ABSTRACT

Due to increased potentialities of computers, it is nowadays possible to perform dynamic non-linear computation of structures to evaluate their ultimate behavior under seismic loads using refined finite element models. Nevertheless, one key parameter for such complex computations is the input load (i.e. input time histories) which may lead to important discrepancies in the results and therefore difficulties to deal with for engineering purpose (variability, number of time histories to use ...). In this situation, the number of accelerograms to be used and the way to deal with the results is to be carefully assessed.

The objective of this study is to give some elements concerning (i) the number of accelerograms to be used for transient non-linear computations and (ii) the way to account for scattering of results. For this purpose, some simplified non-linear models are used. These models represent characteristic types of non-linearities such as:

- Reinforce concrete (RC) structure model (with plastic non-linearity),
- PWR core model (with impact non-linearity).

For each type of non-linearity, different sets of accelerograms are used (artificial and natural ones). Each set is composed of a relatively high number of accelerograms in order to get proper trends. The results are expressed in term of average and standard deviation values of the characteristic parameters for each non-linearity (i.e. ductility drift for RC structure model and impact force for PWR core model).

The results show that, a relatively large number of time histories may be necessary to get proper predictions of the average value of the characteristic non-linear parameter under consideration. In that situation, it should be difficult to deal with such a result for complex studies on reel structures.

Nevertheless, it may be necessarily to perform transient non-linear seismic computations for design analyses but with a reduced number of calculations. For this purpose, the previous results are analyzed once again but using a statistical approach in order to get a conservative evaluation of the value of the non-linear parameter but using a reduced number of calculations. Student statistical estimator is used for this purpose. The previous results are then used to estimate the average value of the characteristic non-linear parameter with a confidence level of 95%. The evolution of the estimation is then compared, depending on (i) the sets of accelerograms used and (ii) the number of accelerograms used.

These results show that a conservative estimation of the average value of the non-linearity can be determined. This value remains conservative (with a confidence level of 95%) and allows to use a relatively low number of accelerograms (typically 5 to 10). This method leaves a relative freedom to the designer for engineering purpose.

This study will be extended in the future with other earthquake characteristics and with other types on non-linearities (uplift, sliding and rocking of rigid bodies ...).

Keywords: Earthquake; Non-linearity; Variability; Scattering; Accelerogram
1 INTRODUCTION

The French Basic Safety Rule concerning seismic design of civil structures (RFS V.2.g, [5]) is currently under revision. In that situation, the potential use of dynamic transient non-linear computation for specific cases is under assessment.

Independently on methods of solutions, which have to be used within their domain of qualification, the question of the input loads (i.e. time histories) is to be assessed in detail in terms of (i) representativeness, (ii) number of accelerograms to be used and (iii) resulting scattering (variability) of results.

In that context, the objective of this study is to give some elements concerning the influence of time histories on results of dynamic transient non-linear seismic computation. The specific points more specifically analyzed are:

- the number of accelerograms to be used for transient non-linear computations,
- the way to account for scattering of results.

2 DESCRIPTION OF TOOLS AND INPUT DATA

2.1 Seismic input motion

2.1.1 Natural time histories selection : Strong Motion Data Based

The European Strong Motion Data Base (ESMDB) used in this study was established by several European scientific organisms [1]. This SMDB is made of 965 seismic records (horizontal and vertical ones) mainly recorded in Europe. Each record is characterized by local magnitude $M$ of the corresponding earthquake, focal distance $D$ from the source and soil characteristics of the station. This ESMDB covers approximately earthquakes from magnitude 4.5 to 7.3 and focal distance 7 km to 100 km.

This ESMDB was used to establish a regression relationship to determine the Ground Response Spectrum (GRS) of an earthquake based on its magnitude $M$, its focal distance $D$ and soil characteristics of the site (i.e. rock : $V_s > 800$ m/s or medium : $300$ m/s < $V_s$ < 800 m/s). This regression relationship is used in application of the French Basic Safety Rule, which defines the method to be used for the evaluation of seismic hazard for nuclear facilities (RFS 2001-01, [5]).

