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VIBRATION CONTROL OF NUCLEAR REACTOR CORE

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ABSTRACT
A set of hyperbolic equations is given to describe the vibration of the nuclear reactor core. The vibration propagates in the core in the form of the traveling wave. According to the minimum of the traveling wave energy, an active control law is given.

Keywords: vibration, fuel bundle, active control

1. INTRODUCTION

Nuclear reactor core is composed of a large number of fuel rods, which are immersed in an acoustic fluid. The core or bundle can be regarded as a heterogeneous medium of periodic microstructure with a single rod and its surrounding fluid identified as a repeating element. By means of the asymptotic homogenization method, a 3-D continuum model for the bundle has been given by Zhang (1998a, 1998b, 1998c). This model is based on the Euler-Bernoulli beam theory.

In order to use the traveling wave control approach presented by Hagedorn (1989) to suppress vibrations of the bundle, a hyperbolic equation system is developed based on the Timoshenko beam theory.

The energy flow in any fuel rod is formulated. This makes it possible to obtain the control law by minimizing the traveling wave energy.

2. HOMOGENIZED MODEL

2.1 Basic equations

Fluids in the fuel bundle are usually regarded as compressible and non-viscous. The motion is governed by the continuity equation

$$\dot{\rho}_f + \overline{\rho}_f \nabla_i v_i = 0 , \tag{1}$$

the momentum equilibrium equation

$$\overline{\rho}_f \dot{v}_i = G_i - \nabla_i p , \tag{2}$$

and the equation of state

$$\dot{p} = c_f^2 \dot{\rho}_f . \tag{3}$$

In above, ρ_f is the variation of the fluid density, $\overline{\rho}_f$ the averaged fluid density (constant), v_i the fluid velocity component, *p* the pressure, G_i the fluid body force per unit volume, and $\nabla_i = \partial / \partial x_i$ with x_i being the global coordinate established on the fuel bundle. We assume x_3 is in the direction of the beam axis and upward (Figure 1). The dot over a variable indicates the derivative with respect to time *t*. In this paper, the Latin subscript assumes the value of 1, 2, or 3, while the Greek subscript assumes the value of 1 or 2. Also, repeated indices in the same term indicate summations.

Fuel rods in the bundle are treated as elastic beams. Their motion satisfies the following equation of motion

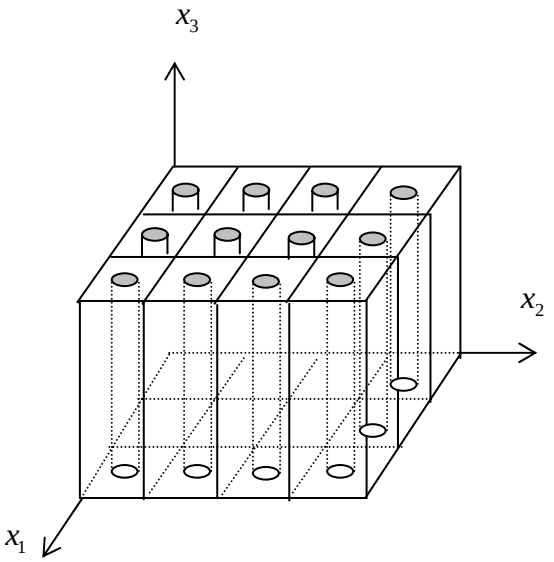


Fig. 1 Schematic diagram of the tube bundles

$$\nabla_j \sigma_{ij} + F_i = \overline{\rho}_s \ddot{w}_i, \tag{4}$$

the strain displacement relation

$$\varepsilon_{ij} = \frac{1}{2}\left(\nabla_j w_i + \nabla_i w_j\right) \tag{5}$$

and the continuity equation

$$\dot{\rho}_s + \overline{\rho}_s \nabla_i \dot{w}_i = 0, \tag{6}$$

in which, σ_{ij}, ε_{ij}, w_i are the stress, strain and displacement components, respectively, F_i the rod body force per unit volume, $\overline{\rho}_s$ the averaged mass density of the rod (constant), ρ_s the variation of the mass density.

Because only body forces due to gravity are considered, thus only F_3 and G_3 exist and remain constants.

In addition, there are the normal velocity and stress continuity conditions at each interface between rods and fluid:

$$\sigma_{\alpha\beta} n_\beta = -p\, n_\alpha \quad \text{and} \quad v_\alpha n_\alpha = \dot{w}_\alpha n_\alpha, \tag{7}$$

in which, n_α the unit normal vector of the interface, which is positive when pointing from the fluid into the rod.

