Transactions of the **14th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 14), Lyon, France, August 17-22, 1997 **B05/6**

A FEM algorithm for analysis of hysteretic systems by using N-node elements

Hosseini M.(1), **Ansari F.**(2)
(1) International Institute of Earthquake Engineering and Seismology, Iran
(2) Itok-Iran, Iran

ABSTRACT

This paper presents a finite element analysis algorithm for static and dynamic analysis of hysteretic systems by using elements which can have any arbitrary number of nodes. Nonlinear behavior of the system due to either large deformation or material plasticity is taken into account by : 1) considering the higher order derivatives of the deformation functions, and 2) computing the response for the deformed shape of the system rather than its initial unreformed shape by modifying the nodal coordinates in each time step.

INTRODUCTION

Many components of the NPP facilities subjected to severe loadings such as earthquake induced forces show hysteretic behavior. On the other hand, there are only a few FEM computer programs, which are capable to deal with analysis of hysteretic systems. Besides, these programs are usually based on some assumptions, specially in the case of dynamic response analysis, which decrease the precision of the response values, so that they can not be used for systems with severe nonlinearity.

Recently, some researchers have tried to present more precise algorithms, which are specially required for the study of impact, contact and crash problems, [1] and [2]. Also, many investigators have tried to develop some computer programs for the study of severely nonlinear problems, [3], but still there are some deficiencies in the precision of the results.

In this paper an algorithm is presented which has the capability of considering large deformations of the system in both elastic and inelastic ranges with a high precision, even in the case of severe nonlinearity. In this algorithm there is no limitation on the number of nodes in finite elements, which means no limitation on the shape of elements as long as they are not concave. This capability usually makes it possible to consider only one element for many of the structural members in a system, although all kinds of commonly used elements such as bar elements, plane stress and plane strain elements, and also solid elements can be used easily. Considering the higher order derivatives of the deformation functions, and computing the response for the deformed shape of the system rather than its initial unreformed shape are the two main features of the proposed algorithm.

To show some of the capabilities of the introduced algorithm a computer program has been prepared and the results of some numerical examples are presented in this paper. The program is capable to combine the results of static analysis with those of dynamic analysis. It can also be used for the push-over as well as support excitation analyses. The results of analyses in all of these cases are shown graphically for more convenience.

METHOD

To consider the hysteretic behavior of the system elements a bilinear load-deformation model has been used, although any other nonlinear relationship can be used instead. In the case of elasto-plastic behavior every one of the failure criteria such as Tresca, von Mises, Mohr-Coulomb, Drucker-Prager and Rankine can be used [4]. The shape functions used for the system elements are defined by using Lagrange expansions, which make possible the consideration of N-node elements. The integrals required for stiffness calculation of elements are evaluated by Gaussian Integration Technique. The precision level of integration can be controlled by the user, and the response accuracy depends on the number of nodes defined by the user for different elements.

The system governing equations of deformation in static state and those of displacement and deformation in dynamic state are solved by Cholesky method using LU decomposition and stored in the computer memory by the sky-line method. For solving the nonlinear equations, governing the system behavior the BFGS method, which is one of the Quasi-Newton methods, is used. This method have been shown to result in high efficiency in both static and dynamic analyses, [5]. Finally, by taking the advantages of some programming skills such as using the virtual memory, the authors have tried to increase the efficiency of the proposed algorithm. The flowchart of the program for the case of nonlinear dynamic analysis is shown in figure 1.

