 % | 31.999 | 259.8 | 69.2 % | 16.933 | 13385 | 418288 | 289602 |
| *Sum or Average:* | 100 % | 12.9050 $= \mathbf{M_1}$ | | 100 % | 24.458 $= \rho_1$ | | | 1041000 $= \mathbf{h_1^0}$ |

**Gas Composition After Detonation**

Hydrogen and oxygen are not completely burned during an explosion. Gas mole fractions after detonation of hydrogen and oxygen in a stoichiometric relation and without steam are given in the literature. These given mole fractions will be reduced by the concentration of steam before burning, which is 14,9 % mole fraction. Considering steam, hydrogen and oxygen in a composition before burning according to table 1, table 2 gives the composition of the burned gas.

**Table 2: Mole Fractions and Molar Mass After Detonation**

| $H_2$ and $O_2$ in a stoichiometric relation before explosion | $H_2O$ | $H_2$ | $O_2$ | OH | H | O |
|---|---|---|---|---|---|---|
| Only $H_2$ and $O_2$ before explosion [1, Tabelle 2.3-3] | 54.3 % | 16.5 % | 5.1 % | 12.5 % | 7.8 % | 3.8 % |
| $H_2$ and $O_2$ with 14.9 % $H_2O$ before explosion | 61.11 % | 14.04 % | 4.34 % | 10.64 % | 6.64 % | 3.23 % |
| Molar mass of gas mixture after detonation, $\mathbf{M_m}$ **= 15.0743 kg/kmol** | | | | | | |

First, the enthalpies of the gases before and after the explosion are compared for a state at same pressure $p_1$, mass m, and volume V. By utilizing the two thermal equations of state a temperature $T_{after\text{-}pressure\text{-}volume\text{-}as\text{-}before}$ will be obtained:

$$\frac{p_1 \cdot V \cdot M_1}{m \cdot R \cdot T_1} = \frac{p_1 \cdot V \cdot M_m}{m \cdot R \cdot T_{after-pressure-volume-as-before}} \tag{1}$$

$$T_{after\text{-}pressure\text{-}volume\text{-}as\text{-}before} = \frac{M_m}{M_1} \cdot T_1 = \frac{15.0743}{12.905} \cdot 455.15\ K = 531.7\ K\ ;\ \vartheta_{after\text{-}pressure\text{-}volume\text{-}as\text{-}before} = 258.5\ °C \tag{2}$$

Table 3 lists enthalpies of the six involved gases at 531.7 K = 258.5 °C.

**Table 3: Burned Gas at 70.6 bar and 531.7 K**

| Gas | Mole Fraction | Molar Mass kg/kmol | Mass Fraction $q_{mass\text{-}fraction\text{-}after}$ | Enthalpy $H^0$ J/mol | $h^0$ J/kg | $q_{mass\text{-}fraction\text{-}after} * h^0$ J/kg |
|---|---|---|---|---|---|---|
| $H_2O$ | 61.11 % | 18.015 | 73.0 % | 17961 | 996995 | 728124 |
| $H_2$ | 14.04 % | 2.016 | 1.9 % | 15276 | 7577916 | 142271 |
| $O_2$ | 4.34 % | 31.999 | 9.2 % | 15767 | 492751 | 45397 |
| OH | 10.64 % | 17.007 | 12.0 % | 16098 | 946549 | 113601 |
| H | 6.64 % | 1.008 | 0.4 % | 11051 | 10963733 | 48661 |
| O | 3.23 % | 15.999 | 3.4 % | 11739 | 733707 | 25183 |
| *Sum or weighted average:* | 100 % | 15.0743 | 100 % | $h^0_{after-pressure-volume-as-before}$ | | **= 1103000** |

The difference of the two enthalpies from table 3 and table 1 is

$$\Delta h^0 = h^0_{after\text{-}pressure\text{-}volume\text{-}as\text{-}before} - h^0_1 = (1103000 - 1041000)\ J/kg = 62000\ J/kg \tag{3}$$

Heat up at constant volume is assumed. Specific heat for constant volume and enthalpy of formation are listed in table 4. The specific heat $c_V$ is the mean value for heat up from 531.7 K to 3100 K [2].

