Fatigue monitoring system using the inverse problem method and extended Green’s function method

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ABSTRACT
A precise thermal fatigue monitoring method was developed for the locations which are exposed to heavy fatigue damage or have no data base of temperature change. The temperature distribution on the inner surface was estimated from that on the outer surface as input data by the inverse problem method on thermal transfer analysis. Using these data, thermal stress change was calculated on real time by the extended Green’s function method. The calculated stress agreed well with the stress by 3D finite analysis.

1. INTRODUCTION
Thermal fatigue is one of the most important degradation factor of nuclear power plants. For the countermeasure of thermal fatigue, European countries and USA developed fatigue monitoring systems [1], [2], [3] and the authors also developed the similar system named FAMS which can evaluate the stress change caused by the thermal stratification phenomenon [4]. These systems basically uses the data from the process computer control system such as pressure, temperature and valve open-close signals and calculate the thermal stresses and fatigue usage by Green’s function method. The systems which adopt these method do not require the hardware change because of no additional sensors, on the other hand, they need a lot of data base and complex estimation logic which has some conservatism.

Therefore in the case of detecting some thermal fatigue damage using conventional system or no database near the evaluation point, there might be necessity of further detailed evaluation by measuring the outer surface temperature on the concerned location. There is an example of thermal stress calculation from outer surface temperature [5], but it just evaluated the stress under the static state condition and there is no example of quantitative stress evaluation considering rapid fluid temperature distribution change.

In this paper, a precise stress calculation method was developed based on the measured temperature distribution change of the pipe cross section on the outer surface. In order to confirm the validity of this method, the calculated stress was compared with the stress from three dimensional finite element analysis.

2. DEVELOPMENT OF THERMAL STRESS CALCULATION METHOD
2.1 Inverse method of heat transfer analysis
In the case of the pipe heat transfer problem, the equations and boundary conditions are as
follows;

Governing Equation
\[
\frac{\partial T}{\partial t} = \nabla \cdot \nabla T \quad \text{in } \Omega
\]  \hspace{1cm} (2.1)

Initial condition
\[
T(0, x) = f(x) \quad \text{in } \Omega
\]  \hspace{1cm} (2.2)

Boundary condition on the outer surface
\[
-\lambda \frac{\partial T}{\partial n} = q_{\text{out}}(t, x) = h_{\text{out}}(T - T_{\text{out}}) \quad \text{on } \Gamma_{\text{out}}.
\]  \hspace{1cm} (2.3)

Boundary condition on the inner surface
\[
-\lambda \frac{\partial T}{\partial n} = q_{\text{in}}(t, x) \quad \text{on } \Gamma_{\text{in}}
\]  \hspace{1cm} (2.4)

Outer surface temperature (known)
\[
T(t, x_i) = T_m(t, x_i) \quad \text{on } \Gamma_{\text{out}}
\]  \hspace{1cm} (2.5)

where \( t \): time, \( x \): coordinate, \( T \): temperature (suffix ‘m’ means measured), \( f \): initial temperature distribution, \( q \): heat flux, \( h \): heat transfer coefficient, \( \Omega \): domain, \( \Gamma \): boundary, and \( \lambda \): thermal conductivity.

Temperature distribution is expressed by the following initial and boundary conditions.

\[
T(t, x) = \int_{\Omega} G(t, x; 0, \xi)f(\xi)d\xi - \int_{0}^{t} \int_{\Gamma_{\text{out}}} G(t, x; \tau, \xi)q_{\text{out}}(\tau, \xi)d\xi d\tau
\]  \hspace{1cm} (2.6)

\[- \int_{0}^{t} \int_{\Gamma_{\text{in}}} G(t, x; \tau, \xi)q(\tau, \xi)d\xi d\tau
\]

where \( t \) and \( \tau \) mean time variables, \( x \) and \( \xi \) mean space coordinate and \( G(t, x; \tau, \xi) \) means Green’s function which express the effect of the flux change at time \( \tau \) and coordinate \( \xi \) on the temperature change at time \( t \) and coordinate \( x \).

Equation (2.6) was made discrete dividing the term with known variables and the term with unknown heat flux.

\[
T(t, x) = \bar{T}(t, x) + \sum_{j} \sum_{k} q^k_j \cdot \varphi_j(t - t_k, x)
\]  \hspace{1cm} (2.7)

where \( \bar{T}(t, x) \) is temperature by the known condition, \( q^k_j \) is heat flux on \( k \) time step and coordinate point \( j \) and \( \varphi_j \) means discrete Green’s function.

