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SINGLE-NARY PHILOSOPHY FOR NON-LINEAR STUDY OF MECHANICS OF MATERIALS

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ABSTRACT:
Non-linear study of mechanics of materials is formulated in this paper as a problem of meta-intelligent system analysis. Non-linearity will be singled out as an important concept for understanding of high-order complex systems. Through single-nary thinking, which will be represented in this work, we introduce a modification of Aristotelian philosophy using modal logic and multi-valued logic (these logics we call 'high-order' logic). Next, non-linear cause - effect relations are expressed through non-additive measures and multiple-information aggregation principles based on fuzzy integration. The study of real time behaviors, required experiences and intuition, will be realized using truth measures (non-additive measures) and a procedure for information processing in intelligence levels.

Keywords: Binary philosophy, single-nary philosophy, high-order complex system, truth-variant system, meta-intelligent system, meta-knowledge, truth dimension, high-order logic.

1. INTRODUCTION:

Most approaches in system analysis use the study of the relationship between a system's properties – a system has the property x, then it has the property y. It illustrates a general mechanism which leads from the cause to the effect. It basically means that the total effect is the sum of the effects of the individual causes – the additive law of elementary properties. This general principle is confined in the framework of traditional Aristotelian philosophy (the Greek philosopher 384-322 years B.C.) based on binary relations (two-valued logic): "to be or not to be". It requires idealized assumptions such as independence and equiprobability of evidence, mutual exclusiveness and exhaustiveness of hypotheses. This trend towards simplicity of truth, by true and false, is considered to be the strength of two-valued logic. It allows analysis and computation of a system excluding those parts which are either unimportant, unknown or intractable. However, in the framework of two-valued logic, we have to choose either 'true' of 'false' when we are not sure if they are really true or false. The limit of these binary possibilities has led to uncertainties and many complicated approaches coping with them.

In other approach, a Chinese philosopher named Trang Chau in 369-298 B. C., had a belief which contradicted the true - false view of the world. He stated that true and false are one and undivided – a single-nary philosophy. He advised that we should go out beyond the framework of binary thinking: true - false, black - white etc. in order to better understand the nature of things. By single-nary thinking, which will be represented in this work, we introduce a modification of Aristotle's philosophy using modal logic and multi-valued logic (these logics we call 'high-order' logic). Next, non-linear cause - effect relations are expressed through non-additive measures and multiple-information aggregation principles based on fuzzy integration. In this study, non-linearity will be singled out as an important concept for understanding high-order complex systems. The study of real time behaviors required experiences and intuition, will be realized using truth measures (non-additive measures) and a procedure for information processing in intelligence levels. Here, emphasis is put on a multidisciplinary approach using a single-nary philosophy based on high-order logic (modal logic and multi-valued logic) and fuzzy arithmetic. Non-linear study of mechanics of materials, in this paper, is formulated as a problem of meta-intelligent system analysis.

2. SYSTEM ANALYSIS

The systems approach has, from Ludwig vol Bertalanffy's [2] times, been interested in the integration of the analytic and the synthetic methods to create universal principles for all systems, whether they are physical, chemical, biological, metal or social. A collection of component systems is characterized by the system's '*distinction*' and their interaction with each other is characterized by the system's '*connection*', into a whole that can be seen as a system. The whole of this system is considered as *more than the sum* of its components. To study the relationship between a system's components, different equations:

$$\frac{dQ_i}{dt} = f_i(Q_1, Q_2, ..., Q_n), i = 1,2,..., m \tag{1}$$

are usually constructed. It provides a measure of generality in suggesting approaches of system analysis. However, we have to ignore those aspects of system which are irrelevant to the studied purposes. Then, if we know the precise relations of all elements in the system we should be able to understand how that system functions (reductionism view). The laws of statistics enable us to understand the behavior of a multitude of disorganized complexities. To predict the properties of a system under different conditions, the law of the additivity:

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i); \quad E_i \cap E_j = \varnothing, \quad i \neq j \tag{2}$$

must be invoked. It illustrates a general simplified mechanism with idealized assumptions such as independence and equiprobability of evidence, mutual exclusiveness and exhaustiveness of hypotheses. Today, the study of systems, from soil to aerospace, to micro electromechanical devices, is characterized by a collection of multi-agents, high dimensional properties, nonlinear interactions and unanticipated global behaviors resulting from the interaction of agents, has become recognized as a new interdisciplinary science. The introduction of electrical, electronics and computer science into engineering systems has allowed these systems to adapt. Consider a cantilever beam of length L, for example, with a force P(x, t) applied at a distance x from its fixed end at time t, resulting in a deflection y(x, t) of the beam. It is shown in the following figure:

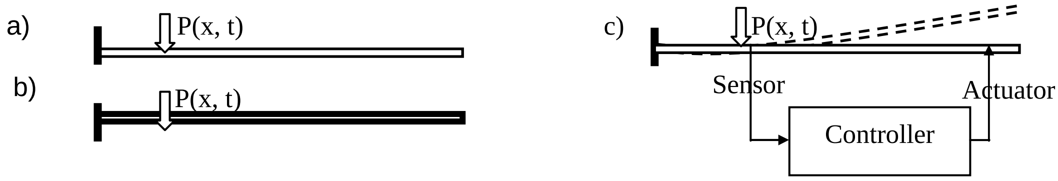


Fig. 1 Vibration control structure (a), passive control (b), active control (c)

Traditional passive control of vibration suppression consists of mounting passive material on the beam (Fig. 1b) i.e. by increasing lateral stiffness through bracing. Active vibration control (Fig. 1c) consists of rather artificially generating a canceling source, processed by the self-turning control algorithm and fed to the actuator in order to destructively interfere with the unwanted source detected by the sensor when an earthquake or other shock occurs. It started with the introduction of optical fibers and piezoelectric materials and shape memory alloys in order to create adaptive systems in robotics, aircraft, large cable bridge, tall buildings, and so on,

