

MITIGATION OF ELEPHANT-FOOT BULGE FORMATION IN SEISMICALLY-EXCITED STEEL STORAGE TANKS

MEDHAT A. HAROUN

*Dean and AGIP Professor, School of Sciences and Engineering
The American University in Cairo, Egypt,
and Professor Emeritus, University of California, Irvine, USA*
maharoun@aucegypt.edu

ABSTRACT

This paper summarizes the findings of the first phase of a comprehensive study aimed at mitigating the effects of earthquakes on steel liquid storage tanks using hybrid fiber reinforced polymer laminates. In assessing the safety of tanks under seismic loading, the capacity of the shell against buckling must be evaluated. There are two types of shell “buckling”: membrane and elastic-plastic. Although the latter is not accounted for in most codes, it may limit the seismic design especially at higher values of total liquid pressure. The elastic-plastic buckling capacity is nearly depleted when the ratio of total-to-hydrostatic pressures is equal to the design factor of safety. In order to relieve the shell from excessive hoop stresses, it is suggested to apply hybrid fiber reinforced polymer laminates, with the fibers aligned along the circumference of the shell. The combined system of carbon and glass fibers must circumvent the galvanic reaction with steel, in addition to exhibiting thermal balance and appropriate durability properties. It has been confirmed theoretically that the proposed system is viable for reducing seismic vulnerability of tanks; experimental verification is underway.

Keywords: hybrid FRP system, shell buckling, hydrodynamic pressure, storage tanks, seismic retrofit

1. INTRODUCTION

Liquid storage tanks are important elements of lifeline and industrial facilities. The evolution of codes and standards for the seismic design of these structures has relied greatly on observations of tank damages during past earthquakes, yet the time lag between acquiring the information and implementing the findings in practice has remained relatively long. Even though current codes and standards reflect a mature state of knowledge for tank design, recent earthquakes as well as advanced state-of-the-art analyses continued to point out to a few overlooked issues. Failure modes of ground-based tanks included, among others, shell buckling which is typically characterized by diamond-shaped buckles or “Elephant’s Foot” bulges which appear a short distance above the base.

For steel tanks, demand requirements arise from the hydrodynamic forces and should be less than the capacity. The buckling capacity is the lesser of the membrane buckling stress or the elastic-plastic buckling stress, yet only the former is used in current codes. The cause of the “Elephant’s Foot” bulge formation in many tanks has been misdiagnosed. Accordingly, most retrofit techniques employed over the past three or so decades have failed to eliminate the problem. It is postulated that the problem is a plastic flow from excessive hoop stresses resulting from the hydrodynamic pressures; this is consistent with field observations in past earthquakes. Once the cause of the problem has been correctly diagnosed, then limiting the increase of circumferential hoop stresses in the steel tank shell due to hydrodynamic pressures delays the onset at which plastic flow occurs. This can effectively be accomplished by applying fiber composite laminates to the shell. Carbon-fiber laminates applied to the bottom section of the tank can provide the needed circumferential confinement. However, they cannot be applied directly to

steel shells due to galvanic reactions. This problem can be avoided by a hybrid system using an E-glass layer to isolate the carbon from the steel shell.

The main goal of the present study is to devise a simple and cost-effective system to retrofit deficient steel liquid storage tanks using fiber reinforced polymers. The study is divided to a theoretical phase in which the viability of this approach is assessed, and an experimental verification phase in which the hybrid FRP system is tested for mechanical and durability properties, and the effectiveness of the installed FRP system on tanks is evaluated by shaking table tests. The theoretical approach is further divided to two main tasks: The first identifies all pertinent factors affecting the formation of “Elephant’s Foot” bulges. These include tank properties such as height, radius, and thickness; liquid-shell interaction which depends on liquid height and density; tank support whether anchored or unanchored; and ground motion characteristics such as peak horizontal accelerations, ratio of peak vertical/peak horizontal accelerations, and ground motion frequency content. The second task implements the new hybrid composite system of carbon and E-glass laminates on a numerical model of the shell to demonstrate its effectiveness in retrofitting deficient tanks.

2. SHELL BUCKLING CAPACITY

A critical aspect in the earthquake resistant design or retrofit of steel, cylindrical tanks has been “Elephant’s Foot” buckling. It occurs near the tank base predominantly due to axial stress in the tank wall, though it is significantly affected by the circumferential membrane stress caused by hydrostatic and hydrodynamic pressures. The axial compressive stress developed at the base of tanks under seismic excitations must be less than an allowable buckling stress to preclude the occurrence of shell buckling. The allowable stress in current codes and standards is basically specified for a uni-directional stress state whereas the actual stress state at the shell bottom is bi-axial. Figure 1 shows the axial and hoop stress distributions along the tank height (H) for anchored and unanchored tanks.