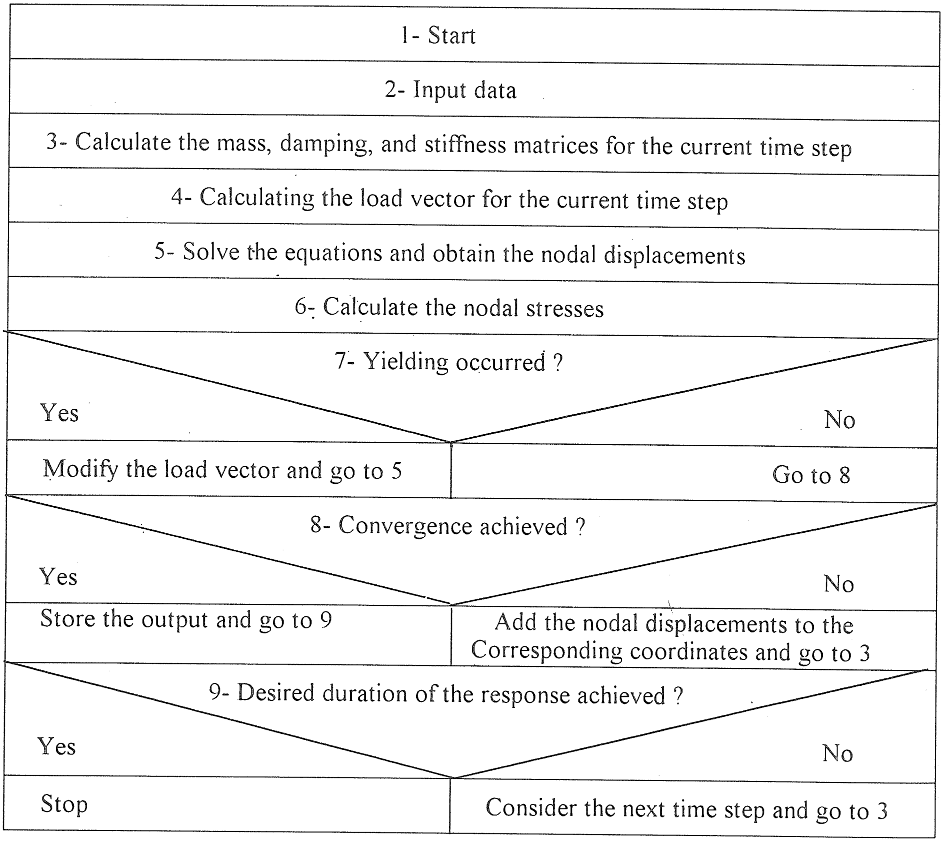
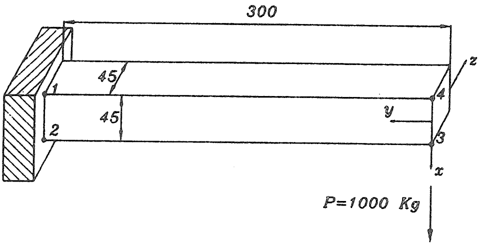


Fig. 1: The flowchart of the program for the case of nonlinear dynamic analysis

NUMERICAL RESULTS

Because of the large volume of the numerical results of the finite element problems, two simple and relatively brief examples are presented here, which have been picked up from one of the authors' recent works, [6], where the reader can find more examples. As the first example, a cantilever beam which has been modelled by only one plane strain element, once with 4 nodes, then with 9 nodes and finally with 16 nodes under a point load at its end is considered. The results of these three cases are shown in figures 2 to 4 respectively along with the results of exact solution to show the capability of the algorithm in defining only one element for one structural member and also to show the effect of the number of nodes considered for elements in increasing the accuracy of the results. It can be seen that in all of these three cases the reactions are exactly the same for calculated and actual results, but in the case of 4-node element the displacements and stresses are quite different from the exact values, in the case of 9-node element they have less error and in the case of 16-node element they are of satisfactory precision, and the error is about 10-13% for displacements and major stresses. The differences between the stress values in other directions rather than the major direction in comparison with the exact solution is due to defining a plane strain element for the beam, which is a little different from the real state.

Cantilever beam is modeled with 4 nodes plane strain element



NOTE :

REACTIONS & DEFLECTIONS

Actual results are based on nutral axis of sections

$R=V=P$

$M_Y = PY$

$\delta_Y = P/6EI(2L^3-3L^2Y+Y^3)$

STRESSES

Actual results are based on AIRY stress function

$\sigma_{YY} = -3/2(Pxy/h^3)$ $\quad \sigma_{XX} = 0$

$\sigma_{XY} = 3P/4h(X^2/h^2-1)$ $\quad \sigma_{XZ} = 0$ $\quad ; h=45/2$

$\sigma_{ZZ} = \nu(\sigma_{XX}+\sigma_{YY})$ $\quad \sigma_{YZ} = 0$

Calculated Results

```
Node Displacement                              time :  0

Nodes          ---- displacements ----
 No.           X            Y            Z

 1          0.00000      0.00000      0.00000
 2          0.00000      0.00000      0.00000
 3          0.00694      0.00077      0.00000
 4          0.00694     -0.00077      0.00000
```

```
Reactions & App. Forces                        time :  0

Nodes            ------- Forces -------
 No.             X            Y             Z

 1            -500.00     -6666.67         0.00
 2            -500.00      6666.67         0.00
 3             500.00         0.00         0.00
 4             500.00        -0.00         0.00
```

```
Existing Node Stresses                  time :  0

Nodes no.      X          Y          Z          XY

 1           0.309      0.721      0.309      1.867
 2          -0.309     -0.721     -0.309      1.867
 3           0.309      0.721      0.309     -0.880
 4          -0.309     -0.721     -0.309     -0.880
```