**Table 4: Specific Heat for Constant Volume and Enthalpy of Formation**

| Gas | Mole Fraction | Mass Fraction | Mean Specific Heat, $c_V$ J/(kg*K) | Molar Mass kg/mol | Enthalpy of Formation at 182 °C for Single Gas J/mol | Single Gas J/kg | Gas Mixture J/kg |
|---|---|---|---|---|---|---|---|
| $H_2O$, not burned | 14.9 % | 17.8 % | 2216 | 0.01802 | 0 | 0 | 0 |
| $H_2O$, burned | 46.2 % | 55.2 % | 2216 | 0.01802 | - 243382 | -13510000 | -7461000 |
| $H_2$ | 14.0 % | 1.9 % | 12420 | 0.00202 | 0 | 0 | 0 |
| $O_2$ | 4.3 % | 9.2 % | 894 | 0.03200 | 0 | 0 | 0 |
| OH | 10.6 % | 12.0 % | 1487 | 0.01701 | + 39364 | 2315000 | 278000 |
| H | 6.6 % | 0.4 % | 12373 | 0.00101 | + 218975 | 217248000 | 964000 |
| O | 3.2 % | 3.4 % | 787 | 0.01600 | + 250208 | 15639000 | 537000 |
| Sum, enthalpy of formation for gas mixture, $\Delta h_{Explosion}$ = | | | | | | | **- 5682000** |

**Deflagration Temperature And Pressure**

With temperature $\vartheta_{after\text{-}pressure\text{-}volume\text{-}as\text{-}before}$, mass fractions $q_{i,mass\text{-}fraction\text{-}after}$, enthalphy difference $\Delta h^0$, enthalpy of formation $\Delta h_{Explosion}$, and mean specific heats $c_{V,i}$, given by eq. (2), table 3, eq. (3), and table 4, the deflagration temperature, $\vartheta_{Deflagration}$ or $T_{Deflagration}$, is determined by

$$\left|\Delta h_{Explosion}\right| - \Delta h_0 = (\vartheta_{Deflagration} - 258.5\ °C) \cdot \sum_{i \in Gases}\left(q_{i,mass-fraction-after} \cdot c_{v,i}\right) ,$$

$$\vartheta_{Deflagration} = 2820\ °C , \qquad T_{Deflagration} = 3093\ K \quad . \tag{4}$$

The mass and the volume before and after the explosion are the same. The deflagration pressure is calculated from the two thermal equations of state, before and after deflagration:

$$p_{Deflagration} = \frac{p_1 \cdot T_{Deflagration} \cdot M_1}{T_1 \cdot M_m} = \frac{7.06 \cdot 3093 \cdot 12.905}{(182 + 273.15) \cdot 15.0743}\ MPa = 41.1\ MPa . \tag{5}$$

The deflagration pressure is $p_{Deflagration}/p_1 = 41.1/7.06 = 5.8$ times the pressure before the explosion.

## DETONATION

Figure 2 shows a plot of Hugoniot curves [1, Abb. 60a] for two enthalpies of formations, Q = 0 and Q = $|\Delta h_{Explosion}|$. The enthalpy $\Delta h_{Explosion}$ is given by table 4.

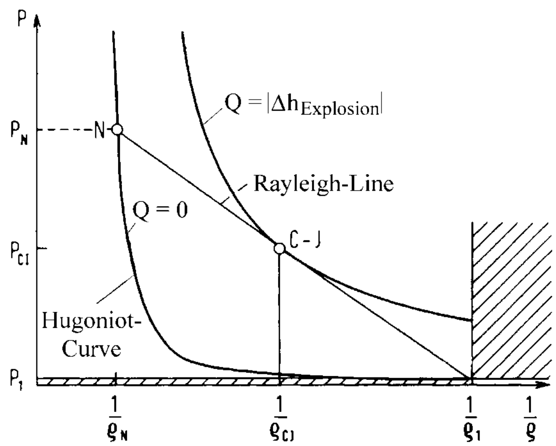


**Figure 2: Hugoniot-Curves for Q = 0 and for Q = $|\Delta h_{Explosion}|$**

The pressure $p_N$ in figure 2 is the peak pressure at detonation. The specific heat ratio κ for the gas after detonation at 3200 K is:

$$\kappa = 1.21 \tag{6}$$

For Q = 0 and for Q = $|\Delta h_{Explosion}|$ the pressures p at the Hugoniot curves are [3, eq. (4.14)]

$$p_{Q=0}(\rho) = p_1 \cdot \frac{\rho \cdot (\kappa+1) - \rho_1 \cdot (\kappa-1)}{\rho_1 \cdot (\kappa+1) - \rho \cdot (\kappa-1)} , \tag{7}$$

$$p_{Q=\left|\Delta h_{Explosion}\right|}(\rho) = \frac{2 \cdot \rho \cdot \rho_1 \cdot (\kappa-1) \cdot \left|\Delta h_{Explosion}\right| + p_1 \cdot [\rho \cdot (\kappa+1) - \rho_1 \cdot (\kappa-1)]}{\rho_1 \cdot (\kappa+1) - \rho \cdot (\kappa-1)} . \tag{8}$$

The specific heat ratio κ, the pressure $p_1$ and density $\rho_1$ before detonation, and the enthalpy of formation $\Delta h_{Explosion}$ are given by eq. (6), figure 1, table 1, and table 4. The Rayleigh-line passes the point $(1/\rho_1 , p_1)$ and is tangent to the Hugoniot-curve $p_{Q=\left|\Delta h_{Explosion}\right|}(1/\rho)$. The peak pressure $p_N$ at detonation is the point of intersection of the Rayleigh-line with the Hugoniot-curve $p_{Q=0}(1/\rho)$. With the known values as listed in table 1, table 4, and eq. (6) the peak pressure at detonation is determined to

$$p_N = 131.1\ MPa \quad . \tag{9}$$

For this example the peak pressure at detonation is $p_N/p_1$ = 18.6 times the pressure before the explosion. The density of the detonated gas is got by substituting $p_N$ for $p_{Q=|\Delta h_{Explosion}|}(1/\rho)$ from eq. (9) in eq. (8) and solving it for ρ:

$$\rho_{Detonation} = 70.16 \text{ kg/m}^3 \tag{10}$$

The thermal equation of state then provides the temperature at detonation :

$$T_{Detonation} = \frac{p_N \cdot M_m}{\rho_{Detonation} \cdot R} = \frac{131.1 \cdot 10^6 \cdot 15.0743}{70.16 \cdot 8314.41} = 3388 \text{ K} \tag{11}$$

## PIPE EXPANSION AND BURST

### Calculation Steps

A stainless steel pipe at 182 °C of $d_i$ = 103.1 mm inside diameter and $s_c$ = 5.1 mm wall thickness will be subjected to the detonation pressure $p_N$ = 131.1 MPa. The modulus of elasticity of the material E = $183 \cdot 10^9$ N/m², the tensile strength $R_m = 391 \cdot 10^6$ N/m², the rupture strain $A_5$ = 35 %, and density $\rho_{steel}$ = 8000 kg/m³. The calculation of radial expansion and subsequent burst distinguish three distinct courses. They are listed below in three action steps arranged in chronological order:

1. The cylinder will expand radially and linear-elastic until the material is at its tensile strength.
2. The pipe section expands ideal plastic to the rupture strain of the material.
3. The cylindrical pipe bursts in two half cylinders. The high pressure accelerates the half cylinders further.

### Linear Elastic Expansion

To simplify the calculation it is assumed that during the time from t = 0 to t = $t_{El}$ the pipe will expand linear elastic to a stress intensity equal to the tensile strength $R_m = 391 \cdot 10^6$ N/m². The radius $r_m(t_{El})$ at tensile strength is determined from the following correlations:

$$R_m = \sigma_{hoop} + \frac{p_N}{2} = E \cdot \varepsilon + \frac{p_N}{2} = E \cdot \frac{r_m(t_{El}) - r_m(0)}{r_m(0)} + \frac{p_N}{2} \quad , \tag{12}$$

$$r_m(t_{El}) = r_m(0)\left(1 + \frac{2 \cdot R_m - p_N}{2 \cdot E}\right) = \frac{0.1031 + 0.0051}{2}\left(1 + \frac{2 \cdot 391 - 131.1}{2 \cdot 183000}\right) \text{m} = 0.05419621 \text{ m} . \tag{13}$$