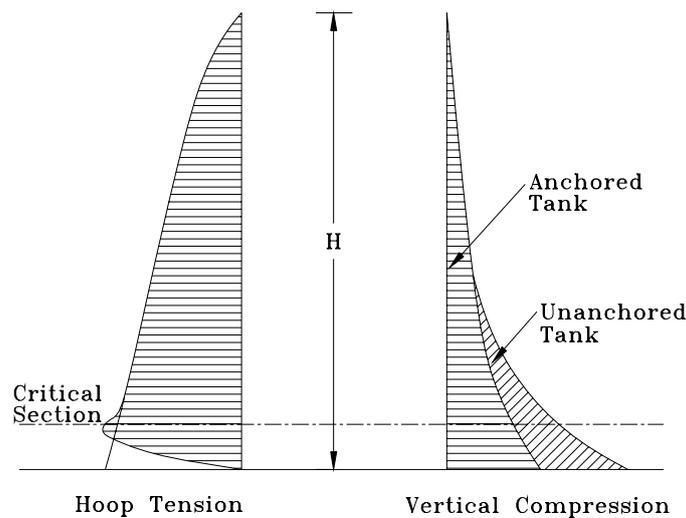


Figure 1. Stress Distribution along the Height of Liquid Storage Tanks.

2.1 Elastic Buckling Stress

The axial membrane stress needed to induce buckling in a shell depends on the internal pressure, circumferential variation of axial stress, and amplitude of imperfection in the shell. The latter tends to decrease the buckling stress to a fraction of the classical buckling stress. A knock-down factor can be estimated based on the amplitude of imperfection, which in turn depends on the quality of construction, tank radius, and shell thickness. The internal pressure reduces the effective imperfection amplitude, and therefore, increases the buckling stress. Circumferential variation of the axial stress reduces the probability of coincidence of the maximum stress and the maximum imperfection, again increasing the buckling stress. The classical buckling stress, f_{cb} , is defined as

$$f_{ct} = 0.6E_s \frac{t}{R} \quad (1)$$

where E_s is the modulus of elasticity of steel, t is the shell thickness, and R is the shell radius. A knock-down factor is usually assumed to estimate the actual capacity; in this study, an average conservative knock-down factor of 5 is assumed. Thus, the allowable (knocked-down) membrane buckling stress, f_{mb} , can be computed.

2.2 Elastic-Plastic Buckling Stress

The bottom of the shell is normally subjected to a bi-axial stress state consisting of hoop tension and axial compression. Radial deformations under the internal pressure create additional eccentricity, tending to induce the commonly observed “Elephant’s Foot” buckling as shown in Figure 2.



Figure 2. “Elephant’s Foot” Buckling.

Elastic-plastic collapse adjacent to the boundary may be assessed by using the relation presented in the New Zealand guidelines (Priestley, 1986). It assumes a quadratic reduction of the classical buckling stress depending on the factor $(PR/t\sigma_y)$ where P is the internal pressure and σ_y is the yield stress of steel. This factor essentially represents the effect of shell yielding in the circumferential direction due to internal pressures on the buckling capacity of the shell. Although this relation was developed from predictions of collapse adjacent to pinned-base in ground silos, fixed-base details are slightly stronger.

2.3 Buckling in Current Standards

The allowable buckling stress in AWWA (American Water Works Association) and API (American Petroleum Institution) standards is based on the classical value of buckling stress under axial load, significantly reduced by a large knock down factor due to shell imperfections and also increased to account for the effects of internal liquid pressure. In the New Zealand guidelines, classical buckling stress in membrane compression is calculated and corrected according to imperfection amplitude and internal pressure. In addition, an elastic-plastic collapse stress is also calculated. The lower of the two stresses is used in the computation of the margin of safety.

2.4 Advanced Finite Element Analysis of Shell Buckling

An advanced nonlinear finite element analysis was performed by Haroun and Bhatia (1997) to evaluate the buckling capacity of a variety of liquid-filled shells. Hydrostatic and hydrodynamic pressures on the tank wall cause circumferential (hoop) tensile stresses in the shell. Earthquake data have always indicated that no seismically induced failures due to hoop stress have occurred, but the primary effect of the circumferential hoop stress is to influence the axial buckling capacity of the tank wall. The finite element model must have a sufficient mesh density

to correctly capture the buckling behavior. The program “MARC” was used for the analysis with a high mesh density, and large displacement analysis was carried out. The shell was loaded incrementally until radial displacements increased rapidly. The hydrostatic and hydrodynamic pressures were applied as pressure values at the element integration points through a user subroutine, and were integrated using the element shape functions to obtain nodal forces. The tank model was first subjected to hydrostatic pressures, and then pseudo-hydrodynamic loads were incrementally added until buckling was initiated. The axial stress necessary to cause buckling for various height-radius ratios and for different shell thicknesses was compared to that obtained using the following simplified formula

$$f_{pb} = f_{cl} \left(1 - \left(\frac{PR}{t\sigma_y} \right)^2 \right) \quad (2)$$

Throughout this study a yield stress of steel $\sigma_y = 250$ MPa (36 Ksi) is used. Sample values of the buckling stress obtained by the finite element analysis are shown in the last column of Table 1.