Actual Results

```
Node Displacement                              time :  0

Nodes          ---- displacements ----
 No.           X            Y            Z

 1          0.00000                   0.00000
 2          0.00000                   0.00000
 3          0.12542                   0.00000
 4          0.12542                   0.00000
```

```
Reactions & App. Forces                        time :  0

Nodes            ------- Forces -------
 No.             X            Y             Z

 1            -500.00     -6666.67         0.00
 2            -500.00      6666.67         0.00
 3             500.00         0.00         0.00
 4             500.00        -0.00         0.00
```

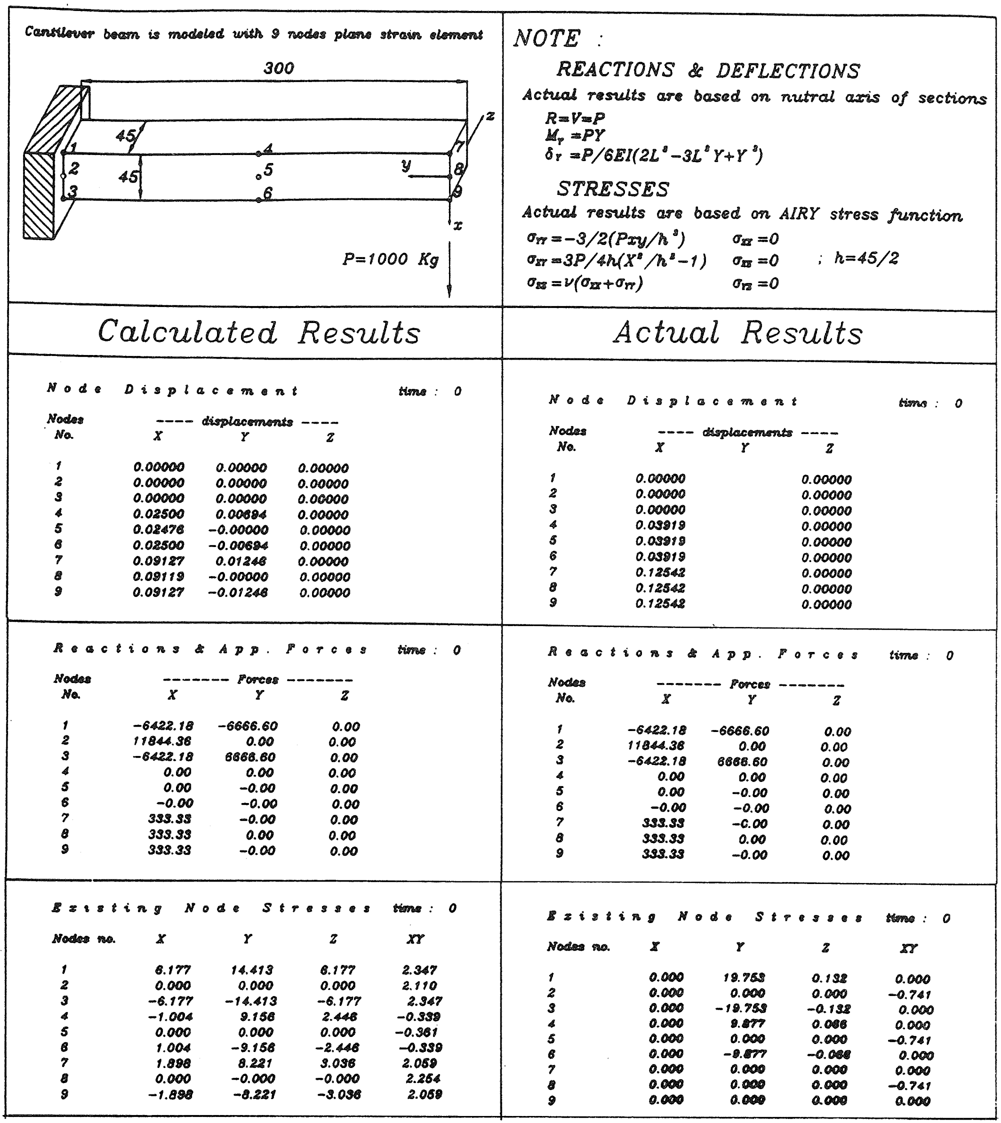
```
Existing Node Stresses                  time :  0

Nodes no.      X          Y          Z          XY

 1           0.000     19.753      0.132      0.000
 2           0.000    -19.753     -0.132      0.000
 3           0.000      0.000      0.000      0.000
 4           0.000      0.000      0.000      0.000
```

Fig. 2: The cantilever beam under point load modelled by one 4-node element

As the second example the bent structure of an existing bridge, shown in figure 5, under its weight and subjected to the lateral support excitation is considered. The results of analyses are shown in figures 6 to 9.