Here we solve unknown heat flux which becomes minimum difference between the calculated outer surface temperature from the eq. (2.7) and the measured data. Using the least square method and adding regularization term by Tikhonov’s method [6] to the object functional, the following equation was obtained.

\[
\min \Pi = \int_{0}^{t} \int_{\Gamma_{\text{out}}} (T(t, x) - T_m(t, x))^2 d\sigma d\tau + \alpha_1 \int_{0}^{t} \int_{\Gamma_{\text{in}}} \left( \frac{\partial q}{\partial s} \right)^2 d\sigma d\tau
\]  \hspace{1cm} + \alpha_2 \int_{0}^{t} \int_{\Gamma_{\text{in}}} \left( \frac{\partial q}{\partial s} \right)^2 d\sigma d\tau
\]  \hspace{1cm} (2.8)

where \( \alpha_1 \) is an regularization parameter relating to space, and \( \alpha_2 \) is an regularization parameter relating to time and \( s \) is the coordinate system along the inner surface.

Substituting the eq.(2.7) into the eq. (2.8), we have the regularization equation relating to
unknown heat flux \( q \). Neglecting the small influence terms on time variable, we consider the time range from the critical time step \( M \) to the maximum time step \( M+\tau \) and regularize only \( q_M \). [7]

Based on the temperature distribution on the time step \( M \), that on the time step \( M \approx M+\tau \) is expressed by the following equation:

\[
T\left( (M+k)\Delta t, x \right) = \hat{T}_M(k \cdot \Delta t, x) + \sum_{j=1}^{n_k} \sum_{l=1}^{n_s} q_j^{M+n_l} \varphi_j \left( (k-n+1) \cdot \Delta t, x \right)
\] (2.9)

where \( \Delta t \) is the time step width, \( k \) is the concerning time step \( (1-\tau) \) and \( \hat{T}_M(k \Delta t, x) \) is the temperature in the condition that \( q \) equals to zero after the time step \( M \).

Therefore the regularization function of the eq. (2.8) becomes the eq.(2.10).

\[
\min \Pi = \int_{t_m}^{t_{M+\tau}} \int_{r_m}^{r_{M+\tau}} \left( T(t, x) - T_m(t, x) \right)^2 ds \, d\tau + \alpha_1 \int_{t_m}^{t_{M+\tau}} \int_{r_m}^{r_{M+\tau}} \left( \frac{\partial q}{\partial s} \right)^2 ds \, d\tau
\]
\[
+ \alpha_2 \int_{t_m}^{t_{M+\tau}} \int_{r_m}^{r_{M+\tau}} \left( \frac{\partial q}{\partial r} \right)^2 ds \, d\tau
\] (2.10)

The calculation algorithm is as follows.

STEP 1 From the eq.(2.10) \( q_M, q_{M+1}, \ldots, q_{M+\tau} \) are obtained.

STEP 2 Using \( q_M, \) temperature on step \( M+1 \) is calculated from the eq. (2.9).

STEP 3 Advancing the time step \( M \) by \( 1 \), \( T_m(k \Delta t, x) \) is modified.

2.2 Extended Green's function method

The Green's function used for fatigue monitoring systems which were reported in the papers were obtained in the assumption of the axial symmetric model. On the other hand, in order to deal with thermal stratification, Green's function has to be prepared for three dimensional model.

Superimposing temperature in the circumference direction of a pipe, the stress calculation equation is as follows:

\[
\sigma(\alpha, t) = \int_{0}^{t} \int_{0}^{t} G(\theta, t - \tau, \alpha) \frac{\partial T_m(\theta, t)}{\partial \tau} \, d\tau \, d\theta
\] (2.11)

Here \( \alpha \) is the angle from the top on the concerned cross section, \( t \) is time, \( \theta \) is the angle from the top on the circumferential angle integral, \( \tau \) is a variable on the time integral. \( T_m(\theta, \tau) \) is the inner surface temperature at the angle \( \theta \) and the time \( \tau \). \( G(\theta, t, \alpha) \) is the stress change at the angle \( \alpha \) and the time \( t \) after the temperature change of \( 1^\circ \text{C} \) at the angle \( \theta \).

Dividing the time range of time integral at \( t_0 \) after which the Green's function becomes constant value \( G_0 \),

\[
\sigma(\alpha, t) = \int_{0}^{t_0} \int_{0}^{t} G_0(\theta, \alpha) \frac{\partial T(\theta, t)}{\partial \tau} \, d\theta \, d\tau + \int_{0}^{t} \int_{0}^{t} \left\{ G(\theta, t - \tau, \alpha) - G_0(\theta, \alpha) \right\} \frac{\partial T(\theta, \tau)}{\partial \tau} \, d\tau \, d\theta
\]
\[
= \int_{0}^{t_0} G_0(\theta, \alpha) \left( T(\theta, t) - T(\theta, 0) \right) d\theta + \int_{0}^{t} \int_{0}^{t} \hat{G}(\theta, t - \tau, \alpha) \frac{\partial T(\theta, \tau)}{\partial \tau} \, d\tau \, d\theta
\] (2.12)

where

\[
\hat{G}(\theta, t - \tau, \alpha) = G(\theta, t - \tau, \alpha) - G_0(\theta, \alpha)
\] (2.13)
\[
\int_0^t \frac{\partial T(\theta, t)}{\partial t} \, dt = T(\theta, t) - T(\theta, 0)
\]  
(2.14)

3. ANALYSIS

3.1 Analysis conditions

For confirmation of the method mentioned above, the analysis was carried out on the subject of a horizontally straight pipe with thermal stratification. Pipe model was assumed to be a pressurizer spray pipe.