Table 1. Comparison of Classical and Elastic-Plastic Buckling Stress

(H/R)	(t/R)	f_{cl} (MPa) [Equation 1]	f_{pb} (MPa) [Equation 2]	f_{pb} (MPa) [Haroun & Bhatia (1997)]
1.0	0.001	124.1	0*	19.12
1.5	0.001		0*	24.20
2.0	0.001		0*	29.32
1.5	0.002	248.2	34.18	34.96
2.0	0.002		37.11	47.19
2.5	0.002		38.28	50.04
2.5	0.003	372.3	78.25	64.73

*Internal pressure causes hoop stress exceeding σ_y

It should be noted that elastic buckling is critical only for low values of membrane circumferential stress (PR/t) coupled with high compression stress in the shell. This combination is expected to be rare. With an increase in the membrane circumferential stress, elastic-plastic buckling becomes the dominant mode. The stresses obtained from the finite element analysis clearly point towards elastic-plastic buckling in the analyzed cases. A comparison between the finite element results and those of the simple formula shows a close correlation, with the values of the elastic-plastic buckling stress well below the elastic buckling stress. It should be noted that the pressure used in the elastic-plastic buckling equation is the sum of the static and pseudo-dynamic pressures.

2.5 Enhancement of Buckling Stress Computation

The nonlinear finite element study performed by Haroun and Bhatia (1997) showed that “Elephant’s Foot” buckling is the most likely mode in a typical tank. It generally confirmed the accuracy of the approximate formula for buckling capacity (Equation 2). Neither the API nor the AWWA standards consider elastic-plastic buckling which has been typically observed as elephant-foot buckling at the base of earthquake vulnerable tanks. These codes should adopt the proven simple formula for elastic-plastic buckling to minimize the occurrence of such damage in future seismic events.

3. SEISMIC BEHAVIOR OF ANCHORED TANKS

The seismic response of fluid-filled tanks to earthquake excitations should combine its response to both horizontal and vertical components of ground motion. The horizontal component of the earthquake produces lateral shear force and overturning moments on the tank. However, the vertical component results in an axisymmetric increase or decrease of the hydrodynamic pressure with no additional lateral forces. Thus, unlike other structures, vertical

earthquake motion may play a measurable role due to the development of additional hydrodynamic pressures which, in turn, induce hoop stresses in the shell.

3.1 Tank Under Horizontal Excitation

The tank under consideration is a circular cylindrical steel tank of radius R and thickness t , filled with liquid to a height H . Tank response to a horizontal component of an earthquake can be reasonably represented by the mechanical model presented by Haroun (1983). There are three modes of response. The first is a low-frequency mode due to liquid sloshing (convective mode) in which the contained liquid sloshes within the tank with negligible interaction with the deformation of the tank shell. The other two modes are the impulsive-flexible component which represents the interaction between the liquid and tank deformation, and the impulsive-rigid component in which part of the tank and the contained fluid move as rigid body with no contribution from shell deformation. The mechanical analog thus contains three equivalent masses m_i (convective mass), m_o' (impulsive mass), m_f (impulsive mass associated with wall flexibility) along with their heights, and two equivalent springs with periods representing sloshing and liquid-shell vibrations. Overturning moment exerted on the tank, neglecting the small contribution of the convective mass, can be expressed as

$$M_{\max(f)} = m_o' h_o' \ddot{u}_g + m_f h_f S_f \quad (3)$$

where S_f is the spectral acceleration corresponding to the impulsive-flexible vibration mode. The axial stress developed at the base of the shell is therefore calculated from

$$\sigma_{(f)} = \frac{W_s}{A} + \frac{M_{\max(f)}}{\pi R^2 t} \quad (4)$$

3.2 Vertical Excitation

The vertical component of an earthquake has a negligible contribution to the base shear and overturning moment, which are mainly caused by the two horizontal components. However, the vertical component of an earthquake amplifies the axisymmetric pressure. The peak hydrodynamic pressure occurs near the base of the tank and is approximately equal to a fraction of the peak hydrostatic pressure. This is computed by replacing the acceleration of gravity by the spectral vertical acceleration corresponding to the period of vibration of shell under axisymmetric vibration.