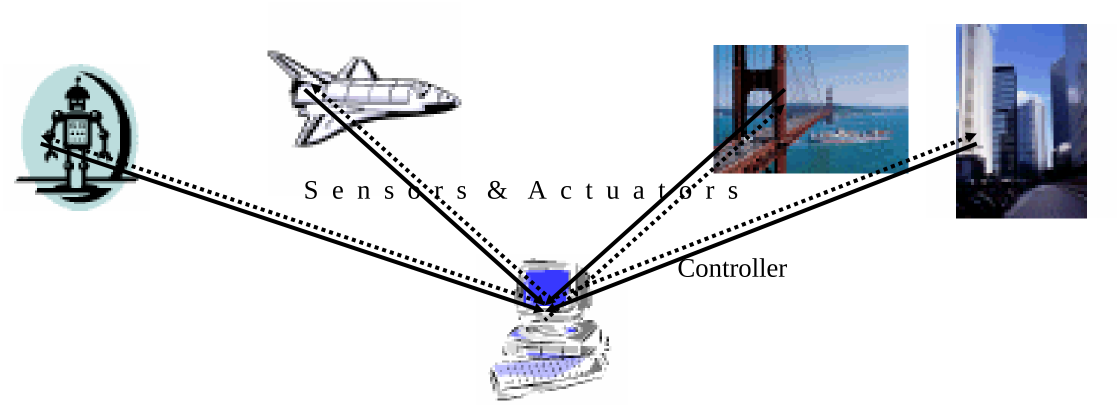


Fig. 2 Smart systems

which called 'smart systems'. The behavior of these systems tends to arise more from the interaction, in real time, among agents than from any constraints imposed onto the structure of whole system. Modern studies in complexity leads to the determination of rules that can modify the system. It generated new kinds of modeling paradigms such as cellular automata, Wolfram [33] using the production rules:

$$\{state(x, y, t) = 1\} \wedge \{state(x, y - 1, t) = 1\} \wedge \{state(x, y + 1, t) = 1\} \wedge \\ \wedge \{state(x', y', t) = 0 \text{ for the other cells}(x', y') \text{ in vicinity}\} \tag{3}$$

or 'if – then' rule for fuzzy systems, (Wang L. & Mendel JM. [30]):

$$R^j: if \{(x_1 = A_1^j) \wedge (x_2 = A_2^j) \wedge ... \wedge (x_n = A_n^j)\}\ then\ (y = B^j) \tag{4}$$

and others resulted from expert experiences. It is, however, very difficult to avoid making contradictory statements when the initial information is incomplete. The complexity of systems increases when their properties cannot be determined precisely by common measurements such as geometrical, physical measures but rather by various types of information with different metrics about vibration, noise evolved in dynamical systems such as airplane, for example, or behaviors of explored equipment in the complex environments such as the Cone Penetration Test (CPT) for example. These information can be determined depending on what is distinguished by the controller, i.e., depend on interaction between controller and controlled system, which creates a more complex system - we call it a high-order complex system/intelligent system.

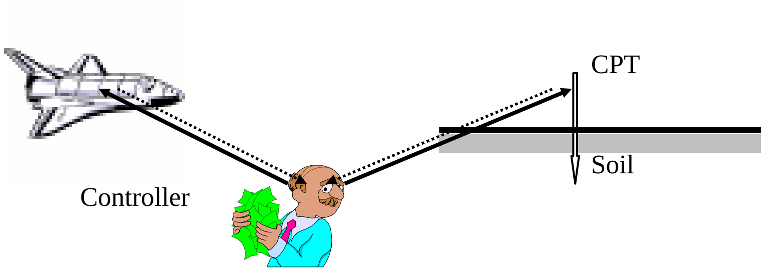


Fig. 3 Intelligent systems: a) aircraft, b) Cone Penetration Test (CPT)

There are principles and laws which can be used as interdisciplinary to unify the understanding of these seemingly disparate types of intelligent systems. Different methods have been imported from mathematics, physics and informative communications. One hope that the speed of computer components and networks allow the representation of great complex systems. It is, however, hardly to verify that a given sequence of controls produces the desired state in the presence of perturbations. Consider the problem of landing an airplane, for example, although the computer's predictions are accurate and the effect of a possible sequence of controls can be determined; to determine, for all possible unpredictable perturbations, a set of controls that lands the aircraft safely is a difficult task. One of the most useful of control techniques is Lyapunov's direct method, in which the mentioned problem is reduced to finding sequences of inputs that guarantee the uniform convergence of suitably chosen scalar functions to a fixed point in the sense of Banach. This equilibrium state to which all other states converge can be seen as an attractor. As non-linear systems have, in general, several attractors. To find an attractor in which the system will end up is difficult because small, unobservable fluctuations can effect the outcome of the process even it is deterministic in principle. The modeling of such non-linear systems in physics has led to the concept of chaos. One hope that random perturbations will help a self-organizing system to find more stable states in its fitness. It requires the power of both computer simulation and intelligent computing. Most popular computing applications derive from ideas originally proposed by cyberneticists, which are distinguished from other, more mechanistic approaches such as control engineering and computer science disciplines through their emphasis on autonomy, self-organization, cognition and the role of the controller in modeling a system. Wiener Norbert et. all [32], for example, developed automatic range finders for antiaircraft guns. It is able to predict the trajectory of an airplane by taking into account the elements of past trajectories and to act with the controller's brain through a negative feedback loop to permit construction of adaptively valuable behavior. Theirs appeared to be 'intelligent' behavior because they dealt with the controller's perception resulting from experience in predicting the future. The creation of a closed loop of information in order to correct any action indicates that the state of an intelligent system may be changed depending on the controller's belief, i.e., *the truth of the information has been changed in time and space*. An emphasis on human perception was also formed by Rosenblueth [22]. His purpose was to approach the study of living organisms from the point of view of a servomechanisms engineer and to consider servomechanisms with the engineer's experience. Generalization of this discovery in terms of the human organism indicates that the study of a complex system is not considered to be a study of a given physical entity, but rather a model of an intelligent system constructed by scientists. It we call 'meta-intelligent' system:

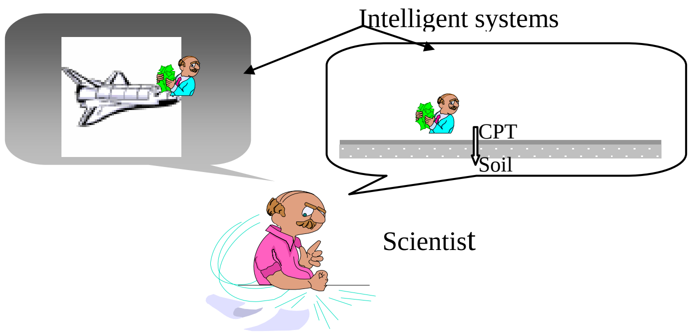


Fig. 4 Meta-intelligent systems.