CONCLUSION

Based on the numerical results of several various examples it can be concluded that the proposed algorithm is an efficient algorithm for static and dynamic analysis of hysteretic systems, specially for study of severely nonlinear cases such as post-yield behavior and very large deformations of structures and their components.



Cantilever beam is modeled with 9 nodes plane strain element

NOTE :

REACTIONS & DEFLECTIONS

Actual results are based on nutral axis of sections

$R=V=P$

$M_Y=PY$

$\delta_Y=P/6EI(2L^3-3L^2Y+Y^3)$

STRESSES

Actual results are based on AIRY stress function

$\sigma_{YY}=-3/2(Pxy/h^3) \qquad \sigma_{XX}=0$

$\sigma_{XY}=3P/4h(X^2/h^2-1) \qquad \sigma_{XZ}=0 \qquad ; \ h=45/2$

$\sigma_{ZZ}=\nu(\sigma_{XX}+\sigma_{YY}) \qquad \sigma_{YZ}=0$

Calculated Results

```
N o d e   D i s p l a c e m e n t                    time :  0

Nodes          ---- displacements ----
 No.           X             Y            Z

 1         0.00000      0.00000      0.00000
 2         0.00000      0.00000      0.00000
 3         0.00000      0.00000      0.00000
 4         0.02500      0.00694      0.00000
 5         0.02476     -0.00000      0.00000
 6         0.02500     -0.00694      0.00000
 7         0.09127      0.01246      0.00000
 8         0.09119     -0.00000      0.00000
 9         0.09127     -0.01246      0.00000
```

```
R e a c t i o n s  &  A p p .  F o r c e s         time :  0

Nodes          ------- Forces -------
 No.           X             Y            Z

 1        -6422.18     -6666.60       0.00
 2        11844.36         0.00       0.00
 3        -6422.18      6666.60       0.00
 4            0.00         0.00       0.00
 5            0.00        -0.00       0.00
 6           -0.00        -0.00       0.00
 7          333.33        -0.00       0.00
 8          333.33         0.00       0.00
 9          333.33        -0.00       0.00
```

```
E x i s t i n g   N o d e   S t r e s s e s      time :  0

Nodes no.      X           Y           Z          XY

 1           6.177      14.413       6.177       2.347
 2           0.000       0.000       0.000       2.110
 3          -6.177     -14.413      -6.177       2.347
 4          -1.004       9.156       2.446      -0.339
 5           0.000       0.000       0.000      -0.361
 6           1.004      -9.156      -2.446      -0.339
 7           1.898       8.221       3.036       2.059
 8           0.000      -0.000      -0.000       2.254
 9          -1.898      -8.221      -3.036       2.059
```

Actual Results

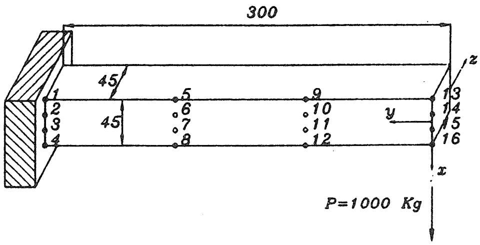
```
N o d e   D i s p l a c e m e n t                    time :  0

Nodes          ---- displacements ----
 No.           X             Y            Z

 1         0.00000                   0.00000
 2         0.00000                   0.00000
 3         0.00000                   0.00000
 4         0.03919                   0.00000
 5         0.03919                   0.00000
 6         0.03919                   0.00000
 7         0.12542                   0.00000
 8         0.12542                   0.00000
 9         0.12542                   0.00000
```

```
R e a c t i o n s  &  A p p .  F o r c e s         time :  0

Nodes          ------- Forces -------
 No.           X             Y            Z

 1        -6422.18     -6666.60       0.00
 2        11844.36         0.00       0.00
 3        -6422.18      6666.60       0.00
 4            0.00         0.00       0.00
 5            0.00        -0.00       0.00
 6           -0.00        -0.00       0.00
 7          333.33        -C.00       0.00
 8          333.33         0.00       0.00
 9          333.33        -0.00       0.00
```

```
E x i s t i n g   N o d e   S t r e s s e s      time :  0

Nodes no.      X           Y           Z          XY

 1           0.000      19.753       0.132       0.000
 2           0.000       0.000       0.000      -0.741
 3           0.000     -19.753      -0.132       0.000
 4           0.000       9.877       0.066       0.000
 5           0.000       0.000       0.000      -0.741
 6           0.000      -9.877      -0.066       0.000
 7           0.000       0.000       0.000       0.000
 8           0.000       0.000       0.000      -0.741
 9           0.000       0.000       0.000       0.000
```