The natural time histories used in this study are selected among the ESMDB. In order to get a set of natural time histories consistent with each-other, they have been selected based on the following process:

1) Determination of an earthquake characteristic (called SSE) in terms of Magnitude $M_{SSE}$ and a focal distance $D_{SSE}$ (this typically allows us to work on a low magnitude - near field earthquake or a high magnitude – far field earthquake),

2) Selection of a set of natural time histories among the ESMDB based on the following criteria:

$M_{SSE} - 0.5 < $ Magnitude of the selected earthquake < $M_{SSE} + 0.5$

$D_{SSE} - 10$ km < Focal Distance of the selected earthquake < $D_{SSE} + 10$ km

2.1.2 Artificial time histories génération : Regression relationships for Ground Motion Parameters

In addition to the regression relationship on the GRS defined by the French Basic Safety Rule (RFS 2001-01), EDF and CEA (Commissariat à l’Energie Atomique – France) have carried out a study [2] to calculate regression relationships for classical Ground Motion Parameters (GMP). The same type of relationship than the one used for the ground response spectrum was selected and the ESMDB was also used to remain fully consistent with the RFS 2001-01 GRS methodology. This study showed that the regression relationship could be expressed in the following form:

$log_{10} GMP = \alpha M + \gamma log_{10}R + \delta$ $ +/- \sigma$

The selected GMP are the following:

$A$ (maximal acceleration or peak ground acceleration),

$V$ (maximal velocity or peak ground velocity),
D (maximal displacement or peak ground displacement),
A/V ratio,
CAV (Cumulative Absolute Velocity) equal to \( \int |\gamma(t)| dt \) where \( \gamma(t) \) is the time history.

Ia (Arias Intensity) equal to \( \frac{\pi}{2g} \int \gamma(t)^2 dt \).

T (duration defined as duration from 5% to 95% Ia).

In table 1 are given calculated \( \alpha, \gamma, \delta \) and \( \sigma \) coefficients for each GMP.

<table>
<thead>
<tr>
<th>GMP</th>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (m/s²)</td>
<td>0.295</td>
<td>-0.985</td>
<td>1.575</td>
<td>0.309</td>
</tr>
<tr>
<td>V (m/s)</td>
<td>0.432</td>
<td>-0.945</td>
<td>-2.468</td>
<td>0.340</td>
</tr>
<tr>
<td>D (cm)</td>
<td>0.570</td>
<td>-0.585</td>
<td>-2.236</td>
<td>0.470</td>
</tr>
<tr>
<td>A/V (s⁻¹)</td>
<td>-0.137</td>
<td>-0.041</td>
<td>2.043</td>
<td>0.197</td>
</tr>
<tr>
<td>CAV (m/s)</td>
<td>0.324</td>
<td>-0.308</td>
<td>-1.058</td>
<td>0.230</td>
</tr>
<tr>
<td>Ia (m/s)</td>
<td>0.495</td>
<td>-0.912</td>
<td>-2.517</td>
<td>0.424</td>
</tr>
<tr>
<td>T (s)</td>
<td>0.164</td>
<td>0.324</td>
<td>-0.483</td>
<td>0.204</td>
</tr>
</tbody>
</table>

The artificial time histories used in this study are generated using POWERSPEC [3] computer program. In order to get a set of artificial time histories consistent with each-other, they have been generated based on the following process:

1) Determination of an earthquake characteristic (SSE) in terms of Magnitude \( M_{\text{SSE}} \) and a focal distance \( D_{\text{SSE}} \),
2) Determination of the GRS and GMP of the given SSE based on the previous regression relationships,
3) Generation of a set of artificial time histories based on the following criteria:

\[ \text{Average ground response spectrum of the 30 accelerograms} > \text{GRS}_{\text{SSE}} \]
\[ \text{GMP}_{\text{SSE}} - \sigma < \text{GMP}_{\text{artificial accelerogram}} < \text{GMP}_{\text{SSE}} + \sigma \]

### 2.1.3 Seismic ground motion characteristics

For this study, one case has been selected, representative of a typical SSE for a French nuclear Pressurized Water Reactor. The SSE characteristics are the following.

Magnitude \( M = 5.5 \)  
Focal distance \( R = 9 \text{ km} \)  
Rock soil (\( V_s > 800 \text{ m/s} \))

The corresponding ground response spectrum (GRS) is determined based on RFS2001-01 regression law. The GRS is shown on figure 1 (maximum ground acceleration is 0.2g).