3.3 Liquid Pressure

The total pressure at the tank base, including the hydrostatic pressure and that resulting from the horizontal and vertical components of an earthquake, is given by

$$P = P_s + P_d \quad (5)$$

The hydrostatic pressure is expressed as

$$P_s = \gamma_w H \quad (6)$$

where γ_w is the unit weight of liquid (water).

The pressure at the base of the tank due to the horizontal earthquake component is made up of both impulsive and sloshing components. Because the maxima of the impulsive pressure, convective pressure, and the pressure due to the vertical components of the earthquake do not occur at the same time, it is estimated that the maximum value of the total dynamic pressure is

$$P_d = \sqrt{P_i^2 + P_c^2 + P_v^2} \quad (7)$$

The second term of Equation 7 is the contribution of the convective mode which is small and may be neglected. The impulsive pressure for flexible tank is proportional to the spectral horizontal acceleration and can be calculated from

$$P_{if} = q_o \gamma_w H A_h \frac{\ddot{u}_g}{g} \quad (8)$$

where A_h is the normalized spectral horizontal acceleration and the coefficient q_0 may be obtained from the chart displayed in Figure 3.

The pressure due to the vertical component of the earthquake can be evaluated from

$$P_{vf} = 0.5\gamma_w H A_v \frac{\ddot{u}_g}{g} \quad (9)$$

in which it was assumed that the peak vertical ground acceleration is half of the peak horizontal ground acceleration, and A_v is the normalized spectral vertical acceleration.

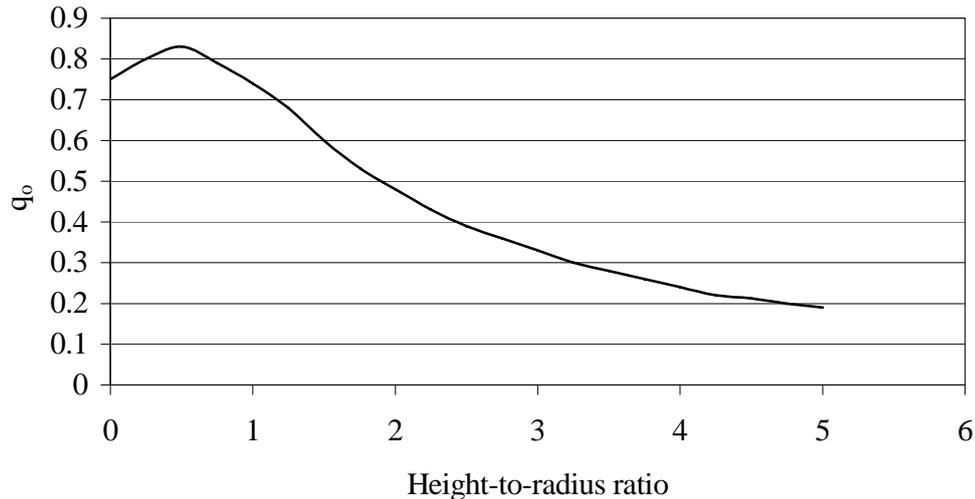


Figure 3. Pressure Coefficient (q_0).

4. THICKNESS OF TANK SHELL

The thickness of the tank shell plays a dominant role in determining the shell buckling capacity. It is primarily evaluated from consideration of allowable hoop stress under hydrostatic pressure, and therefore, it is dependent on tank radius, liquid height, yield stress of steel, and the factor of safety. The latter is viewed as a reduction from the yield stress of steel to provide the allowable hoop stress. The equation for calculating the thickness is

$$t = \gamma_w H R r / \sigma_y \quad (10)$$

where r is the reduction factor as defined above.

5. DESIGN OF A NEW TANK SHELL FOR SEISMIC LOADS

Shell membrane buckling limits the design at low total liquid pressure but the effect of elastic-plastic buckling becomes apparent with the increase of earthquake acceleration accompanied by higher (H/R) values. High dynamic pressures cause the elastic-plastic buckling stress to dip below the knocked-down membrane buckling stress. Therefore, the problem arising from such a low elastic-plastic buckling stress can be controlled by selecting an appropriate reduction factor that ensures the elastic-plastic buckling does not limit the design. This is satisfied if

$$\left(\frac{P/P_s}{r} \right)^2 \leq 0.8 \quad (11)$$

where the ratio between the total to static pressures equals to

$$\frac{P}{P_s} = \left(1 + \left(\frac{\ddot{u}_g}{g} \sqrt{(A_h q_o)^2 + (0.5A_v)^2} \right) \right) \quad (12)$$

For a given reduction factor, one can calculate the above ratio and use Equation 11 to test for elastic-plastic buckling. If Equation 11 is not satisfied, one may need to increase the reduction factor. It should be noted that using Equation 11 does not guarantee a safe design; however, it ensures that elastic-plastic buckling is controlled especially at higher earthquake accelerations.