An intelligent system has two interlocking components: 1) controller's perception/judgments and logic of his/her mind and 2) the structure of the electromechanical systems including their environment in which the controller's mind operates. On the one hand, standard mathematical models of dynamical electromechanical systems consist of a number of variables and their relations to one another determined in a predictable way. Then complexity of a system has been produced in traditional time-space dimension illustrated by "structural", "functional", "structural hierarchical", "functional hierarchical". In this dimension, facts have been treated as true-invariant for all time, no attention has been paid to possible changes in the state of system when new facts become very true or fairly true, and others become false, very false or fairly false. On the other hand, controllers looking at these systems which include their environment, however do not conform to true-invariant view. Their belief may change depending on information processing. Unfortunately, without fundamentals of formal logic for reasoning, a controller might, in many cases, use informal logic. There is usually a contradiction of logical laws underlying the model of a system using mathematical methods and laws underlying the logic of a controller. It may result in a set of incomplete decisions, which may be contrary to solutions resulting from mathematical models. A lack of general principles to combine two types of information (objective and subjective) and human inferences including uncertainties forces us to go beyond the information given in the traditional dimension and to modify the "true or false" philosophy.

3. PHILOSOPHY

Truth measures: Since time immemorial, assigning real numbers to lengths, areas, volumes and time was firstly conceived as a comparison with a standard unit. It was revealed later that measurements must involve infinite sets and infinite processes (see Wang Z. et. all [31]). A probability measure (Kolmogorov [15]), first of all, is a set function that assigns 0 to an empty set and a non-negative number to any other set and that is additive. Although additivity characterizes many types of measurements well under ideal, error-free conditions, it is not fully adequate to characterize most measurements under real conditions, when measurement errors are unavoidable (see Viertl [28]). In an experimental argument, a student laboratory test of the filter factor (k) of soil, was conducted at the University WM of Olsztyn, Poland. Consider, for example, two unconnected events, A and B shown in the Fig. (5) (water-output measured in the time (T) of 30 second: Q_{30}) and adjoined event $A \cup B$ (water-output measured in the time (T) of 60 second: Q_{60}).

$$\overleftrightarrow{\quad A \quad} + \overleftrightarrow{\quad B \quad} \quad = \quad Q_{30} + Q_{30} = Q^{I}_{60}$$

$$\overleftrightarrow{\qquad A \cup B \qquad} \quad = \quad Q^{II}_{60}$$

Fig. 5. An example illustrating the violation of the additive axiom of probability theory

As a result, the typical value of the relative difference of filter factor: $\Delta\xi = |k^{I} - k^{II}| / ((k^{I} + k^{II})/2) \approx 3\%$; $k^{j} = Q^{j} / (i\ A\ T)$, where, i denotes hydraulic gradient of water, A denotes cross-section surface of soil sample, $j = I, II$. This indicates that the same observations produce more evidence for the single event $A \cup B$ than for the two unconnected events $A + B$. Hence, the degree of belief in $A \cup B$ (probability of $A \cup B$) should be rather than the sum of the degrees of belief in A and B (probabilities of A and B). That is

$$P(A \cup B) \neq P(A) + P(B) \tag{5}$$

i.e., the additive axiom is violated. In addition, human measurement and measurements involving non-repeatable experiments are intrinsically non-additive. Hence, many non-additive measurements have been created. The first non-additive measures were originated by Dempster [10] and Shafer [24] which are called belief measure, Bel(.) - super-additive measures:

$$Bel\left(\bigcup_{i=1}^{n} E_i\right) \geq \sum_{I\subset\{1,2,\ldots n\}, I\neq\varnothing} (-1)^{|I|+1} Bel\left(\bigcap_{i\in I} E_i\right), \quad E_i \subset X \tag{6}$$

and Pl(.) - sub-additive measures:

$$Pl\left(\bigcap_{i=1}^{n} E_i\right) \leq \sum_{I\subset\{1,2,\ldots n\}, I\neq\varnothing} (-1)^{|I|+1} Pl\left(\bigcup_{i\in I} E_i\right), \quad E_i \subset X \tag{7}$$

Other measures proposed by Zadeh [34], Dubois and Prade [11] in possibility theory are called: necessity measures, Nec(.) and possibility measures, Pos(.) and are presented as:

$$Nec\left(\bigcap_{i\in I} E_i\right) = \inf_{i\in I} Nec(E_i); \quad Pos\left(\bigcup_{i\in I} E_i\right) = \sup_{i\in I}(E_i) \tag{8}$$

and other theories, such as fuzzy set theory, rough set theory etc. have been created. These represent an important point in the evolution of the modern concept of subjective element which was presented by American philosopher Max Black (1937) [4]. On the one hand, belief and plausibility measurements in the theory of evidence have a natural interpretation as lower and upper probabilities, respectively. Intervals between belief and plausibility values may be viewed as ranges of admissible probabilities. On the other hand, necessity measures, Nec(.) and possibility measures, Pos(.) have emerged from the context of fuzzy sets and the context of evidence theory. There are also connections between these measures and modal truth values:

$$Nec(E) = 1 - Pos(\overline{E}) \text{ corresponding to } \Box E = \neg\Diamond\neg E \text{ (in modal logic)} \tag{9}$$

$$Pos(E) = 1 - Nec(\overline{E}) \text{ corresponding to } \Diamond E = \neg\Box\neg E \text{ (in modal logic)} \tag{10}$$

It suggests that we can use truth as a new non-additive measurement based on modal logic to represent the degree of human belief. In fact, we recognize the real world through our perception measured, first of all, by discrete values of truth. Primitive man described discrete values of truth, 'the being' and 'the non-being' through the perceptible flame. Later, binary philosophy - a syllogism distinguishing any things existing and non-existing in respect to the real world was discovered by Aristotle (the Greek philosopher 384-322 years B.C.). He described it in his metaphysics: *"…it is possible for anything, at the same time to be or not to be"* (see Ross [21], Costescu [7]). Or, according to Heraclitus [13]. *"It is impossible to stop twice in the same river"*, which can be represented logically as follows:

$$(x \in X) \vee (x \notin X) \tag{11}$$

Since civilization began, numbers, 1 = 'the being' and 0 = 'the not being', have been regarded as a symbol of truth which characterizes our logical reasoning. Hence, Boolean algebra invented by George Boole [5] and set theory today both play important roles in the footsteps of mathematics and physics. Hence, the famous classical law of the excluded middle was invented:

$$p \vee \neg p = 1; \quad p \in \{0 \;\; or \;\; 1\} \tag{12}$$

It is so clear as rives can't travel in an opposite direction to their sources and this has become a basic law in science and technology today. Any logical function $\nu(p_1, p_2 ..., p_m)$; (p_i, i = 1, 2, ... m are logic variables) represents a mapping

$$\Re_n \text{ x } \Re_n \text{ x...x } \Re_n \rightarrow \Re_n \tag{13}$$

where n = 2 denotes two-valued logic. Then, each variable with two coordinates (x, y) with their truth values defined through any logic function will be determined in a linear plane, which is shown in Fig. 6:

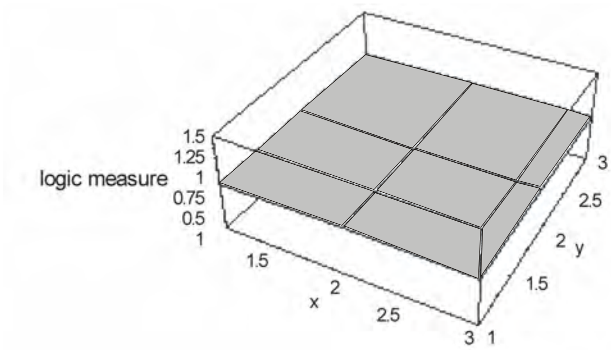


Fig. 6 A truth linear plane in two-valued logic

When truth-values 0 or 1 have been attached to all of variables, a mathematical model has been formulated under the assumption that all of the data and the structure of the study problem are uniquely determined with true-invariant for all time, then the truth dimension disappears completely. In practice, however, the human perception of surrounding events rarely takes place through 'true' or 'false' values. Even Aristotle himself, who developed the theory of the syllogism based on true or false philosophy in the perfect form, also worked on a theory of modal syllogism, in which premises and conclusions may contain the terms "*necessary*, $\Box$" and '*possible*, $\Diamond$' (see Boolos [6]). Later, modal logic was introduced as formalized deductive systems of proposition calculus by Lewis and Langford [17], in which two readings, $^{+}\Box$ and $^{-}\Box$ are intended as kinds of universal quantifiers over time, i.e. truth may be changed depending on time. Philosophical aspects of this logic are discussed in the same book and in the book written by Beth [3]. Modal logic, in fact, 'four-valued' ($\Box, \neg\Box, \Diamond, \neg\Diamond$) logic is a bridge between two-valued logic and multi-valued logic. For example: let return to the early '*black or white*' reasoning beginning with Aristotle's syllogistic, i.e. using the two symbols: T (true) and F (false), we represent 'black' by two possibilities ($B, \neg B$) and '*white'* by, ($W, \neg W$). We can obtain 4 possible cases:

$$\text{'black or white'} = \{B\,W,\ B\,\neg W,\ \neg B\,W,\ \neg B\,\neg W\} \tag{14}$$

Now, from modal logic using box connective, $\Box$ - 'necessary', and diamond connective, $\Diamond$ - 'possible', we can represent '*black*' by 4 symbols, ($\Box B, \Diamond B, \neg\Box B, \neg\Diamond B$); '*white*' by ($\Box W, \Diamond W, \neg\Box W, \neg\Diamond W$). We can obtain:

$$\begin{aligned}\text{'black or white'} = \{&\Box B\,\Box W,\ \Box B\,\Diamond W,\ \Box B\,\neg\Box W,\ \Box B\,\neg\Diamond W,\ \Diamond B\,\Box W,\ \Diamond B\,\Diamond W,\ \Diamond B\,\neg\Box W,\ \Diamond B\,\neg\Diamond W,\\ &\neg\Box B\,\Box W,\ \neg\Box B\,\Diamond W,\ \neg\Box B\,\neg\Box W,\ \neg\Box\,\neg\Diamond W,\ \neg\Diamond B\,\Box W,\ \neg\Diamond B\,\Diamond W,\ \neg\Diamond B\,\neg\Box W,\ \neg\Diamond B\,\neg\Diamond W\}\end{aligned} \tag{15}$$

Thus, many possible cases, which may be more practical, may be lost in reasoning based on two-valued logic. It supports the effect of truth measures of 'four-valued' logic. In fact, the basic assumption upon which two-valued logic is based has been questioned since Łukasiewicz (1920) [18], [19]. Propositions about future events, he maintains, are neither actually true nor actually false, as their truth-value is undetermined. Then, several *n*-valued logics were developed in the 1930. The classical law of the excluded middle becomes:

$$p \vee \neg p = \tau; \quad (\tau, p) \in [0,1] \tag{16}$$

The truth measure on (X, $\aleph$) is a mapping:

$$\tau: \aleph \to [0, 1]. \tag{17}$$

in which X be a nonempty and finite set, and $\aleph$ a non-empty class of subsets of X. It exactly represents the amount of the controller's belief contained in his/her perception of a real system. They satisfy the requirements:

a) $\tau(\emptyset) = 0$ and $\tau(X) = 1$ (boundary requirements). Here, the empty set does not contain any element, obviously, it cannot contain the element of our interest. The finite set X containing all elements under consideration must contain our element as well.

b) $E \in \aleph$, $F \in \aleph$ and $E \subset F$ imply $\tau(E) \le \tau(F)$ - when we know: any element belongs to a set, then our belief that it belongs to a larger set containing the former set can be greater or equal, but can't be smaller.

The truth measure with a monotonic condition (b), instead of the additive condition, is considered to be a non-additive measure. Thus, each variable determined by two coordinates *x*, *y* that their truth values are defined through any logic function will be determined in a non-linear plane which is shown in the Fig. 7. Here, truth and space are intimately linked like body and soul in a non-linear phenomenal.

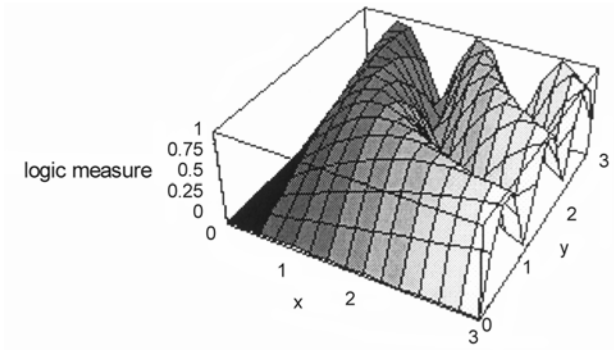


Fig. 7 A truth non-linear plane in a truth-space dimension

It may express, for example, the changing of the controller's belief in the closed loop of information discovered by Wiener Norber, changing of optimisation of the truth dimension presented by Chi Tran [8], [9]. Consequently, each point A determined traditionally by (x_A, y_A) will be expressed by:

$$A = \{x_A \mid \tau,\ \ y_A \mid \tau\},\ \ \tau \in [0,\ 1] \tag{18}$$

We can use a special linguistic term called linguistic hedges (or simply hedges) generated in fuzzy logic, by which other linguistic terms are modified, to modify fuzzy estimates, e.g.,

"ϕ is big is true", "ϕ is big is very true", "ϕ is big is fairly true" (19)

Any hedges "true", "very true" or "fairly true" are interpreted as unary operations:

$$True(\tau) = \tau,\ very\ true(\tau) = \tau^2,\ fairly\ true(\tau) = \tau^{1/2,}\ \tau \in [0, 1]. \tag{20}$$

From mapping (13) with n = 2, for m logic variables or functions we have:

$$\Re_2^{\,1} \text{ x } \Re_2^{\,2} \text{ x...x } \Re_2^{\,m} \rightarrow \Re_2 \tag{21}$$

for m = 1 there are (2^z, $z = 2^1$) = 4 truth-functions and for m = 2 there are (2^z, $z = 2^2$) = 16 different functions. Generally, in n-valued logic for m arguments we have $\Im$ truth-functions which are obtained from:

$$\Im = n^z,\ z = n^m \tag{22}$$

All of these functions (in the binary logic), for example, are shown in Table 1.