The corresponding radial velocity $\dot{r}_m(t_{El})$ will be determined next. The expanding pipe section is shown.

$$r_m(0) = \frac{d_i + s_c}{2}$$

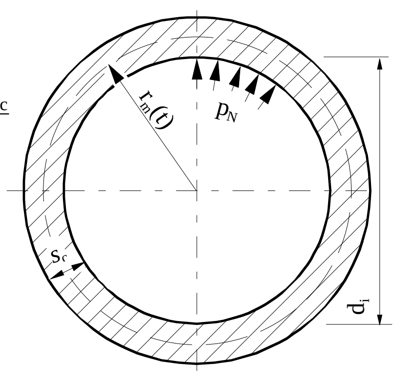


**Figure 3: Linear Elastic Pipe Expansion**

For a pipe with unit length, the differential equation of linear elastic pipe expansion is obtained from

$$\ddot{r}_m \cdot 2 \cdot \pi \cdot r_m \cdot s_c \cdot \rho_{steel} \cdot 1 = p_N \cdot \pi \cdot (2 \cdot r_m - s_c) \cdot 1 - \sigma_{hoop} \cdot 2 \cdot \pi \cdot s_c \cdot 1 \quad .$$

Substituting $\sigma_{hoop} = E \cdot \varepsilon = E \cdot \frac{r_m - r_m(0)}{r_m(0)}$, the differential equation becomes

$$\ddot{r}_m = \frac{p_N \cdot (2 \cdot r_m - s_c)}{2 \cdot r_m \cdot s_c \cdot \rho_{steel}} - \frac{E \cdot [r_m - r_m(0)]}{\rho_{steel} \cdot r_m \cdot r_m(0)} \quad . \tag{14}$$

With values given above the numerical solution of the differential equation (14) is

$$t_{EL} = 8.099 \cdot 10^{-6} \text{ s} \quad ; \quad \dot{r}_m(t_{EL}) = 22.74 \text{ m/s} \quad . \tag{15}$$

**Ideal Plastic Expansion**

In the second step of radial expansion the hoop stress will remain constant. The ideal plastic expansion continues until the circumference has expanded by $A_5$ = 35 %, at which rupture is assumed to occur. The constant hoop stress is

$$\sigma_{hoop} = R_m - \frac{p_N}{2} = \left(391 - \frac{131.1}{2}\right)\cdot 10^6 \frac{N}{m^2} = 325.45 \frac{N}{mm^2} \quad . \tag{16}$$

At hoop strain $A_5$ = 35 % the mean radius $r_m\left(t_{rupture}\right)$ = 0.0541·1.35 m = 0.073035 m. Figure 4 illustrates plastic expansion of pipe.

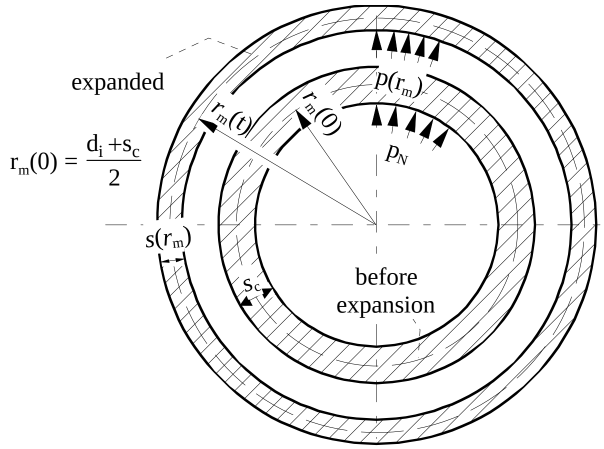


**Figure 4: Plastic Pipe Expansion**

For a cylinder of unit length, the differential equation of ideal plastic expansion is

$$\ddot{r}_m \cdot 2\cdot\pi\cdot r_m \cdot s(r_m)\cdot\rho_{steel}\cdot 1 = p(r_m)\cdot\pi\cdot[2\cdot r_m - s(r_m)]\cdot 1 - \sigma_{hoop}\cdot 2\pi\cdot s_c\cdot 1 \tag{17}$$