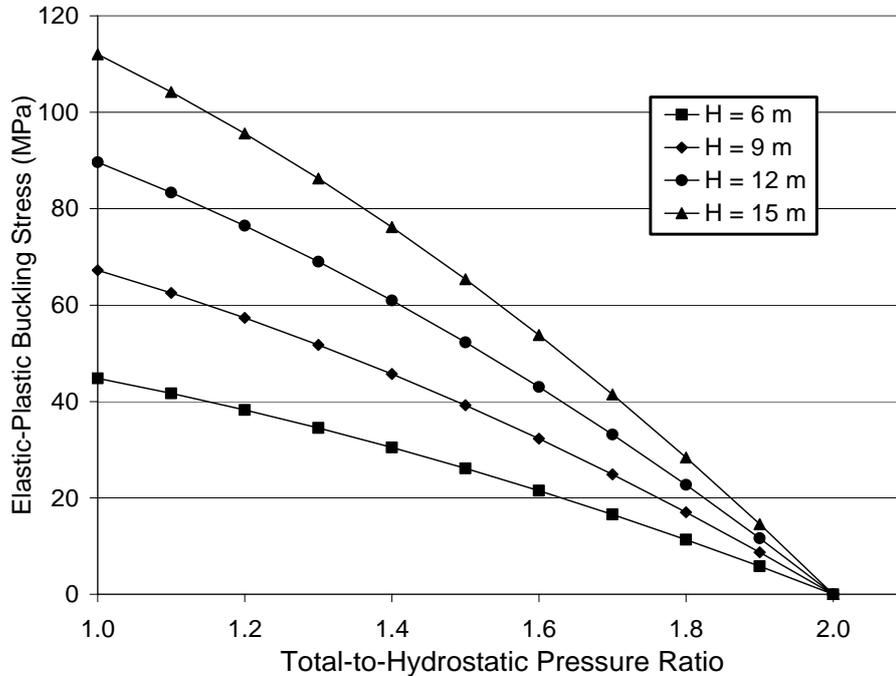


Figure 4. Variation of Elastic-Plastic Buckling Stress (Reduction Factor $r = 2$).

Different tank dimensions were chosen and analyzed for varied earthquake loading intensities (Haroun and Al-Kashif, 2005). The parameters used were $H = 6, 9, 12,$ and 15 m (20, 30, 40, and 50 ft), $R = 6, 9, 12,$ and 15 m (20, 30, 40 and 50 ft), and peak horizontal accelerations of 0.2g, 0.3g, 0.4g and 0.5g. The tanks were analyzed, the developed stresses were calculated and the margin of safety for each case was inspected. Unsafe tanks were identified and an increase of the reduction factor was introduced to re-design each of those unsafe tanks.

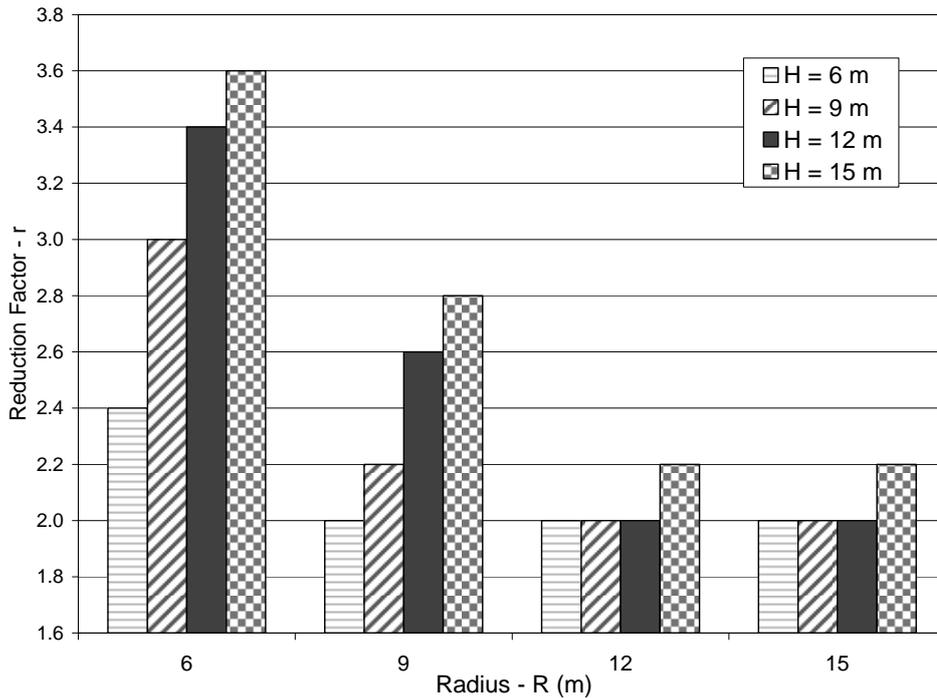


Figure 5. Reduction Factors to Provide Safe Tanks ($\ddot{u}_g = 0.4g$).