Fig. 3: The cantilever beam under point load modelled by one 9-node element

Cantilever beam is modeled with 16 nodes plane strain element



NOTE :

REACTIONS & DEFLECTIONS

Actual results are based on nutral axis of sections

$R=V=P$
$M_Y=PY$
$\delta_Y=P/6EI(2L^3-3L^2Y+Y^3)$

STRESSES

Actual results are based on AIRY stress function

$\sigma_{YY}=-3/2(Pxy/h^3)$ $\sigma_{XX}=0$
$\sigma_{XY}=3P/4h(X^2/h^2-1)$ $\sigma_{XZ}=0$; $h=45/2$
$\sigma_{ZZ}=\nu(\sigma_{XX}+\sigma_{YY})$ $\sigma_{YZ}=0$

Calculated Results

Node Displacement time : 0

Nodes No.	X	Y	Z
1	0.00000	0.00000	0.00000
2	0.00000	0.00000	0.00000
3	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000
5	0.01678	0.00677	0.00000
6	0.01649	0.00223	0.00000
7	0.01649	-0.00223	0.00000
8	0.01678	-0.00677	0.00000
9	0.05782	0.01105	0.00000
10	0.05772	0.00367	0.00000
11	0.05772	-0.00367	0.00000
12	0.05782	-0.01105	0.00000
13	0.11197	0.01258	0.00000
14	0.11188	0.00415	0.00000
15	0.11188	-0.00415	0.00000
16	0.11197	-0.01258	0.00000

Reactions & App. Forces time : 0

Nodes No.	Forces X	Forces Y	Forces Z
1	-6061.49	-3970.27	0.00
2	5561.49	-8089.20	0.00
3	5561.54	8089.19	0.00
4	-6061.54	3970.27	0.00
5	0.00	-0.00	0.00
6	-0.00	-0.00	0.00
7	-0.00	0.00	0.00
8	0.00	-0.00	0.00
9	-0.00	-0.00	0.00
10	-0.00	0.00	0.00
11	0.00	-0.00	0.00
12	-0.00	0.00	0.00
13	250.00	-0.00	0.00
14	250.00	0.00	0.00
15	250.00	-0.00	0.00
16	250.00	0.00	0.00

Existing Node Stresses time : 0

Nodes no.	X	Y	Z	XY
1	9.618	22.443	9.618	0.750
2	3.110	7.257	3.110	0.140
3	-3.110	-7.257	-3.110	0.140
4	-9.618	-22.443	-9.618	0.750
5	-1.504	12.199	3.209	-0.108
6	-0.489	4.096	1.082	0.804
7	0.489	-4.096	-1.082	0.804
8	1.504	-12.199	-3.209	-0.108
9	0.724	7.103	2.348	0.040
10	0.251	2.389	0.792	0.536
11	-0.251	-2.389	-0.792	0.536
12	-0.724	-7.103	-2.348	0.040
13	-2.501	-0.905	-1.022	-0.147
14	-0.940	-0.549	-0.447	0.975
15	0.940	0.549	0.447	0.975
16	2.502	0.905	1.022	-0.147

Actual Results

Node Displacement time : 0

Nodes No.	X	Y	Z
1	0.00000		0.00000
2	0.00000		0.00000
3	0.00000		0.00000
4	0.00000		0.00000
5	0.01858		0.00000
6	0.01858		0.00000
7	0.01858		0.00000
8	0.01858		0.00000
9	0.06503		0.00000
10	0.06503		0.00000
11	0.06503		0.00000
12	0.06503		0.00000
13	0.12542		0.00000
14	0.12542		0.00000
15	0.12542		0.00000
16	0.12542		0.00000