Using $s(r_m) \approx s_c \cdot r_m(0)/r_m$ the differential equation is transformed to

$$\ddot{r}_m = \frac{p(r_m)\cdot\left[2\cdot r_m^2 - s_c\cdot r_m(0)\right]}{2\cdot s_c\cdot r_m(0)\cdot r_m\cdot\rho_{steel}} - \frac{\sigma_{hoop}}{\rho_{steel}\cdot r_m(0)} \quad . \tag{18}$$

For isentropic gas expansion the pressure is [4, eq. (4.31)]

$$p(r_m) = p_N \cdot \frac{\left[2r_m(0) - s_c\right]^{2\kappa}}{\left[2r_m - s_c\cdot\frac{r_m(0)}{r_m}\right]^{2\kappa}} \approx p_N\cdot\left[\frac{r_m(0)}{r_m}\right]^{2\kappa} \quad . \tag{19}$$

Inserting eq. (19) in the differential equation (18) above gives

$$\ddot{r}_m = \frac{p_N\cdot\left(\frac{r_m(0)}{r_m}\right)^{2\kappa}\cdot\left[2\cdot r_m^2 - s_c\cdot r_m(0)\right]}{2\cdot s_c\cdot r_m(0)\cdot r_m\cdot\rho_{steel}} - \frac{\sigma_{hoop}}{\rho_{steel}\cdot r_m(0)} \quad . \tag{20}$$

With detonation temperature from eq. (11) and universal gas constant R = 8314.41 J/(kmol·K), temperature [4, equation (4.30)] and density are

$$T = T_{Detonation}\cdot\left(\frac{p(r_m)}{p_N}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} \overset{(19)}{\approx} T_{Detonation}\cdot\left[\left(\frac{r_m(0)}{r_m}\right)^{2\kappa}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} = T_{Detonation}\cdot\left(\frac{r_m(0)}{r_m}\right)^{2(\kappa-1)} , \tag{21}$$

$$\rho = \frac{m}{V} = \frac{p\cdot M_m}{R\cdot T} \overset{(\text{Table 2})}{=} \frac{p\cdot 15.0743}{T\cdot 8314.41}\,\frac{s^2\cdot K}{m^2} \quad . \tag{22}$$

Table 5 gives solutions of eq. (20), (21), and (22). Initial conditions are given by eq. (13), (15), and (16).

**Table 5: Solutions for the Differential Equation Describing Ideal Plastic Pipe Expansion**

| t | $\dot{r}_m(t)$ | $r_m(t)$ | ε(t) | s(t) | p(t) | ρ(t) | T(t) |
|---|---|---|---|---|---|---|---|
| $8.099\cdot10^{-6}$ s = $t_{El}$ | 22.74 m/s | 0.054196 m | 0.1778 % | 0.00509095 m | 130.54 MPa | 69.9 kg/m³ | 3385 K |
| $20\cdot10^{-6}$ s | 50.0 m/s | 0.0546 m | 0.98 % | 0.00505 m | 128.0 MPa | 68.8 kg/m³ | 3374 K |
| $80\cdot10^{-6}$ s | 174.8 m/s | 0.0615 m | 13.7 % | 0.00449 m | 96.1 MPa | 54.3 kg/m³ | 3210 K |
| $132.8\cdot10^{-6}$ s = $t_{rupture}$ | 257.2 m/s = $\dot{r}_m(t_{rupture})$ | 0.07304 m = $r_m(t_{rupture})$ | 35 % = $A_5$ | 0.003778 m = $s(t_{rupture})$ | 63.41 MPa = $p(t_{rupture})$ | 38.49 kg/m³ = $\rho(t_{rupture})$ | 2987 K = $T(t_{rupture})$ |

**Accelerating Half Cylinder**

According to the last row of table 5, at the time $t_{rupture} = 132.8\cdot10^{-6}$ s the has pipe has expanded to rupture strain $A_5$ = 35 %. It is assumed, that at this point the pipe breaks up under circumferential stress in two half cylinders. As the gas escapes, the half cylinders are further accelerated by the high gas pressure. Figure 5 shows an accelerating half cylinder while gas releases.