Table 1 Truth-functions in two-valued logic with one (p) and two arguments (p, q)

p	I	¬	1	0	\	p	q	⇔	⊕	∧	∨	→	←	\	/	↑	↓	⊥	T	-|	|-	1	0
0	0	1	1	0	/	0	0	1	0	0	0	1	1	0	0	1	1	0	1	0	1	1	0
1	1	0	1	0	\	0	1	0	1	0	1	1	0	0	1	0	1	0	1	1	0	1	0
					/	1	0	0	1	0	1	0	1	1	0	0	1	1	0	0	1	1	0
					\	1	1	1	0	1	1	1	1	0	0	0	0	1	0	1	0	1	0

With n = 3, m = 2, we have (3^z, $z = 3^2$) = 19638 possible functions. When the human perceptions are quantified in the terms of truth values, the flexibility and the wealth of logic structures of multi-valued logic form the basis of the human thinking based on single-nary philosophy.

Single-nary philosophy: In spite of the constant research of scientists, the additive principle is still being applied, more and less, to information combinations. The additive model , for example,

$$E_w = \sum_{i=1}^{n} w_i f(x_i) \tag{23}$$

requires that the weights, $0 \le w_i \le 1$, i = 1, 2, …, n, and $w_1 + w_2 + \ldots + w_n = 1$ corresponding to the individual factors be given before the evaluation is made; $\{f(x_i)\}$, i = 1, 2, …, n denote scores given by an expert. It is based on the implicit assumption that the factors $x_1, x_2, \ldots, x_n$ are independent of one another, their effects are additive. Although this model simplifies the complexity of any system, its additivity supports reasoning which is not completely based on a ‘true or false’ philosophy. In fact, since Aristotle’s times, the Chinese philosopher (see [27]) Trang Chau (369-298 BC), had developed an alternative to the binary idea of Confucianism; against the “the being or the non-being” or "true or false" confirmation. In his philosophy, true contains an amount of false and false contains an amount of true; they have common sense which is subjective and approximate in nature. Thus, true and false are united and undivided in the term of truth, that is to say – a single-nary philosophy which is represented graphically as:

a. b.

Fig. 8 Binary philosophy (a) and single-nary philosophy (b)

According to the most important idea of Buddhism:

辨不辨 不辨辨

i.e., "discrimination non-discrimination, non-discrimination discrimination", the single-nary philosophy of Trang Chau can be represented logically by *"to be AND not to be"* as:

$$(x \in X) \wedge (x \notin X) \tag{24}$$

This deep philosophical paradox was described by D. T. Suzuki [26], [27]. According to Trang Chau’s philosophy we need:

„overpass”

a) 知忘是非 b) 大知

i.e., (a) knowledge, which is beyond the knowledge resulting from "the being or the non-being" thinking, called "great knowledge" (b) or “Bat Nha” according to Buddhism. Here, the term “overpass” (i.e., go beyond) supports, in the mind, the sense that this meta-knowledge/"great knowledge" is evolved from “being or non-being” knowledge. That is, we should first understand binary philosophy in order to understand single-nary philosophy – we can't just go beyond nothing. He claimed that, for using single-nary philosophy, we need meta-knowledge, which represents the human perception resulting from the physical processes inside someone’s head. It is characterized today by engineering experiences. The problem of "car packing into place between two cars" is an example of using meta-knowledge rather than knowledge resulted from the laws of mathematics. It may take place, for example, in the closed loop of information of Wiener et al.’s example in order to create an adaptive behavior. In this paper, meta-knowledge is quantified by physical measurements in the truth dimension using truth values, τ, of multi-valued logic. More information about meta-knowledge and how we can determine it will be represented in other papers. In fact, the semantics of modal logic presented by S. Kripke (see Boolos [6]), in which Leibniz's fantasy of the actual world seems as one "possible world" among others, represents Trang Chau's idea through the following triple formula:

$$K = \{W, R, \Phi\}; \quad R \subset W^2$$
$$\bigwedge_{w_i, w_j \subset W} \Phi(A, w_i) = true \;\; iff \;\; \{[\Phi(A, w_j) = true] : \forall w_j (w_i R w_j)\}, \;\; i, j \in \{1, 2, \ldots, n\}, i \neq j \tag{25}$$

This means that sentence A is necessary true in the world w_i if it is possible in every possible world w_j so that $w_i R w_j$; where, $w_i R w_j$ denotes: w_j is possible on account of w_i; W denotes the set of possible worlds; a relation Φ

between worlds and sentence letters specifying which sentence letters are true in which worlds. Thus, we can say A is possible to be in $w_j \rightarrow$ not to be in w_i, i.e, A is possible to be and not to be in w_i. Here, we must understand "to be" and "not to be" in the sense of 'possible to be'. Thus, we have: *it is possible for anything, at the same time to be AND not to be*, which exactly represents Trang Chau's single-nary philosophy. It indicates that we are allowed to reason in the framework of this logic about facts that are believed to be true by one expert and false or possibly true by another, in which the commitment of belief to a subset does not force the remaining belief to be committed to its complement. This is shown by:

$$E \in W, F \in W \text{ and } E \subset F \text{ imply } \tau(E) + \tau(F - E) \leq 1 \tag{26}$$

This non-additive set function τ is seen as an important framework of meta-knowledge. It is clearly illustrated by the decision-making based on the expert's experiences as follows: suppose we have to make decision A2 when x = x1, i.e., x1 $\in$ G $\Rightarrow$ A2 or A1 when x = x2 = 10.5, (x2 >> 1.5) i.e., x2 $\notin$ G $\Rightarrow$ A1. When x = x3 = 1.6, which is shown in the following figure:

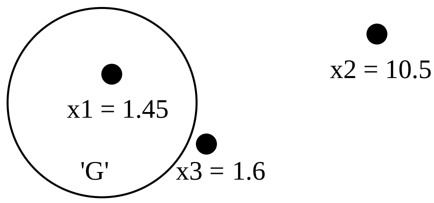


Fig. 9 Decision-making depending on fuzzy conditions

By the way, based on reasoning in the framework of two-valued logic we have: let *p*: x $\in$ G, $\overline{p}$: x $\notin$ G, deduction process based on binary philosophy is represented as function 27, in which, the Δ notation on the right indicates that the associated clauses are in the initial database, and the numbers indicate the clauses from which the associated clause is derived.