Reactions & App. Forces time : 0

Nodes No.	Forces X	Forces Y	Forces Z
1	-6061.49	-3970.27	0.00
2	5561.49	-8089.20	0.00
3	5561.54	8089.19	0.00
4	-6061.54	3970.27	0.00
5	0.00	-0.00	0.00
6	-0.00	-0.00	0.00
7	-0.00	0.00	0.00
8	0.00	-0.00	0.00
9	-0.00	-0.00	0.00
10	-0.00	0.00	0.00
11	0.00	-0.00	0.00
12	-0.00	0.00	0.00
13	250.00	-0.00	0.00
14	250.00	0.00	0.00
15	250.00	-0.00	0.00
16	250.00	0.00	0.00

Existing Node Stresses time : 0

Nodes no.	X	Y	Z	XY
1	0.000	19.753	0.132	0.000
2	0.000	6.584	0.044	-0.658
3	0.000	-6.584	-0.044	-0.658
4	0.000	-19.753	-0.132	0.000
5	0.000	13.169	0.088	0.000
6	0.000	4.390	0.029	-0.658
7	0.000	-4.390	-0.029	-0.658
8	0.000	-13.169	-0.088	0.000
9	0.000	6.584	0.044	0.000
10	0.000	2.195	0.015	-0.658
11	0.000	-2.195	-0.015	-0.658
12	0.000	-6.584	-0.044	0.000
13	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	-0.658
15	0.000	0.000	0.000	-0.658
16	0.000	0.000	0.000	0.000

Fig. 4: The cantilever beam under point load modelled by one 16-node element

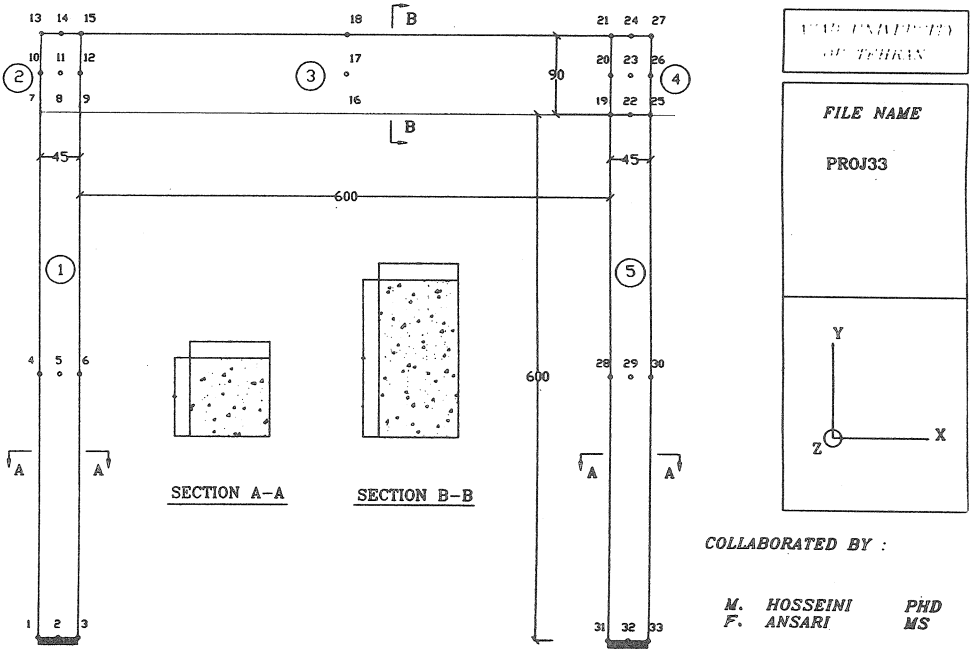


Fig. 5: Bent structure of an existing highway bridge modelled by five 9-node elements