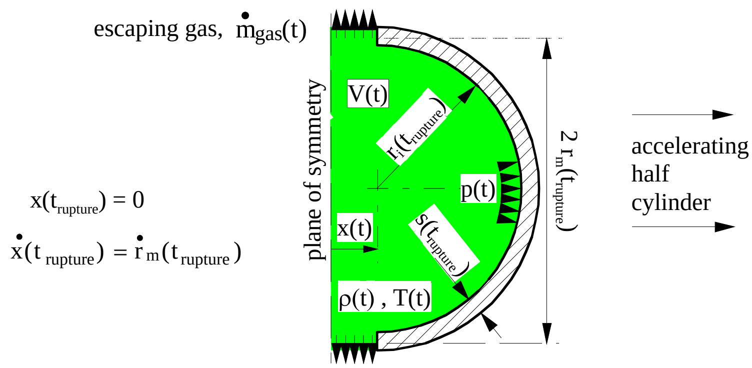


**Figure 5: Accelerating Half Cylinder After Pipe Rupture**

Two coupled differential equations are established from the releasing gas mass $\dot{m}_{gas}(t)$ and from the half cylinder acceleration $\ddot{x}(t)$. The gas pressure still acts on the half cylinder inner surface. The formulas for gas releasing through an opening [5] and for an accelerating half cylinder with unit length are:

$$\left.\begin{aligned} \dot{m}_{gas}(t) &= -2\cdot(1\,\mathrm{m})\cdot x(t)\cdot\left(p(t)-1\,\mathrm{bar}\right)\cdot\sqrt{\frac{2\cdot M_m}{R\cdot T(t)}}\cdot\left(\frac{2}{1+\kappa}\right)^{\frac{1}{\kappa-1}}\cdot\sqrt{\frac{\kappa}{1+\kappa}} \\ \ddot{x}(t)\cdot\rho_{steel}\cdot\pi\frac{(d_i+2s_c)^2-d_i^2}{8}\cdot(1\,\mathrm{m}) &= \left(p(t)-1\,\mathrm{bar}\right)\cdot d_i(t_{rupture})\cdot(1\,\mathrm{m}) \end{aligned}\right\} \tag{23}$$

Gas pressure p(t) and temperature T(t) drop while gas is being released. For isentropic gas expansion with volume V(t) given in figure 5 it is

$$\left.\begin{aligned} p(t)\cdot V(t) &= p(t)\cdot\left(d_i(t_{rupture})\cdot x(t)+\frac{d_i^2(t_{rupture})\cdot\pi}{8}\right)\cdot(1\,\mathrm{m}) = \frac{m_{gas}(t)\cdot R\cdot T(t)}{M_m}\;, \\ \frac{T(t)}{T(t_{rupture})} &= \left(\frac{p(t)}{p(t_{rupture})}\right)^{\frac{\kappa-1}{\kappa}} \;. \end{aligned}\right\} \tag{24}$$

From eq. (24), solutions for p(t) and T(t) can be expressed as follows:

$$\left[\frac{p(t)}{p(t_{rupture})}\right]^{\frac{1}{\kappa}} = \left[\frac{T(t)}{T(t_{rupture})}\right]^{\frac{1}{\kappa-1}} = \frac{m_{gas}(t)\cdot\frac{R}{M_m}\cdot T(t_{rupture})}{\left[d_i(t_{rupture})\cdot x(t)+\frac{d_i^2(t_{rupture})\cdot\pi}{8}\right]\cdot(1\,\mathrm{m})\cdot p(t_{rupture})} \tag{25}$$

The two functions for p(t) and T(t) given by eq. (25) are to be inserted in the differential equations (23). For the sake of convenience the transformed differential equations (23) are not shown here. The differential equations (23) are solved numerically. Starting values are the values at rupture time $t_{rupture}$ which are given in the last row of table 5. Table 6 lists solutions for the differential equations (23).