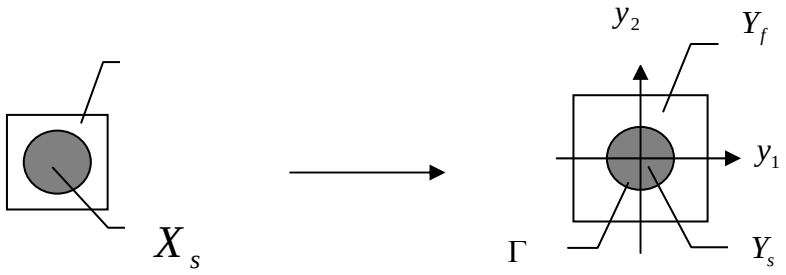
The above fundamental equations do not include any stress-strain relation. The Hook's stress-strain relation together with the beam assumption will be introduced later.

Equations (1) to (7) can be used to establish a homogenized model for the fuel bundle.

2.2 Asymptotic Expansion

The cross section of a fuel bundle can be divided into many unit cells of the same shape. Each unit cell consists of the cross section of a fuel rod and the surrounding fluid (Figure 2). Since the number of rods in a fuel bundle is very large, the dimension of the unit cell is much smaller than the characteristic size of the cross

section of the fuel bundle. Thus, the size ratio of the unit cell and the bundle is a dimensionless small parameter denoted by ε.



(a) unit cell *(b) magnified unit cell*

Fig.2 Unit cell and local coordinates system

All unknown functions can be expanded in terms of the polynomials of the small parameter ε, for example:

$$p = p^{(0)} + \varepsilon\, p^{(1)} + \varepsilon^2 p^{(2)} + \ldots \tag{8}$$

Also, on the unit cell, one can introduce following two-dimensional local coordinates:

$$y_\alpha = x_\alpha / \varepsilon \tag{9}$$

The local coordinate is shown in Figure 2.

The location of any point in the fuel bundle is described by the global coordinate $x = (x_1, x_2, x_3)$ and the local coordinate $y = (y_1, y_2)$. The global coordinate indicates the unit cell on which the point is located, while the local coordinate indicates the exact location of the point on that unit cell. For example, the pressure at a point, denoted by $p(x)$ originally, will be represented by $p(x,y)$, due to this notation. The derivatives will change also as in:

$$\frac{\partial}{\partial x_\alpha} p(x) = \frac{\partial}{\partial x_\alpha} p(x,y) = \frac{\partial p}{\partial x_\alpha} + \frac{\partial p}{\partial y_\alpha}\frac{\partial y_\alpha}{\partial x_\alpha} = \left(\frac{\partial}{\partial x_\alpha} + \frac{1}{\varepsilon}\frac{\partial}{\partial y_\alpha} \right) p(x,y)$$

That is, all derivatives in the fundamental equations should be expanded according to:

$$\nabla_\alpha \Rightarrow \nabla_\alpha + \frac{1}{\varepsilon}\frac{\partial}{\partial y_\alpha}, \quad \nabla_3 \text{ no change} \tag{10}$$

Applying the two procedures indicated by Eqs. (8) and (10) to Eqs. (1-7), setting equal the coefficients of the terms of same order in the polynomials for the small parameter ε, one obtains a series of expansion equations.

Dealing with these equations in the level of first order approximation, namely, only remaining all the ε^0 order unknown functions and eliminating all the high order unknown functions in the ε^0 order and ε^1 order expansion equations, after some long derivation, results in the two homogenized equations:

$$\left(\frac{\lambda}{c_f^2} + \frac{1-\lambda}{\kappa c_s^2} \right) \ddot{p}^{(0)} - A\nabla_\alpha \nabla_\alpha p^{(0)} - \lambda \nabla_3 \nabla_3 p^{(0)} + \bar{\rho}_f B \nabla_\alpha \ddot{w}_\alpha^{(0)} = 0 \tag{11}$$

and $$M\ddot{w}_{\alpha}^{(0)} + B\nabla_{\alpha} p^{(0)} - \nabla_3 \left(\frac{1}{|Y|} \int_{Y_s} \sigma_{3\alpha}^{(0)} dy \right) = 0 \tag{12}$$

Here $\lambda = \left|X_f\right| / |X| = \left|Y_f\right| / |Y|$ is the fluid volume fraction or porosity of the bundle. $\left|X_f\right|$ and $\left|Y_f\right|$ are the areas of fluid in the unit cell and in the magnified unit cell respectively; $|X|$ and $|Y|$ are the areas of the unit cell and the magnified unit cell respectively. Y_s is the rod domain in the magnified unit cell. c_f is the speed of sound in fluid. c_s is the speed of sound in rods. $\kappa = \overline{\rho}_s / \overline{\rho}_f$ is density ratio.