The margin of safety is the ratio between the allowable buckling stress and the maximum actual axial stress. When the margin of safety is less than 1, then the design is unsafe. Several of the tanks investigated in this study showed a margin of safety smaller than 1 when a uniform safety factor was applied initially to determine the shell thickness. Tanks subjected to higher earthquake intensities or having higher (H/R) ratios were mostly unsafe. The redesign process involves an iterative approach by increasing shell thickness through increasing the reduction factor. Figure 5 and 6 show examples of design charts to select the appropriate reduction factor for peak ground accelerations of 0.4g and 0.5g, respectively, and for different values of R and H.

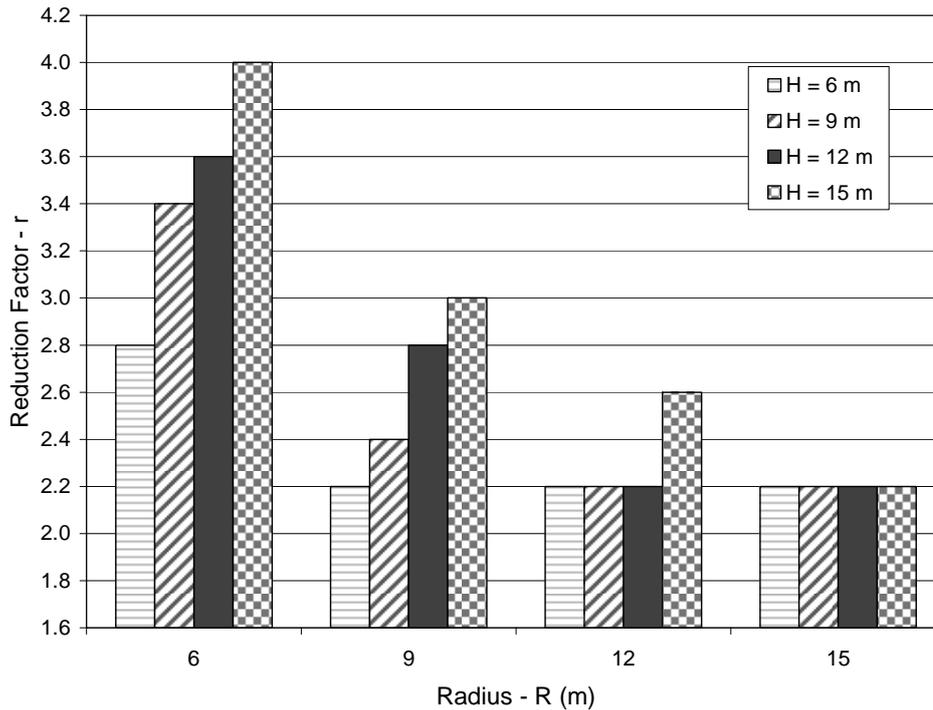


Figure 6. Reduction Factors to Provide Safe Tanks ($\ddot{u}_g = 0.5g$).

6. RETROFIT OF TANK SHELL WITH HYBRID FRP SYSTEM

If an existing tank is evaluated according to the procedure outlined earlier and found unsafe, hybrid FRP laminates may be applied on the shell to effectively participate in carrying the circumferential hoop stress due to the excessive hydrodynamic pressure. It should be noted that the full effectiveness of the composite retrofit can only be realized when it is applied when the tank is empty.

To demonstrate the effectiveness of the proposed retrofit technique, a quasi-static large displacement finite element analysis was performed using the software "MARC" on a tank with $H/R = 2$ and $t/R = 0.001$. The salient features of the finite element analysis were:

- A high-density mesh was devised to correctly model the buckling behavior.
- The aspect ratio of shell elements was close to 1.
- A large displacement analysis method was used instead of the eigenvalue procedure. This approach incrementally loads the structure until displacements increase rapidly and cause divergence. This is more accurate when material nonlinearity may influence buckling.
- A linearly-elastic perfectly-plastic material model is used to model the steel shell. Orthotropic material properties were used to model the composites. The material properties assigned in the analysis are displayed in Table 2. The thickness of each layer is 1 mm (0.04 inch).
- A layered shell element was employed to model the composite layers which were attached to the steel shell. This assumes a perfect bond between the composites and the steel.
- A full Newton-Raphson solution approach was used for its better convergence characteristics when very large displacements are obtained.
- The hydrostatic and hydrodynamic pressures were applied as pressure at the element integration points through a user subroutine and were integrated using the element shape functions to obtain nodal forces.