**Table 6: Solutions of Differential Equations (23), for the Accelerating Half Cylinder**

| t | p | $\dot{x}(t)$ | x(t) |
|---|---|---|---|
| $132.8 \cdot 10^{-6}$ s = $t_{rupture}$ | 63.41 MPa = $p(t_{rupture})$ | 257.2 m/s = $\dot{r}_m(t_{rupture})$ | 0 mm |
| $200 \cdot 10^{-6}$ s | 38.6 MPa | 327 m/s | 19.8 mm |
| $400 \cdot 10^{-6}$ s | 5.2 MPa | 397 m/s | 94 mm |
| $800 \cdot 10^{-6}$ s | 0.19 MPa | 407 m/s = $v_{end}$ | 256 mm |

As shown in table 6, the debris accelerates still after pipe rupture. It achieves the velocity $v_{end}$ = 407 m/s.

## PROTECTION FROM DEBRIS

After a detonation in a pipe, debris with a rectangular area of half pipe circumference and a length equal to the pipe inner circumference were observed. For the pipe given as example above, such a metal piece has a mass

$$m_{debris} = \pi^2 \frac{(d_i + 2 \cdot s_c)^2 - d_i^2}{8} \cdot d_i \cdot \rho_{steel} = \pi^2 \frac{0.1133^2 - 0.1031^2}{8} \cdot 0.1031 \cdot 8000 \,\text{kg} = 2.2 \text{ kg} \; .$$

The next question to be raised is: how thick the steel must be to guard it against not being perforated by the debris. For this purpose the formulas presented in a Japanese publication [6] and in a published report in the UK [7] are adequate to compute the steel wall thickness. Unfavourable is the situation, when the target is hit by the front surface of the half cylinder. For the calculation, a cylindrical projectile with the same circumference as the half cylinder front surface is considered [7]. In the demonstration example as calculated here an equivalent cylindrical projectile with the following diameter is taken into consideration:

$$d_{projectile} = \frac{d_i + 2s_c}{\pi} \cdot \left(1 + \frac{\pi}{2}\right) = \frac{0.1031 + 2 \cdot 0.0051}{\pi} \cdot \left(1 + \frac{\pi}{2}\right) \text{m} = 0.0927 \text{ m} \; .$$

The formula from Ohte el al. [6] has been verified for cylinders hitting steel plates. Using values listed above, the required target thickness is computed with the help of the formula worked out by Ohte et al.:

$$\frac{t_{required}}{m} = 307 \cdot 10^{-9} \cdot \frac{\left(\frac{m_{debris}}{kg}\right)^{0.67} \cdot \left(\frac{v_{end}}{m/s}\right)^{1.33}}{\frac{d_{projectile}}{m}} = 307 \cdot 10^{-9} \cdot \frac{2.2^{0.67} \cdot 407^{1.33}}{0.0927} = 0.017$$

A formula presented in an AEEW report has been verified for cylinders crashing on pipes [7]. With values listed above, and by setting $d_{i\text{-}target}$ = 0.246 m, the required target thickness may be calculated by the formula in the AEEW report as follows:

$$\frac{t_{required}}{m} = \frac{d_{projectile}}{m} \left( \frac{m_{debris} \cdot v_{end}^2}{16 \cdot 10^9 \frac{N}{m^2} \cdot d_{projectile}^3} \right)^{0.588} \cdot \left( \frac{d_{i\text{-}target}}{d_{projectile}} \right)^{0.294} = 0.0927 \left( \frac{2.2 \cdot 407^2}{16 \cdot 10^9 \cdot 0.0927^3} \right)^{0.588} \cdot \left( \frac{0.246}{0.0927} \right)^{0.294} = 0.015$$

Both formulas produce comparable values for the required target thickness. For debris striking a pipe as described in the above example the required wall thickness is 15 mm. When hitting a steel plate the calculated required wall thickness is 17 mm.