There are four micro structure parameters A, B, M and D in the above equations. In fact, these parameters are tensor if the cross section of rods has arbitrary shape. However, as indicated by Zhang (1999) that these tensors are isotropic for the circular, rectangular and hexagonal cross section, and then the parameters become scalar quantity as follows:

$$A = (1-\lambda) - D, \tag{13}$$

$$B = (1-\lambda) + D, \tag{14}$$

$$M = \overline{\rho}_s (1-\lambda) + \overline{\rho}_f D, \tag{15}$$

and $$D = \frac{1}{|Y|} \int_{Y_f} \frac{\partial \chi}{\partial y_1} dy_1 dy_2, \tag{16}$$

Here D represents the added fluid area fraction as indicated by Zhang (1999). χ is called as local function, and can be determined by solving the local problem:

$$\begin{cases} \chi_{,\alpha\alpha} = 0 & in\ Y_f, \\ \chi_{,\alpha} n_{\alpha} = n_1 & on\ \Gamma, \\ \int_{Y_f} \chi dy_1 dy_2 = 0 & \\ \chi \text{ is a doubly periodic function of } y_1 \text{ and } y_2. & \end{cases} \tag{17}$$

Here n_1, n_2 are the components of the exterior normal to the fluid domain on the interface between the fluid and the rod in the magnified unit cell.

3. HYPERBOLIC EQUATIONS BASED ON TIMOSHENKO BEAM THEORY

3.1 Transverse shear force

Now let us return to see the integral term in equation (12). According to the Timoshenko beam theory, there is the relation between shear stress and strain as follows:

$$\sigma_{3\alpha}^{(0)} = G\left(\nabla_3 w_{\alpha}^{(0)} - \vartheta_{\alpha} \right) \tag{18}$$

where G is the shear modulus, ϑ_α is the rotation of the cross section of rods in $x_\alpha - x_3$ plane as shown in Figure 3. Then the integral term in equation (12) can be calculated as follows:

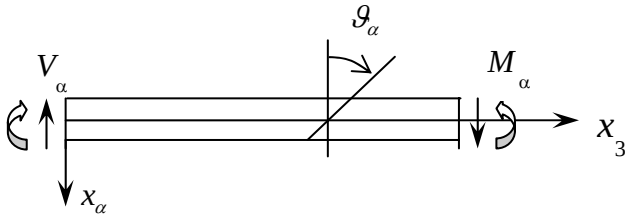


Fig.3 Shear force, bending moment and rotation of cross section in $x_\alpha - x_3$ plane.

$$\frac{1}{|Y|}\int_{Y_s}\sigma_{3\alpha}^{(0)}dy = \zeta G\frac{|Y_s|}{|Y|}\left(\nabla_3 w_\alpha^{(0)} - \vartheta_\alpha\right) = \zeta G(1-\lambda)\left(\nabla_3 w_\alpha^{(0)} - \vartheta_\alpha\right) \tag{19}$$

in which ζ is the shape coefficient of the cross section of rods. Thus, equation (12) becomes

$$M\ddot{w}_\alpha^{(0)} + B\nabla_\alpha p^{(0)} - \zeta G(1-\lambda)\nabla_3\left(\nabla_3 w_\alpha^{(0)} - \vartheta_\alpha\right) = 0 \tag{20}$$

Furthermore, it is not difficult, according to the Timoshenko beam theory, to write the relation:

$$\frac{\bar{\rho}_s I}{|X|}\ddot{\vartheta}_\alpha - \frac{EI}{|X|}(\nabla_3\nabla_3\vartheta_\alpha) - (1-\lambda)\zeta G(\nabla_3 w_\alpha - \vartheta_\alpha) = 0 \tag{21}$$

where EI is the bending stiffness of fuel rods, $|X_s|$ is the cross sectional area of a rod.

3.2 Potential form

Equations (11) and (21) are expressed in terms of the pressure, taken as the fundamental unknown in the fluid region. As a result, non-symmetry of the coefficient matrices is introduced into the corresponding finite element solution. To remove this non-symmetry, a velocity potential rather than pressure is adopted as the fundamental unknown in the fluid region.