Table 2. Material Properties

Material	Modulus of	Yield Stress	Ultimate
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	Elasticity E (GPa)	σ_y (MPa)	Stress σ_u (MPa)
Steel	206.9	250	---
E-Glass*	27.58	---	550
Carbon*	68.95	---	930

*Assumed properties for installed composite laminates.

A plot of the shell radial displacement from the finite element analysis is displayed in Figure 7 at a node in the region where buckling has occurred. It demonstrated at which level the displacement has increased without bound. As it is not specifically defined when actual failure might occur, it is assumed that it occurs when the ratio of radial displacement to thickness reaches 10. The use of an isolation layer of E-glass plus one laminate of carbon increased the axial load capacity by 74% whereas the use of the isolation layer plus four laminates of carbon almost tripled the buckling capacity of the unretrofitted tank.

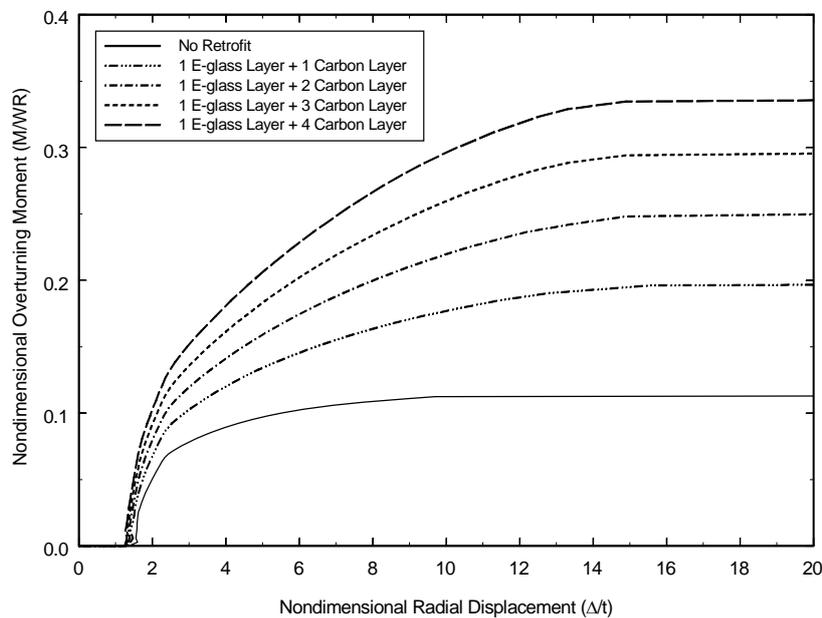


Figure 7. Radial Displacement of Retrofitted Anchored Tank ($H/R = 2$ and $t/R = 0.001$).

“Elephant’s Foot” bulging usually extends completely around the bottom of tanks. It is assumed that when the earthquake acceleration reserves direction, it causes the tank shell to buckle in the portion of the shell opposite to the region of the initial buckle. This hypothesis was confirmed by the finite element analysis. The tank was loaded with the hydrodynamic pressure in one direction until primary buckling was obtained, then the load was reversed until secondary buckling was obtained. The secondary buckle connected up with the primary buckle and produced complete “Elephant’s Foot” bulging all around the circumference as in tanks in the field.

The deformed shape of the tank, illustrated in Figure 8, clearly shows that elephant foot buckling has developed near the base of the tank. Figure 9 shows the distribution of the hoop stress in the FRP laminates and the steel tank shell in the circumferential direction when buckling occurred. It is clear that the hoop stress in the steel is near or at its yield stress, and the stress in the FRP laminates is below and within the range of ultimate stress in the composite materials.