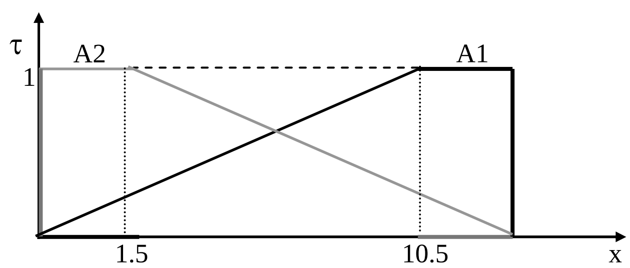
$$\begin{cases} 1.\ p & \Delta \\ 2.\ p \rightarrow A2 & \Delta \\ 3.\ \overline{p} \rightarrow A2 & \Delta \\ 4.\ A2 & (1,2) \\ 5.\ \{\overline{p}\} & (3,4) \\ 6.\ \{.\} & (1,5) \end{cases} \tag{27}$$

That is, resolving two singleton clauses (1, 5) leads to the empty clause (6); i.e. the clause consisting of no literal at all. The derivation of the empty clause means that the database contains a contraction. In the other way, decisions resulted from expert's experiences i.e., meta-knowledge are represented by two fuzzy numbers, we call perceptual information, as follows:

A2: x|τ = {≤1.5|1.0, 1.6|0.90, ..., 3.5|0.45, ..., 8.0|0.25, ≥ 10.5|0.0}

A1: x|τ = {≤1.5|0.0, 1.6|0.05, ..., 3.5|0.45, ..., 8.0|0.65, ≥ 10.5|1.0}

Here, both 'true' (1) and 'false' (0) contained in each single decision (A2 for both x $\in$ G and x $\notin$ G) express what we mean - meta-knowledge is based on single-nary philosophy. In which, τ, represents the degree of match of different values of *x* to response alternatives. In which, $\tau(x3 \in G) + \tau(x3 \notin G) = 0.90 + 0.05 = 0.95 \leq 1$, $x3 = 1.6$ represents its non-additive character. Fuzzy decisions are represented graphically as follows:



Rys.10 The fuzzy decisions in the truth-space dimension

Generally, fuzzy arithmetic operations (see [14]) on fuzzy numbers, A and B, are presented by:

$$(A*B)(z) = \sup_{z=x*y} \min\lfloor A(x),\quad B(y)\rfloor \tag{28}$$

where, * denotes any of the four basic arithmetic operations (+, -, ., /). Thus, we can say that truth-time-space dimension represents an hypothesis space that consists of a set of non-mutually exclusive hypotheses. It helps us to represent the human experience - meta-knowledge without contradiction. That is to say, in this case, 'going in the opposite direction to their sources is impossible for outgoing rivers but it is possible for an outgoing man'. It is regarded as a symbol of Trang Chau's reason of single-nary thinking in respect to binary philosophy.

4. INFORMATION-PROCESSING BASED ON SINGLE-NARY PHILOSOPHY

Modelling of an intelligent system – a meta-intelligent system – represents a model of human information-processing in order to overpass binary thinking, in which both knowledge based on binary philosophy and meta-knowledge based on single-nary philosophy are taken into account. This mental model is described by two temporally overlapping stages of information processing: evaluation and integration. During evaluation, the degree of support of a particular feature for each relevant alternative is expressed in terms of non-additive measures. It has a term with the combination of all elementary truth measures treated as fuzzy measures multiplied by a factor λ:

$$\tau(E \cup F) = \tau(E) + \tau(F) + \lambda.\tau(E)\tau(F), \quad \lambda > -1 \tag{29}$$

as proposed by Sugeno [25], where, λ has an effect similar to a weight factor for interaction between different properties. Fuzzy measures satisfying mentioned condition is called λ-fuzzy measure. While $\lambda = 0$, τ can be used as an additive measure (probability measure). For a set of elements E_i, $E_i \in X$, the relationship (29) can be used recursively:

$$\tau\left(\bigcup_{i=1}^{n} E_i\right) = \frac{1}{\lambda}\left\{\prod_{i=1}^{n}\left[1+\lambda.\tau(E_i)\right]-1\right\}; \lambda \neq 0 \tag{30}$$

As, $\tau(X) = 1$, when, $\bigcup_{i=1}^{n} E_i = X$ we have:

$$\tau\left(\bigcup_{i=1}^{n} E_i\right) = 1 = \frac{1}{\lambda}\left\{\prod_{i=1}^{n}\left[1+\lambda.\tau(E_i)\right]-1\right\}; \lambda \neq 0 \tag{31}$$

Thus, the parameter λ should be obtained by solving the following equation:

$$1+\lambda = \left\{\prod_{i=1}^{n}\left[1+\lambda.\tau(E_i)\right]\right\}; \ \lambda \in (-1, \infty) \text{ and } \lambda \neq 0 \tag{32}$$

Integration of information is performed by the fuzzy integral/Sugeno integral as follows: Let (X, $\wp$) be a measurable space, where X $\in \wp$; $\wp$ is a σ-algebra of sets in the class of all finite subsets of X. A real-valued function f: $X \to (-\infty, \infty)$ on X is called as a measurable function if for any Borel set B:

$$f^{-1}(B) = \{x \mid f(x) \in B\} \in \wp \tag{33}$$

The functional relationship between measurable function, f, and fuzzy measure, τ, is represented by Sugeno's integral. Let $X \in \wp$, $f \in \mathbf{F}$, $\mathbf{F}$ is the class of all finite nonnegative measurable functions defined on (X, $\wp$). The fuzzy integral of $f(x)$ on X with respect to τ, which is denoted by:

$$⨍ f(x)d\tau = \sup_{\alpha\in[0,\infty]} [\alpha \wedge \tau(X \cap \mathbf{F}_\alpha); \ \ \mathbf{F}_\alpha = \{x \mid f(x) \geq \alpha\}] \tag{34}$$

where, $\mathbf{F}_\alpha$ is called an α-cut of f(.); α is the threshold where the assumption if fulfilled. Generally, for m experts, the synthetic evaluation, E_m, is given by:

$$E_m = \int \frac{1}{m} \sum_{j=1}^{m} f_j(x) d\tau \tag{35}$$

5. DETERMINING RELATIVE, D_r, OF SANDS USING CPT DATA

Empirical correlations between the relative density of sands (D_r) and cone-tip resistance (q_c) from cone penetration test (CPT) data considering the effective overburden stress (σ'_v) and soil compressibility are available in the literature. As no single correlation seems to be able to predict correctly D_r for all sands. For example, the correlation proposed by Vilet and Mitchell [29] is applicable to sands with low compressibility; Schmertmann's correlation [23] is applicable to sands with high compressibility and the correlation of Baldi et al. [1] was developed for sands of medium compressibility. A general relationship, D_r - q_c, used in this study was established by Kulhawy et al. [16] based on a database of 24 sands. It is

$$D_r^2 = \left(\frac{1}{Q_F}\right)\left[\frac{q_c / p_a}{(\sigma'_v / p_a)^{0.5}}\right] \tag{36}$$

where, p_a denotes atmospheric pressure (about 100 kPa); Q_F is an empirical constant determined by least-square regression analyses with values of 332, 305 and 278 for normally consolidated (NC) sands of low (r^L = 0), medium (r^M = 0.5) and high (r^H = 1.0) compressibility, respectively. To characterize the sand compressibility, the friction ratio, r:

$$r = \frac{f_s}{q_c} \ [\%] \tag{37}$$

where, f_s denotes the sleeve friction, is used in this study. CPT data used for determining relative density are listed in the following table:

Table 2 CPT data

CPT number	Depth [m]	σ'_v [kPa]	q_c [kPa]	f_s [kPa]
12	6.0	81.0	5030	3

There is a problem in determining D_r for the actual value r_a which is not equal to the standard values (r^L, r^M, r^H) and given new information of soil that we can't mathematically determine its effect on D_r. For example, r_a = 0.06 and information about stress history, mineral type, particle angularity, particle size, particle surface roughness and others.

Traditional method: According to Robertson and Campanella [20], for most normally consolidated (NC) sands, we have the predefined values of r, ($r_L \approx 0\%$, $r_M \approx 0.5\%$, $r_H \approx 1\%$). When the actual values of the friction ratio, r_a = 0.06, it is regarded as r_L (or $r = 0.06 \in r_L$) which is shown in the following figure:

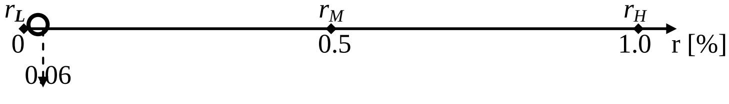


Fig. 11 Three predefined values (r_L, r_M, r_H) represented three compressibility qualifiers and the two actual values of the friction ratio (r_a = 0.06)

According to binary philosophy, the actual value of the friction ratio, r_a, represents either low or medium or high level of compressibility of sands. That is:

$$r \in \{r_L \ \vee \ r_M \ \vee \ r_H\} \tag{38}$$

sand with r_a = 0.06, is considered as sand with low compressibility. Using eq. 36 with Q_F = 332 we can obtain the value of D_r.

A new approach based on fuzzy set theory: Since compressibility is dependent on stress level and mode of shearing, sands with the same mineral type could be placed in different categories of compressibility depending on D_r and stress history and other factors, which are generally qualitative based on an expert's experience. To represent the fuzziness of expert's experience, three fuzzy numbers proposed by Juang et all [12] for three compressibility qualifiers (low, medium and high) are shown in the figure:

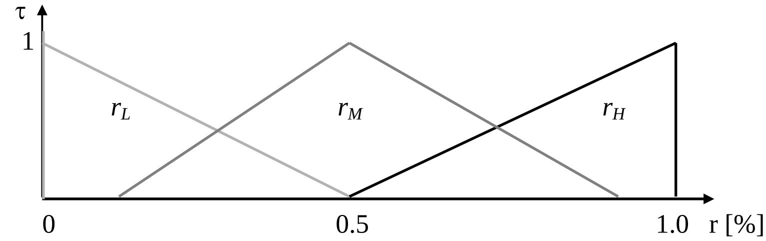


Fig. 12 Fuzzy numbers that represent different levels of sand compressibility

Generally, for each value *r* we have (according to fuzzy philosophy):

$$r \in \{ r_L \ \vee \ (r_L \wedge r_M) \ \vee \ (r_M \wedge r_H) \ \vee \ r_H \} \tag{39}$$

To determine D_r, a weighted aggregation technique using additive model is developed by Juang et all (1996) [12] to combine three base correlations:

$$D_r = D_r^L w^L + D_r^M w^M + D_r^H w^H \tag{40}$$

where, D_r^k, $k = L, M, H$ are relative densities, defined by eq.36; w^k, denotes weights which are determined based on a "similarity" measure of three predefined levels of compressibility. This technique is based on the implicit assumption that the effects of the three compressibility levels (L, M, H) are viewed as additive $\{w^L + w^M + w^H = 1$ and $0 \le w^k \le 1\}$.

*New approach based on single-nary philosophy***.** Firstly, three fuzzy numbers (non-linear type) that represent different levels of sand compressibility have been proposed which are generally presented as follows:

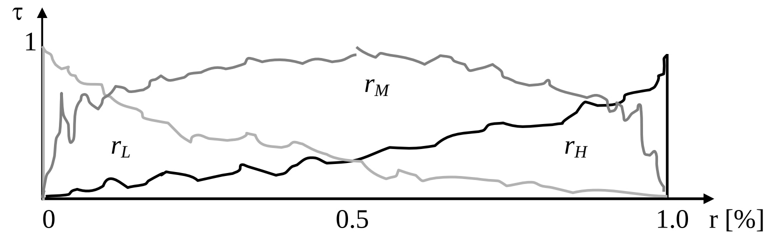


Fig. 13 Fuzzy numbers that represent three predefined levels of sand compressibility

i.e., according to single-nary philosophy, each value of the friction ratio, *r*, represents both low and medium and high level of the compressibility of sands according to different values of truth. It is represented in the form:

$$r \in \{ r_L \ \wedge \ r_M \ \wedge \ r_H \} \tag{41}$$

The measure of "difference" of the actual value r_a in respect of the predefined numbers, r_k, k = L, M, H is defined as follows:

$$diff_{r_a}(k) = \left| r_a - r_k \right| \tag{42}$$

It expresses how close the actual friction ratio, r_a, is to both three predefined numbers, r_k, k = L, M, H. A smaller distance indicates a higher degree of similarity. The compressibility measured by friction ratio corresponding to a higher similarity is assigned a greater value of truth, i.e.,