Thermal and structural boundary conditions are shown in Fig.1. The model geometry and the material constants are shown in Table 1. Assumed fluid temperature change is shown in Fig.2. For FE analysis, three dimensional 20 node solid element type was used. FE code was MARC (Version K5.2) The model was the half of a straight pipe because of symmetry. The pipe cross section was divided by 5 in the axial direction, by 15° interval in the circumferential direction on and by 6 in the thickness direction.

By FE analysis, the temperature distribution change on the outer surface was obtained and it was used for the input data of the inverse analysis of heat transfer.

3.2 Inverse analysis on the inner surface temperature

In the inverse analysis of heat transfer the boundary condition on the outer surface was known condition and that on the inner surface was unknown. The temperature measuring points were assumed on five points on the outer surface of the cross section (\(\theta = 0^\circ, 90^\circ, 120^\circ, 150^\circ\) and \(180^\circ\) from the top of the horizontal pipe.) Time step was 10 sec.

Fig 3 shows the circumferential temperature distribution on the outer surface by the finite element analysis. These results was used as simulated measured data. Fig.4 shows the comparison of the circumferential temperature distribution change on the inner surface by the finite element analysis with that by the inverse analysis. From this figure, there remains small oscillation at \(t=650\) sec and 660 sec, but both of general distribution shape agree well.

3.3 Derivation of Green’s function

In order to derive Green’s function, the same material constants and model geometry were used as mentioned in the section 3.1. Heat transfer coefficient on the inner surface was a large number \(10^6\) kcal/m²h°C in order that the temperature change on the inner surface was nearly equal to the fluid temperature change within the very short time lag. On purpose to increase precision of Green’s function, the fluid temperature was rapidly changed by 10°C on one element mesh in the circumferential direction and the obtained stress change was divided by 10.

3.4 Result

Estimated temperature distribution change on the inner surface by the inverse heat transfer analysis was used for input data of extended Green’s function calculation.

Fig.5 and Fig.6 show comparison between the results of stress calculation by FE analysis and the new method for a fatigue monitoring system. For the inverse analysis and the stress calculation by Green’s function method, EWS SUN Sparc Station630 was used.

From these figure both results agree very well. The stress change was calculated by less than 1 sec for each outer surface temperature input by time interval of 10 sec, so the proposed method is useful even for on-line real time monitoring.
4. CONCLUSION

A precise thermal fatigue monitoring method was developed for the locations which are exposed to heavy fatigue damage or have no data base of temperature change. The temperature distribution on the inner surface was estimated from that on the outer surface as input data by the inverse problem method on heat transfer analysis. Using these data, thermal stress change was calculated on real time be the extended Green’s function method.

The calculated stress agreed well with the stress by 3D finite analysis on the subject of pressurizer spray pipe in consideration of thermal stratification.

REFERENCE

### Table 1
**Analysis conditions**

<table>
<thead>
<tr>
<th>Pipe geometry</th>
<th>Material constants</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong> L</td>
<td>Young's modulus E</td>
</tr>
<tr>
<td><strong>Outer diameter D</strong></td>
<td>Poisson's ratio $\nu$</td>
</tr>
<tr>
<td><strong>Thickness</strong> t</td>
<td>Thermal conductivity $\lambda$</td>
</tr>
<tr>
<td></td>
<td>Density $\rho$</td>
</tr>
<tr>
<td></td>
<td>Specific heat $C_p$</td>
</tr>
<tr>
<td></td>
<td>Thermal expansion $\alpha$</td>
</tr>
</tbody>
</table>

**Thermal boundary condition on the outer surface**
- **Outer temperature** $T_0$ | 20$^\circ$C |
- **Heat transfer coefficient** $h_0$ | 10 kcal/m$^2$h$^\circ$C |

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**Fig. 1 Thermal and structural boundary conditions**

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**Fig. 2 Fluid temperature change**
Fig. 3 Simulated outer surface temperature change
(θ=0°, 90°, 120°, 150°, 180°, Δt=10sec)

Fig. 4 Comparison between the circumferential temperature distribution change on the inner surface by FEM and that by the inverse analysis
Fig. 5 Comparison between the axial stress change on the top of the pipe by FEM and that by the proposed method.

Fig. 6 Comparison between the axial stress change on the bottom of the pipe by FEM and that by the proposed method.