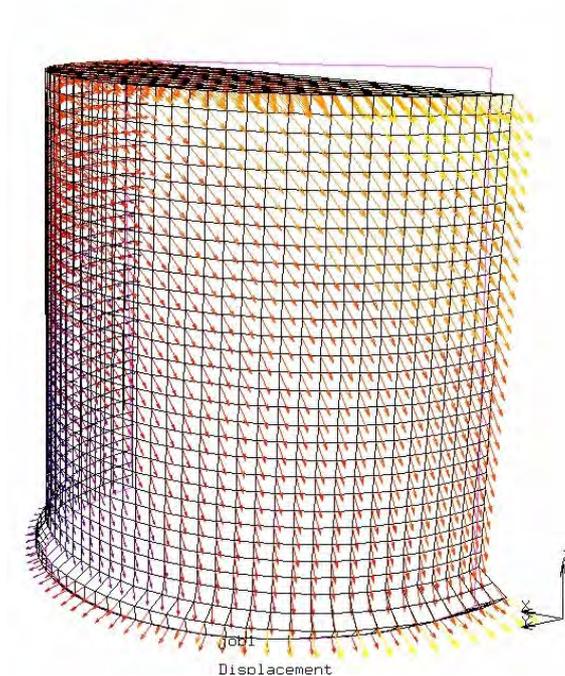


Figure 8. Delayed Buckling of Retrofitted Tank.

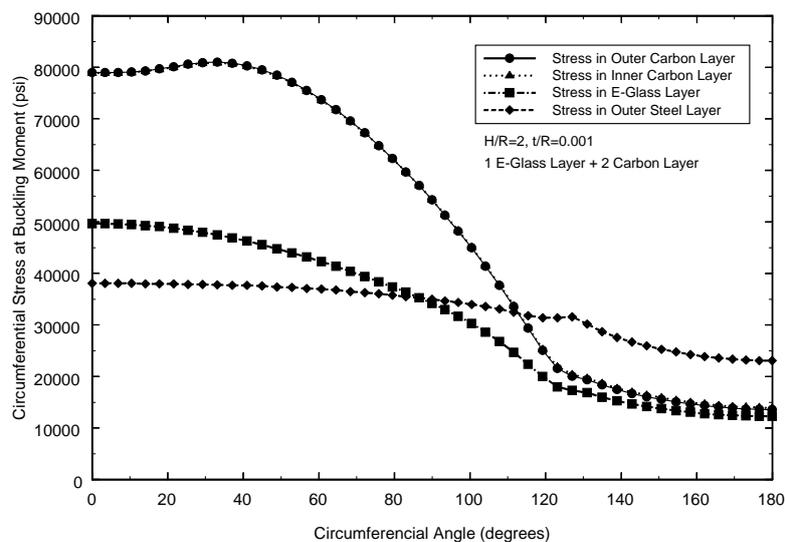


Figure 9. Circumferential Stress Distribution at Buckling (1 Ksi = 6.895 MPa).

7. EXPERIMENTAL RESEARCH

It has been demonstrated theoretically that a hybrid FRP system can effectively increase the buckling capacity of the tank shell by participating in carrying the excessive hoop stress in the steel shell which causes “Elephant’s Foot” bulging. However, the proposed system must undergo experimental verification tests before it is actually recommended in engineering practice. There are two sets of experimental tests that must be carried out: the first pertains to the performance of the hybrid FRP system alone, and the second evaluates the performance of the retrofitted tanks. The former set of tests is underway to determine the durability properties of the hybrid system, and in particular, determine its environmental reduction factors. In addition, the combined system of glass and carbon must be thermally balanced, and for this a sandwich-type laminate system is recommended. Once the properties of

the hybrid FRP system are accurately predicted, a set of shake table tests on retrofitted tanks filled with liquid and subjected to earthquake time histories will be carried out to confirm and calibrate the theoretical findings of this study.

8. CONCLUSIONS

- The elastic-plastic buckling stress may limit the seismic design of liquid storage tanks, especially at higher values of the total liquid pressure. Such an elastic-plastic buckling capacity is depleted when the ratio of total-to-hydrostatic pressures is equal to the factor of safety used in computing shell thickness. It is noted that the elastic-plastic buckling phenomenon is not included in most commonly used standards for liquid storage tanks.
- The factor of safety adopted in current design codes for computing shell thickness must not be applied uniformly to all tanks. Tanks with larger height-to-radius ratio, when subjected to high peak earthquake accelerations, tend to be unsafe if originally designed under such a factor of safety. A formula is suggested to modify the factor of safety (reduction factor) in the initial design phase of the tank to ensure that the elastic-plastic buckling stress does not dip below the knocked-down membrane buckling stress. Such a formula depends primarily on the height-to-radius ratio of the tank and the expected peak earthquake acceleration.
- For existing deficient tanks, a hybrid FRP system of glass and carbon is recommended, based on an advanced finite element analysis, for enhancing the elastic-plastic buckling capacity. The number of laminates needed to resist the excessive hoop stress resulting from the hydrodynamic pressures can be determined based on the required percentage increase in the elastic-plastic buckling stress.
- Subject to experimental verifications of the desired properties of the hybrid FRP system and of the dynamic behavior of retrofitted tanks undergoing shake table tests, the proposed FRP retrofit scheme is promised to be structurally efficient as well as cost effective.

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