It has been shown by Zhang (1999) that the fluid in the bundle is irrotational. Hence a velocity potential exists. Its ε^0 order approximation $\psi^{(0)}$ satisfies

$$p^{(0)} = -\bar{\rho}_f\,\dot{\psi}^{(0)}. \tag{22}$$

Integrating equation (11) with respect to time and introducing (22) into the integration, we obtain the equation

$$\left(\frac{\lambda}{c_f^2} + \frac{1-\lambda}{\kappa c_s^2}\right)\ddot{\psi} - A\nabla_\alpha\nabla_\alpha\psi - \lambda\nabla_3\nabla_3\psi - B\nabla_\alpha\dot{w}_\alpha = 0. \tag{23}$$

Similarly, equation (20) is rewritten as

$$M\ddot{w}_\alpha + (1-\lambda)\zeta G\nabla_3\left(\nabla_3 w_\alpha - \vartheta_\alpha\right) - \bar{\rho}_f\,B\nabla_\alpha\dot{\psi} = 0. \tag{24}$$

Equations (23), (24) and (21) are the final equations. Here the superscript $^{(0)}$ has been omitted for simplicity. It is

a hyperbolic equation system, and then is convenient for solving the control problem.

4. FIRST ORDER SYSTEM

Introduce the parameters

$$\boldsymbol{c_1^2} = \frac{EI}{\overline{\rho}_s I} = \frac{E}{\overline{\rho}_s}, \tag{25}$$

$$\boldsymbol{c_2^2} = \frac{(1-\lambda)\zeta G}{M}, \tag{26}$$

$$\boldsymbol{l^2} = \frac{\overline{\rho}_s I}{M|X|}, \tag{27}$$

$$\gamma = \frac{c_1}{c_2} \tag{28}$$

and the dimensionless variables

$$\tau = \frac{\boldsymbol{c_s}}{\boldsymbol{l}} t, \tag{29}$$

$$\xi_i = \frac{1}{l} x_i, \tag{30}$$

$$\tilde{w}_\alpha(\xi_\alpha, \tau) = \frac{1}{l} w_\alpha(x_\alpha, t), \tag{31}$$

$$\tilde{\psi}(\xi_\alpha, \tau) = \frac{1}{l c_s} \frac{\overline{\rho}_f}{M} \psi(x_\alpha, t), \tag{32}$$

$$\tilde{\vartheta}_\alpha(\xi_\alpha, \tau) = \vartheta_\alpha(x_\alpha, t), \tag{33}$$

Then the equations (23), (24) and (21) can be rewritten in the form:

$$\ddot{\psi} - \tilde{A}\nabla_\alpha \nabla_\alpha \psi - \tilde{\lambda}\nabla_3 \nabla_3 \psi - \beta\tilde{B}\nabla_\alpha \dot{w}_\alpha = 0 \tag{34a}$$

$$\ddot{w}_\alpha - \nabla_3(\nabla_3 w_\alpha - \vartheta_\alpha) - B\nabla_\alpha \dot{\psi} = 0 \tag{34b}$$

$$\ddot{\vartheta}_\alpha - \gamma^2 \nabla_3 \nabla_3 \vartheta_\alpha - (\nabla_3 w_\alpha - \vartheta_\alpha) = 0 \tag{34c}$$

with the dimensionless parameters

$$\tilde{\boldsymbol{c}}_{\boldsymbol{f}} = \frac{c_f}{c_2}, \quad \tilde{\boldsymbol{c}}_{\boldsymbol{s}} = \frac{c_s}{c_2} \tag{35}$$

$$\beta = \frac{\overline{\rho}_f}{M} \tag{36}$$

$$\tilde{A}, \tilde{B}, \tilde{\lambda} = (A, B, \lambda) \Big/ \left(\frac{\lambda}{\tilde{c}_f^{\,2}} + \frac{1-\lambda}{\kappa \tilde{c}_s^{\,2}} \right) \tag{37}$$

In equation (34), the symbols w_α, ψ and ϑ_α are again used for the dimensionless variables for simplicity. Also the symbol $\nabla_i = \dfrac{\partial}{\partial \xi_i}$, the dot over a variable indicates the derivative with respect to the dimensionless time τ.