$$\tau(k) = 1 - diff_{r_a}(k), \text{ k = L, M, H} \tag{43}$$

Sand, which is considered as sand having a compressibility level k, k = L, M, H is assigned the truth value τ(k). Next, we can modify these estimates by using different hedges resulting from our perception. Using "true" hedges, for example, the difference of the actual friction ratio, r_a = 0.06%, in comparison with predefined numbers r_k, for different levels of compressibility is determined using eq.42. The truth, τ(k), assigned for studied sand that is considered as sand with the compressibility levels L, M, H respectively, is determined using eq.43:

$$\tau(\mathrm{k}) = \{0.94, 0.56, 0.06\}, \ \mathrm{k} = \mathrm{L, M, H} \tag{44}$$

e.g. the sand with r_a = 0.06 is considered as sand with low compressibility with the assigned truth: τ(L) = 0.94; medium compressibility with τ(M) = 0.56 and high compressibility with τ(H) = 0.06. As sands with the same mineral type could be under different categories of compressibility depending on new information obtained (stress history, mineral type, particle angularity, particle size, particle surface roughness and others), which are generally descriptive and not readily applicable for mathematically quantifying their compressibility. In this case, expert's evaluations using meta-knowledge are needed. This is supported, for example, by the evaluations of three experts, τ^*(k):

Table 3 Expert's evaluations (initial meta-knowledge)

Expert	τ^*(L) -	τ^*(M) -	τ^*(H) -
1	0.8	0.3	0.1
2	0.8	0.5	0.1
3	0.8	0.3	0.2

(We will represent, how we can determine initial meta-knowledge, in detail in other paper). From this, we can construct a λ-fuzzy measure for all of other subsets of set X, $X = \{L \cup M \cup H\}$. Then, the λ-fuzzy measures for different subsets $\{(L \cup M), (L \cup H)$ and $(M \cup H)\}$ are defined by eq. 32 and the truth of these subsets {τ(L∪M), τ(L∪H) and τ(M∪H)} are defined by eq.29. Next, the value D_r for the studied sand with the actual friction ratio, r_a, is calculated using the Sugeno integral with $\alpha = \{D_r^L, D_r^M, D_r^H\}$, where, D_r^L, D_r^M, D_r^H are determined by eq.36 for sands of low, medium, and high compressibility respectively. Finally,

$$D_r^F = \oint f d\tau = \left[D_r^L \wedge \tau\left(X \cap F_{D_r^L}\right)\right] \vee \left[D_r^M \wedge \tau\left(X \cap F_{D_r^M}\right)\right] \vee \left[D_r^H \wedge \tau\left(X \cap F_{D_r^H}\right)\right] \tag{45}$$

where, '$\wedge$' and '$\vee$' denote 'min' and 'max' operations respectively. This fuzzy integral differs from the above weighted aggregation operator in that both objective evidence supplied by various sources $\{D_r^L, D_r^M, D_r^H\}$ and the expected worth resulted from the expert's perception of subsets of these sources:

$$\left\{\tau\left(X \cap F_{D_r^L}\right), \tau\left(X \cap F_{D_r^M}\right), \tau\left(X \cap F_{D_r^H}\right)\right\}$$

are considered in the aggregated process. The obtained values, D_r^F, are shown in the following table:

Table 4 Results according to expert's evaluations using meta-knowledge.

Expert	$\tau^*(L)$	$\tau^*(M)$	$\tau^*(H)$	$\tau(L \cup M)$	$\tau(L \cup H)$	$\tau(M \cup H)$	D_r^F
1	0.80	0.30	0.10	0.95	0.85	0.38	0.41
2	0.80	0.50	0.10	0.98	0.84	0.56	0.428
3	0.80	0.30	0.20	0.93	0.88	0.46	0.428

To reduce the influence of subjective biases of the individual experts and to obtain a more reasonable evaluation, D_r^*, using an arithmetic average we can obtain:

$$D_r^* = \overline{D_r^F} = \frac{1}{3}(0.41 + 0.428 + 0.428) = 0.422$$

The complete results are listed in the following table

Tab. 5 Final results D_r^* and results obtained from different methods

D_r^L [%]	D_r^M [%]	D_r^H [%]	D_r^J [%]	D_r^* [%]
41.0	42.8	44.8	41.0	42.2

Summary of case study: The predicted values: D_r^k, $k = L, M, H$ are calculated based on a 'true or false' philosophy using a set of three compressibility levels of low, medium and high compressibility respectively according to the values of the friction ratio (r_L, r_M, r_H). Value D_r^J is calculated based on a mixed method (fuzzy philosophy), using both the weighted mean and expert evaluations. However, the equality, $D_r^J = D_r^L$, for r_a = 0.06 indicates that these results are closely dependent on the friction ratio r_L ($r_a = 0.06 \approx 0 = r_L$) without the effect of new soil information (assuming a truth-invariant system). It is the disadvantage of this method. Result D_r^* evolved (overpass) from D_r^k is based on a single-nary philosophy using assumptions of the truth-variant system in which both new information of mineral types of sand and the beliefs of different experts are taken into account.

6. CONCLUSION

We have lived for a long time believing that ‘true or false’ philosophy governs science and technology. The success of science and technology based on this traditional philosophy in the time-space framework today are so clear that we can hardly recognize the need for the flexibility of a single-nary philosophy in the truth-time-space framework for meta-intelligent systems. Without this philosophy, we have to develop either a linearization process, which is a two-edged and dangerous sword that we should have learned from examples in the history of science and technology, or many sophisticated theories and complex procedures which require special knowledge and experience such as probability theory, evidence theory, possibility theory and others.

The success of the engineering approach is, in fact, evident from the history of technology and the applied sciences. It reflects the correctness of the logical principles of engineers in relation to the real world. It indicates that we might shift to an intermediate mode of work that concentrates on studying of the truth-variant states of a system using meta-knowledge rather than on mechanisms of the mind in the framework of two-valued logic. It is vital for a better understanding of Bertalanffy’s idea of “more than of sum” and has significant potential for the development of research capabilities in the longer term of meta-intelligent systems. It is important to note that we can, in addition, create a learning process through interaction between meta-knowledge and databases in order to improve the controller’s experiences and to learn his/her adaptive behaviour. Although this approach leads to expensive computing processes, comprehensive techniques of distributed computing/parallel computing using meta-computers are available today to support it.

Finally, in the modelling of intelligent systems – meta-intelligent systems we are, at the moment, confined to the framework of ‘true OR false’ philosophy. It is only a matter of habit, which we should change through ‘true AND false’ philosophy using meta-knowledge in order to modify our ways of thinking in the future for better understanding and modelling of human intelligence in order to better understand non-linear nature of any material.

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