![Figure 1 – Ground Response Spectrum (M = 5.5 – R = 9 km)](image-url)

According to regression relationships described in chapter 2.1.2, the corresponding GMP are given in table 2.
Table 2 – Ground Motion Parameters (MSSE = 5.5 – DSSE = 9 km)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average value</th>
<th>Average val. – Std deviation</th>
<th>Average val. + Std deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (s)</td>
<td>5.3</td>
<td>3.3</td>
<td>8.6</td>
</tr>
<tr>
<td>V (m/s)</td>
<td>0.101</td>
<td>0.05</td>
<td>0.22</td>
</tr>
<tr>
<td>D (cm)</td>
<td>2.2</td>
<td>0.7</td>
<td>6.5</td>
</tr>
<tr>
<td>AV (s^-1)</td>
<td>18</td>
<td>11</td>
<td>28</td>
</tr>
<tr>
<td>CAV (m/s)</td>
<td>2.7</td>
<td>1.6</td>
<td>4.6</td>
</tr>
<tr>
<td>ia (m/s)</td>
<td>0.22</td>
<td>0.08</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Two examples of selected natural accelerograms are shown in figure 2.

Figure 2 – Example of selected natural time histories

Two examples of generated artificial accelerograms are shown in figure 3.

Figure 3 – Example of generated artificial time histories

2.2 Non-linear models description

2.2.1 Simplified reinforced concrete structure: Plastic non-linearity

In our case, simplified reinforced concrete structures are modeled as a single degree of freedom (SDOF) oscillator governed by bilinear Takeda’s constitutive relationship that takes into account stiffness degradation. Post-elastic behavior can be modeled to reproduce typical behavior of a (shear) wall type RCS or a frame type RCS, as shown in figure 4.
For this study, the parameters used for the simplified RCS are the following:

- **Frequency**: 5 Hz (typical frequency of a nuclear RC building)
- **Damping**: 5% of critical
- **Elastic threshold**: Defined based on a ground response spectrum identical to the SSE in shape but scaled to a ZPA less than the SSE one (in order to get plastic deformation). Two different design levels (in terms of ZPA) were defined:
  - 0.05 g (i.e. 1/4 of the SSE)
  - 0.1 g (i.e. 1/2 of the SSE)
- **Plastic stiffness**: 0.2 x elastic stiffness

The difference between the wall type structure and the frame type structure comes essentially from the dissipative effects (loading and unloading behavior), cf. figure 4.

The results are expressed in terms of ductility drift. Ductility drift is defined here as the ratio between the maximum non-linear displacement over the elastic threshold displacement. This parameter is admitted to be a representative indicator of damage on RCS.

### 2.2.2 PWR core model: Impact non-linearity

The PWR core finite element model used in this study is composed of a row of fuel assemblies. Each fuel assembly is made of 2 beams. The first one (Guide Tubes beam) accounts for the 24 guide tubes and the instrumentation tube. The second one (Fuel Rods beam) accounts for the 264 fuel rods.

Due to seismic excitation, these fuel assemblies may move in the lateral direction and, due to small gaps, they may impact with each other (or with pressure vessel baffle) at mixing grid location, as shown in figure 5.

The results are expressed in terms of maximum impact forces on grids.
3 NON-LINEAR SEISMIC RESPONSE OF THE STRUCTURES – VARIABILITY QUANTIFICATION

This part gives some elements on the effect of the number of accelerograms used on the results of non-linear computations. For this purpose, the previous two types of non-linearities are analysed. The first one is the impact non-linearity based on PWR core described in paragraph 2.2.2. The second one is the plastic non-linearity based on the simplified RCS described in paragraph 2.2.1.

3.1 PWR core : Impact non-linearity

3.1.1 Case one : In-structure (floor) seismic motion – Artificial time histories – High number of time histories

This case was performed using a high number of artificial accelerograms (500 time histories) all generated with the same input parameters : in-structure (floor) seismic motion. The objective is to show how a result of a non-linear computation may depend on the set of accelerograms used for the calculation.

Figure 6 shows the results in the following form :

- Base case : Population = 500 calculations : This gives the average value of the impact force of the population.
- Additional cases : Selection of different sets of 50 (different) accelerograms (7 sets) taken among the population, and for each set, presentation of:
  - The evolution of the average value of the impact force with the number of accelerogram used to calculate it (from 3 to 50),
  - The comparison of the evolution of the average value of the impact force between sets.