Now only the movement in the $x_1 - x_3$ plane is considered for simplicity. In order to transform equation (34) into a first order system, following new variables are introduced

$$\begin{aligned} \boldsymbol{u^1} &= \dot{\psi}(x_1, x_3, t), \quad u^2 = \sqrt{\tilde{A}} \nabla_1 \psi, \quad u^3 = \sqrt{\tilde{\lambda}} \nabla_3 \psi, \quad u^4 = \dot{w}_1(x_1, x_3, t), \\ u^5 &= \nabla_3 w_1 - \vartheta_1, \quad u^6 = \dot{\vartheta}_1(x_1, x_3, t), \quad u^7 = \gamma \nabla_3 \vartheta_1 \end{aligned} \tag{38}$$

There exist four identical relations between the new variables：

$$\dot{u}^2 = \sqrt{\tilde{A}} \nabla_1 u^1 \tag{39a}$$

$$\dot{u}^3 = \sqrt{\tilde{\lambda}} \nabla_3 u^1 \tag{39b}$$

$$\dot{u}^5 = \nabla_3 u^4 - u^6 \tag{39c}$$

$$\dot{u}^7 = \gamma \nabla_3 u^6 \tag{39d}$$

Now the equation (34) can be rewritten in the first order form as follows：

$$\dot{u}^1 - \sqrt{\tilde{\boldsymbol{A}}} \nabla_1 u^2 - \sqrt{\tilde{\lambda}} \nabla_3 u^3 - \beta \tilde{B} \nabla_1 u^4 = 0 \tag{40a}$$

$$\dot{u}^4 - B\nabla_1 u^1 - \nabla_3 u^5 = 0 \tag{40b}$$

$$\dot{u}^6 - u^5 - \gamma\nabla_3 u^7 = 0 \tag{40c}$$

By introducing an unknown function vector

$$u = \left[\boldsymbol{u^1}, \boldsymbol{u^2}, \boldsymbol{u^3}, \boldsymbol{u^4}, \boldsymbol{u^5}, \boldsymbol{u^6}, \boldsymbol{u^7}\right]^T \tag{41}$$

Equations (39) and (40) can then be written in the matrix form

$$\dot{u} + \boldsymbol{D_1}\nabla_1 u + \boldsymbol{D_3}\nabla_3 u + \boldsymbol{D_0} u = 0 \tag{42}$$

with

$$D_1 = \begin{bmatrix} 0 & -\sqrt{\tilde{A}} & 0 & -\beta\tilde{B} & 0 & 0 & 0 \\ -\sqrt{\tilde{A}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -B & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{43}$$

$$D_3 = \begin{bmatrix} 0 & -\sqrt{\tilde{\lambda}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\tilde{\lambda}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma \\ 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \end{bmatrix} \tag{44}$$

$$D_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{45}$$

The wave equation (42) can further be written in the normal form

$$\dot{V} + \tilde{\boldsymbol{D}}_{\boldsymbol{1}} \nabla_1 V + \tilde{\boldsymbol{D}}_{\boldsymbol{3}} \nabla_3 V + \tilde{\boldsymbol{D}}_{\boldsymbol{0}} V = 0 \tag{46}$$

with

$$\tilde{D}_3 = \boldsymbol{diag}\left(\sqrt{\tilde{\lambda}}, 0, -\sqrt{\tilde{\lambda}}, \gamma, 1, -1, -\gamma\right) \tag{47}$$

$$\tilde{D}_1 = \boldsymbol{T}\boldsymbol{D}_{\boldsymbol{1}}\boldsymbol{T}^{\boldsymbol{-1}} = \begin{bmatrix} 0 & -\sqrt{\frac{\tilde{A}}{2}} & 0 & 0 & \frac{\beta\tilde{B}}{2} & -\frac{\beta\tilde{B}}{2} & 0 \\ -\sqrt{\frac{\tilde{A}}{2}} & 0 & -\sqrt{\frac{\tilde{A}}{2}} & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{\tilde{A}}{2}} & 0 & 0 & \frac{\beta\tilde{B}}{2} & -\frac{\beta\tilde{B}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{B}{2} & 0 & \frac{B}{2} & 0 & 0 & 0 & 0 \\ -\frac{B}{2} & 0 & -\frac{B}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{48}$$

$$\tilde{D}_0 = TD_0T^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \tag{49}$$

The transformation matrix of the linear transformation $V = \mathbf{T}u$ is

$$\boldsymbol{T} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{50}$$

5. CONTROL LAW

Now we consider a fuel rod in the bundle identified by $x_1 = x_1^*$. When a control force $q(t)$ and a control moment $m(t)$ are applied at the axial location $x_3 = a$ as shown in Figure 4, then the following equilibrium conditions are satisfied

$$\boldsymbol{u}^5\left(x_1^*, a^-, t\right) - \boldsymbol{u}^5\left(x_1^*, a^+, t\right) = q(t) \tag{51a}$$

$$\boldsymbol{u}^7\left(x_1^*, a^-, t\right) - \boldsymbol{u}^7\left(x_1^*, a^+, t\right) = m(t)/\gamma \tag{51b}$$