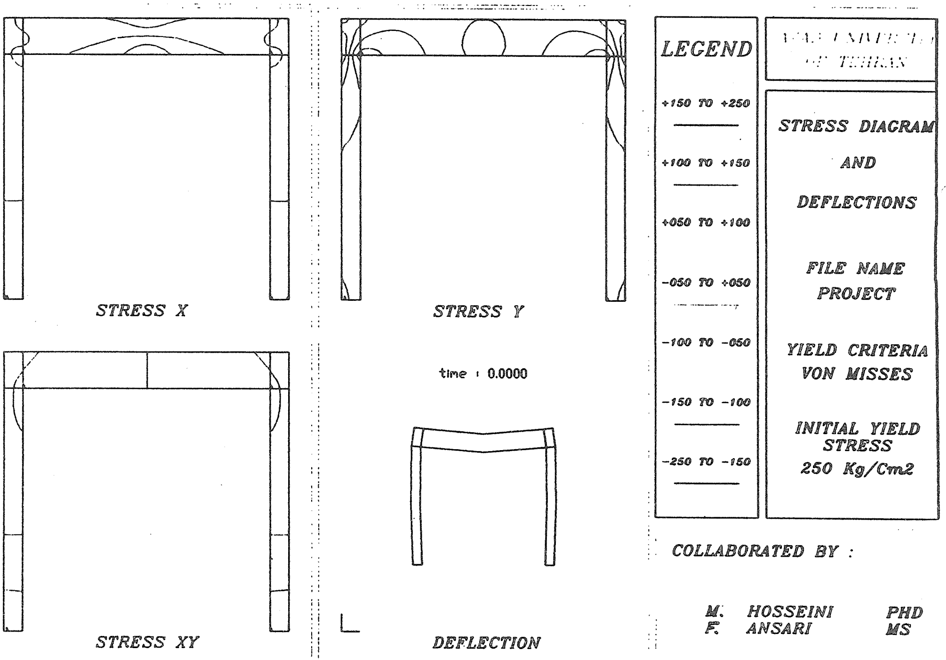


Fig. 6: Stress contours in the bent structure and its deformation under static load

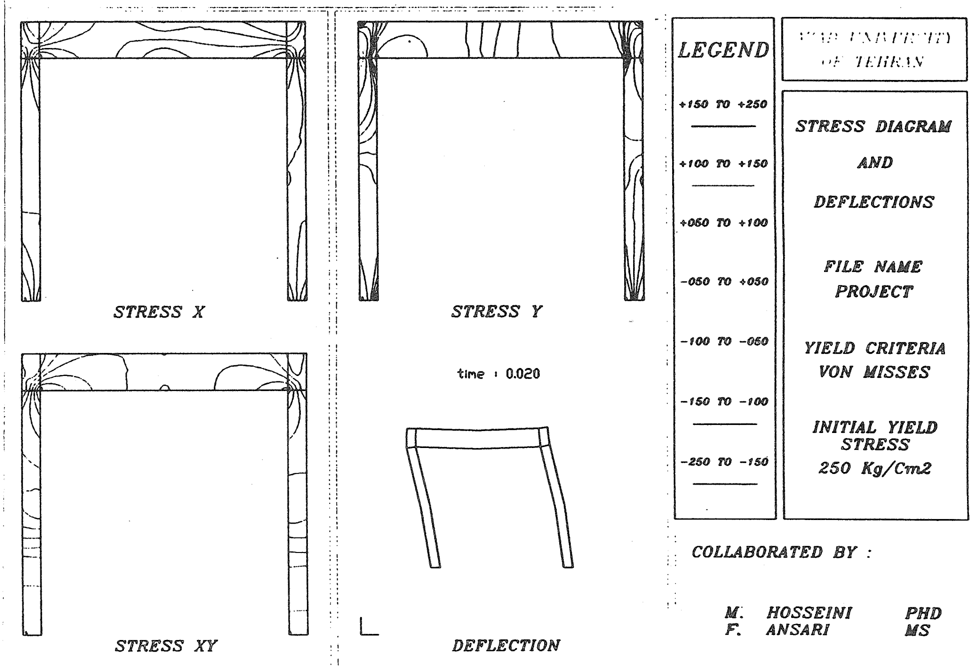


Fig. 7: Stress contours and deformation of the bent in the first time step of support excitation