## NOMENCLATURE

| | | |
|---|---|---|
| $A_5$ | = | rupture strain of pipe material, = 35 % |
| E | = | modulus of elasticity of pipe material, = $183 \cdot 10^9$ N/m² |
| $H^0$ | = | molar basis enthalpy of standard state |
| $M_m$ | = | molar mass of gas mixture after explosion, = 15.0743 kg/kmol |
| $M_1$ | = | molar mass of gas mixture before explosion, = 12.9050 kg/kmol |
| Q | = | enthalpy of formation |
| R | = | universal gas constant, = 8314.41 J/(K·kmol) |

| | | |
|---|---|---|
| $R_m$ | = | tensile strength of pipe material, = $391 \cdot 10^6$ N/m² |
| $T$ | = | absolute temperature, in K |
| $T_{after-pressure-volume-as-before}$ | = | temperature of burned gas with same pressure and volume as before ignition, = 531.7 K |
| $T_{Detonation}$ | = | temperature of detonated gas, = 3388 K |
| $V$ | = | volume, in $m^3$ |
| $c_v$ | = | mean specific heat for constant volume of gas, in J/(kg·K) |
| $d_i$ | = | inner diameter of pipe, = 0.1031 m |
| $d_{i-target}$ | = | inner diameter of target pipe, = 0.246 m |
| $d_{projectile}$ | = | diameter of cylindrical projectile, with same circumference as half cylinder, = 0.0927 m |
| $h^0$ | = | mass basis enthalpy of standard state |
| $h^0_{after-pressure-volume-as-before}$ | = | enthalpy of burned gas, at $p_1$ and $T_{after-pressure-volume-as-before}$, = 1103000 J/kg |
| $h_1^0$ | = | mass basis enthalpy of gas mixture before explosion, = 1041000 J/kg |
| $m_{debris}$ | = | mass of half cylinder, = 2.2 kg |
| $m_{gas}$ | = | mass of gas in half cylinder, in kg/m³ |
| $p$ | = | pressure, in MPa, = N/m² |
| $p_{Deflagration}$ | = | deflagration pressure, = 41.1 MPa |
| $p_N$ | = | detonation pressure, = 131.1 MPa |
| $p_1$ | = | pressure before explosion, = 7.06 MPa |
| $q_{i,\ mass-fraction-after}$ | = | mass fraction of gas i, after explosion |
| $r_m$ | = | mean radius of pipe |
| $s$ | = | wall thickness |
| $s_c$ | = | wall thickness of a pipe before expansion, = 0.0051 m |
| $t_{required}$ | = | required thickness of steel target, in m |
| $t_{EL}$ | = | time at end of linear elastic expansion, = $8.099 \cdot 10^{-6}$ s |
| $t_{rupture}$ | = | time at pipe rupture, after ideal plastic expansion, = $132.8 \cdot 10^{-6}$ s |
| $v_{end}$ | = | velocity of half cylinder debris, = 407 m/s |
| $x$ | = | distance of half cylinder debris, in m |
| $\Delta h_{Explosion}$ | = | enthalpy of formation for gas mixture, = - 5682000 J/kg |
| $\Delta h^0$ | = | $h^0_{after-pressure-volume-as-before} - h^0_1$, = 62000 J/kg |
| $\varepsilon$ | = | strain |
| $\vartheta_{after-pressure-volume-as-before}$ | = | temperature of burned gas with same pressure and volume as before ignition, = 258.55 °C |
| $\vartheta_{Deflagration}$, $T_{Deflagration}$ | = | deflagration temperature, = 2820 °C or 3093 K |
| $\vartheta_1$, $T_1$ | = | temperature before explosion, = 182 °C or 355.15 K |
| $\kappa$ | = | specific heat ratio for gas mixture after detonation, = 1.21 |
| $\rho$ | = | density, in kg/m³ |
| $\rho_{Detonation}$ | = | density of detonated gas , = 70.16 kg/m³ |
| $\rho_{steel}$ | = | density of pipe material, = 8000 kg/m³ |
| $\rho_1$ | = | density of gas before explosion, = 24.458 kg/m³ |
| $\sigma_{hoop}$ | = | hoop stress in pipe |

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