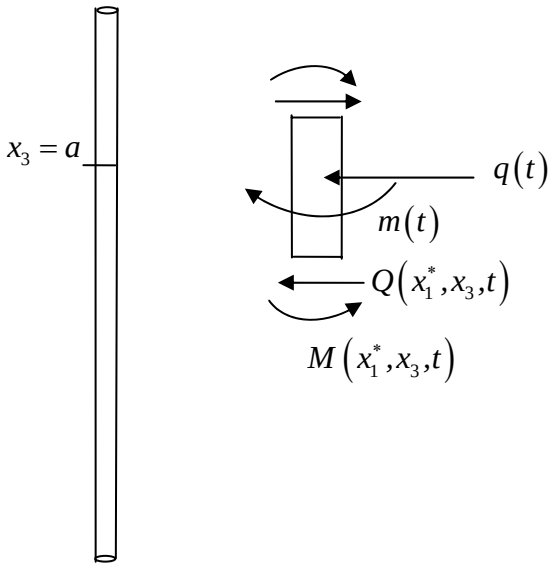


Fig.4 Equilibrium of a rod section

While the continuity leads to

$$\boldsymbol{u}^4\left(x_1^*, a^-, t\right) - \boldsymbol{u}^4\left(x_1^*, a^+, t\right) = 0 \tag{52a}$$

$$\boldsymbol{u}^6\left(x_1^*, a^-, t\right) - \boldsymbol{u}^6\left(x_1^*, a^+, t\right) = 0 \tag{52b}$$

Conditions（51）and（52）can be written in the new variables

$$\boldsymbol{V}_4^- - \boldsymbol{V}_4^+ = \frac{m(t)}{\sqrt{2}\gamma} \tag{53a}$$

$$\boldsymbol{V}_5^- - \boldsymbol{V}_5^+ = \frac{q(t)}{\sqrt{2}} \tag{53b}$$

$$\boldsymbol{V}_6^- - \boldsymbol{V}_6^+ = \frac{q(t)}{\sqrt{2}} \tag{53c}$$

$$\boldsymbol{V}_7^- - \boldsymbol{V}_7^+ = \frac{m(t)}{\sqrt{2}\gamma} \tag{53d}$$

It can be shown that the energy flow along the rod is given by

$$\begin{aligned} P_3 &= \frac{1}{2} V^T \tilde{D}_3 V = \frac{1}{2}\left[\sqrt{\tilde{\lambda}}\left(V_1^2 - V_3^2\right) + \gamma\left(V_4^2 - V_7^2\right) + \left(V_5^2 - V_6^2\right)\right] \\ &= -\tilde{\lambda}\,\dot{\psi}\nabla_3\psi - \gamma^2\dot{\vartheta}_1\nabla_3\vartheta_1 - \dot{\boldsymbol{w}}_1\left(\nabla_3 w_1 - \vartheta_1\right) \end{aligned} \tag{54}$$

The first term in the expression is $\frac{1}{2}\sqrt{\tilde{\lambda}}\left(V_1^2 - V_3^2\right) = -\tilde{\lambda}\,\dot{\psi}\nabla_3\psi = \frac{\tilde{\lambda}}{\overline{\rho}_f}\, pv_3$. It is very small. By neglecting the small term the energy flow along the rod is finally written in the form:

$$P_3 = \frac{1}{2} V^T \tilde{D}_3 V = \frac{1}{2}\left[\gamma\left(V_4^2 - V_7^2\right) + \left(V_5^2 - V_6^2\right)\right] \tag{55}$$

Obviously, the energy flow $p\left(a^-, t\right)$ at $x = a^-$ is maximized if letting

$$\boldsymbol{V}_7^- = 0 \quad \text{and} \quad \boldsymbol{V}_6^- = 0 \tag{56}$$

Furthermore, by introducing (56) into (53c) and (53d) we finally obtain the control laws

$$q(t) = -\sqrt{2}\boldsymbol{V}_6^+ \tag{57a}$$

$$m(t) = -\sqrt{2}\gamma\boldsymbol{V}_7^+ \tag{57b}$$

We guess that the vibration of surrounding fuel rods will be damped, when the control law is applied at a certain rod. The numerical simulation will be given in the next paper.

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