![Impact non-linearity - Average value](attachment:impact.png)

Figure 6 – Evolution of average value with the number of accelerograms used
Dependence on the sets of accelerograms

The first comment is that different sets of accelerograms may lead to significant differences in terms of average values. One can also observe that for low number of accelerograms (typically less than 10), the scattering on the average value of the non-linear parameter (i.e. impact force) may be very important depending on the different sets of accelerograms used for the calculations.

In another hand, the scattering remains significant for high number of accelerograms and the convergence to the average of the population is very low, even for numbers of accelerograms higher than 30.

Although scattering of results is known for such non-linear analyses, the previous results quantify the impact of using of a reduced number of accelerograms for non-linear computation and also illustrates that different sets of accelerograms may lead to different results. As the number of accelerograms has to remain reasonable for design studies, an appropriate method to account for scattering should be applied for engineering purpose (cf. § 4).
3.2  **Simplified reinforced concrete structure : Plastic non-linearity**

3.2.1  **Case two : Seismic ground motion – Natural time histories**

This case was performed using a set of 26 natural time histories.

The figure 7 shows the same type of results than the previous case (case one) :

**Base case =** All the results (from the 26 time histories) ; This case gives the average value of ductility drift of the population,

**Additional cases :** Selection of 2 sets of 13 (different) accelerograms, and for each set, presentation of :
- The evolution of the average value of the ductility drift with the number of accelerogram used to calculate it (from 3 to 13),
- The comparison of the evolution of the average value of the impact force between the 2 sets.

![Figure 7 – Evolution of average value with the number of accelerograms](image)

These results show that the average value of the ductility drift may be slightly different depending on the accelerograms used to calculate it (from 3 to 5 in the left figure ; from 1.5 to 2.7 in the right figure).

3.2.2  **Case three : Seismic ground motion – Artificial time histories**

This case was performed using a set of 30 artificial time histories.

The figure 8 shows the same type of results than the previous ones (cases 1 & 2) :

**Base case =** All the results (from the 30 time histories) ; This case gives the average value of ductility drift of the population,

**Additional cases :** Selection of 2 sets of 15 (different) accelerograms, and for each set, presentation of :
- The evolution of the average value of the ductility drift with the number of accelerogram used to calculate it (from 3 to 15),
- The comparison of the evolution of the average value of the impact force between the 2 sets.

![Figure 8 – Evolution of average value with the number of accelerograms](image)
Once again, the results show that the average value of the ductility drift may be slightly different depending on the accelerograms used to calculate it (from 5 to 6.5 in the left figure; from 2.4 to 2.7 in the right figure).

4 A WAY TO ACCOUNT FOR SCATTERING : STUDENT-FISHER ESTIMATOR

This paragraph proposes a method to account for scattering based on statistical considerations. The objective is to determine a conservative value of a design parameter $D$ based on results obtained from non-linear computations with a set of $N$ accelerograms.

The results obtained from the $N$ computations can be expressed in terms of average value $D_{AV}$ and standard deviation $S_D$, these two parameters being characteristic of the sample (i.e. set of $N$ calculations). To reach the objective expressed previously, a statistical estimator can be used to predict the average value of the population, with an appropriate confidence level.

The statistical estimator used here is the Student-Fisher estimator. The main characteristics of this estimator are described in the next paragraph.

4.1 Description of Student-Fisher estimator

Student-Fisher relationship $S_N$ ($N$ being its degree of freedom) is commonly used in statistics. Its main characteristic is to tend to a Normal relationship when $N$ tends to infinite. This relationship is particularly appropriate to predict the average value of a population $D$ based on the average value of the sample $D_{AV}$ and its standard deviation $S_D$, for a desired confidence level and depending on the size of the sample $N$, considering the following expression.

$$D_{95\%} = D_{AV} + \frac{t_{0.05;N-1}}{\sqrt{N}} S_D$$

Where $t_{0.05;N-1}$ is the Student-Fisher parameter taken for $N-1$ degrees of freedom and for the appropriate probability (0.05 unilateral for 95% confidence level in our case). See table 3 to get characteristic values of Student-Fisher parameter.