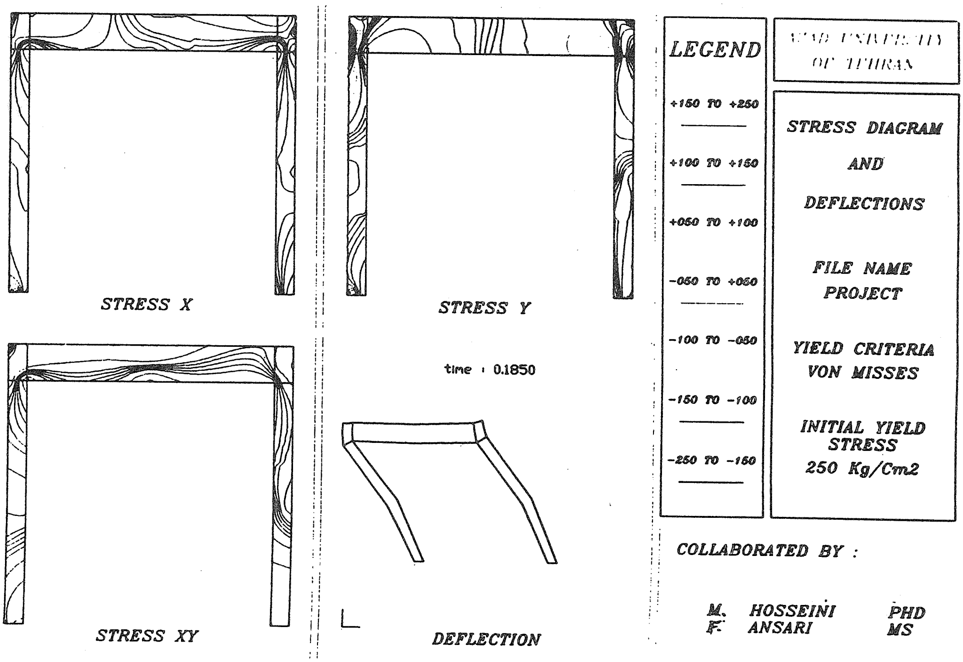


Fig. 8: Stress contours and deformation of the bent after few time steps of support excitation

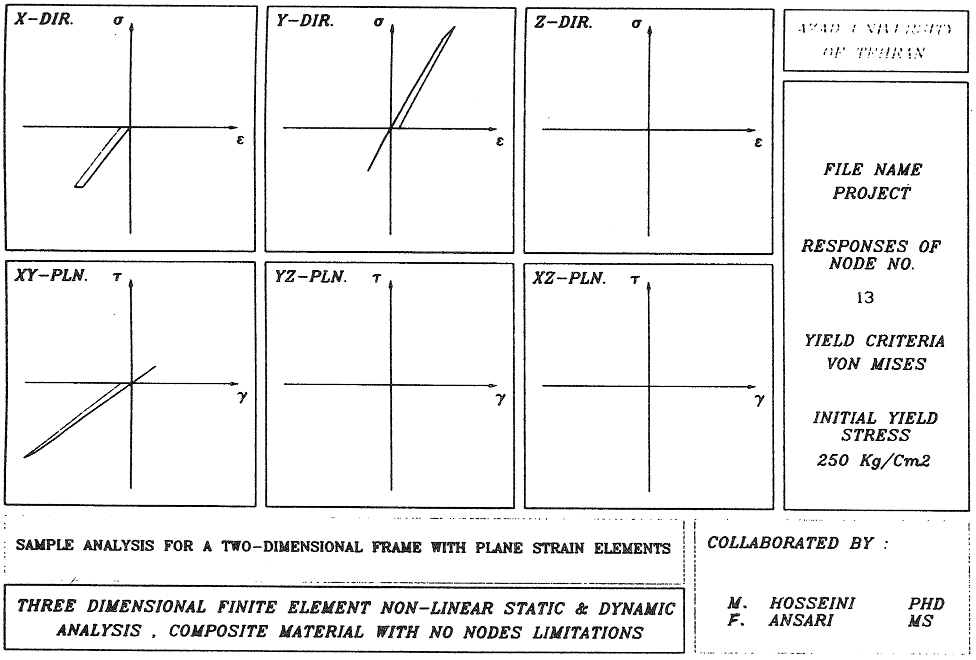


Fig. 9: Inelastic stress-strain curves of the bent along different axes or in different planes for the a few time steps of support excitation

REFERENCES

1. Khalil, T. B., and T. D. Lin. 1994. Simulation of the hybrid III dummy response to impact by nonlinear finite element analysis.
Proc. STAPP Car Crash Conf., SAE, 325-343.

2. Hunek, I. 1993. Explicit calculations of highly nonlinear contact-impact problems.
IMPACT-IV, 12th SMiRT Conf., PS-2.

3. Lyons, L. P. R. et al. 1993. Plasticity and element formulations for impact analysis in LUSAS
FEN (2), 29-35.

4. Chen, W. F. and D. J. Han 1988. *Plasticity for structural engineers.*
Springer-Verlag : New York Inc.

5. Mirdamadi-Khozani, H. M. 1990. Finite element analysis of the continua having geometric and material nonlinearity by using BFGS algorithm.
Master thesis, Civil Eng. Dept., Sharif Univ. of Technology, Tehran, Iran.

6. Ansari F. 1995. Study of the behavior of the bent structures in highway bridges subjected to lateral earthquake loads.
Master Thesis, Civil Eng. Dept., Graduate School of the Tehran South Branch, Islamic Azad Univ., Tehran, Iran.