<table>
<thead>
<tr>
<th>Table 3 – Student-Fisher parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student parameter</td>
</tr>
<tr>
<td>Confidence level : 95% (unilateral)</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

The Student-Fisher gives an appropriate estimator of the average value of the population especially in the following cases :

- the statistical distribution of the population follows a Normal law,
- the standard deviation of the population is unknown,
- the size of the sample is small (typically less than 50).

4.2 Application to non-linear structures

4.2.1 Case one : Impact non-linearity - In-structure (floor) seismic motion – Artificial time histories – High number of time histories

In this part, Student-Fisher estimator is used to predict the average value of the population from the previous results shown in figure 6. The estimation of the average value of the population with a confidence level of 95% is then evaluated and compared to the calculated one (500 calculations). Figure 8 shows the comparison of the
estimated average value of the population using Student-Fisher estimator for the different sets of accelerograms and its evolution with the number of accelerograms used to calculate it within a set.

**Figure 8 – Use of Student estimator - Evolution with the number of accelerograms**

Impact non-linearity

One can see that the use of Student-Fisher estimator (95% confidence level) leads to a conservative prediction of the average value of the population, even with a low number of accelerograms. One can also observe that, the higher the number of accelerometer is, the more accurate the estimation is.

For low number of results (less that 10 in that situation), estimation may be very conservative in some case (+50% approximately), it may also be a little under conservative in some cases (one case in fig. 8, for 3 and 4 calculations). For a high number of results, this estimator tends to a precise estimation of the average value of the population.

4.2.2 Case two : Plastic non-linearity - Seismic ground motion – Natural time histories

The results presented in paragraph 3.2.1 are used once again but with Student-Fisher estimator to show the effect of the estimator on another type of non-linearity. Figure 9 shows the results (calculated ductility drift) obtained for frame type structure with natural accelerograms.

**Figure 9 – Use of Student estimator – RCS plastic non-linearity**

One can observed that in that situation, the use of Student-Fisher estimator (95% confidence level) also leads
to an accurate and conservative prediction of the average value of the population, even with a low number of accelerograms. Once again, the higher the number of accelerogram is, the more accurate the estimation is.

4.2.3 Case three: Plastic non-linearity - Seismic ground motion – Artificial time histories

The results presented in paragraph 3.2.2 are used here but with Student-Fisher estimator to show the effect of the estimator on the prediction of the average value. Figure 10 shows the results (calculated ductility drift) obtained for wall type structure with natural accelerograms.

One can observed that in that situation, the use of Student-Fisher estimator (95% confidence level) also leads to an accurate and conservative prediction of the average value of the population, even with a low number of accelerograms. Once again, the higher the number of accelerogram is, the more accurate the estimation is.

4.3 Analysis of results

In most of cases, the use of Student-Fisher estimator (95% confidence level) leads to a conservative prediction of the average value of the population, even with a low number of accelerograms. This result is confirmed with two different types of non-linearity (impact and plastic non-linearities).

In addition, the results show that the higher the number of accelerogram is, the more accurate the estimation is. In that situation, a relative freedom is left to the designer for engineering purpose (more calculations can be performed to get an optimized estimation of the design parameter or a lower number of calculations is possible but with a more conservative estimation of the design parameter).

5 CONCLUSIONS AND PERSPECTIVES

The objective of this study was to give some elements concerning the number of accelerograms to be used for non-linear computations and the way to account for scattering of results.

Our results show that for low number of accelerograms (typically less than 10), the scattering on the average value of the non-linear parameter may be very important depending on the different sets of accelerograms used for the calculations. In another hand, the scattering decreases but remains significant for high number of accelerograms and the convergence to the average of the population is very low, even for numbers of accelerograms higher than 30.
In that context, a method to account for variability of the results is proposed, based on Student-Fisher statistical estimator and allows to estimate the design parameter with an appropriate confidence level. This method leaves a relative freedom to the designer for engineering purpose (more calculations can be performed to get an optimized estimation of the design parameter or a lower number of calculations is possible but with a more conservative estimation of the design parameter).

This study will be extended in the future with other earthquake characteristics (high magnitude - far field earthquakes; low magnitude - near field earthquake especially) and with other types on non-linearities (uplift, sliding and rocking of rigid bodies ...).

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