Validation of the shakedown route in R5 for the assessment of creep fatigue crack initiation

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ABSTRACT
Validation of the R5 method for estimating crack initiation in a defect-free structure under creep-fatigue loading is presented. Test geometries under both mechanical and thermal loads are considered. Life estimates using R5 are compared with the experimental results. Comparisons are made using the R5 estimates for the cyclic strain range and the stress at the start of the creep dwell, based on simplified shakedown and reference stress techniques, and the results of detailed inelastic calculations. The conservatism of the simplified estimates is shown.

1. INTRODUCTION
R5 [1] is a comprehensive set of procedures developed within the UK for the assessment of the high temperature response of structures. In particular, methods are set out in R5, based on simplified shakedown and reference stress techniques, for calculating life under conditions of combined creep and fatigue. For an initially defect-free component, failure may be defined following Volume 3 of R5 as initiation of a crack of specified depth under creep-fatigue loading. Other volumes of R5 consider propagation of defects. Comparisons between the R5 approach and alternative design code approaches to creep assessments, such as in ASME N-47 [2] and RCC-MR [3], are discussed in [4].

Validation of the R5 methodology is illustrated by comparison of estimated cycles to crack initiation with experimental data for notched beams under cyclic bend loading. Validation by comparison with both laboratory test results and detailed inelastic calculation has been previously fully reported [5] for the particular case of a stainless steel test specimen [6,7] subjected to thermal shock loading and is also summarised here.

2. THE R5 PROCEDURE FOR CREEP-FATIGUE CRACK INITIATION
The strain range due to fatigue, the stress at the start of the creep dwell and the elastic follow-up factor during the creep dwell are estimated at critical points of the component using Volume 2. These quantities are then combined with fatigue
endurance and creep ductility data to estimate fatigue and creep damage, and hence creep-fatigue endurance using an interaction diagram. The strain range calculations are based on elastic analyses modified using the Neuber principle to account for inelastic and volumetric strain enhancements. The methods assume that cyclic loading is such that 'global shakedown' conditions are satisfied. This is the case if no more than 20\% of any structural section cyclically yields in the steady cyclic state. Satisfaction of this shakedown criterion is tested using simplified shakedown analysis based on the superposition of elastic and residual stress fields.

Endurance under creep-fatigue loading is estimated in Volume 3 by calculating accumulated fatigue and creep damage, \( D_f \) and \( D_c \) respectively, and equating the sum of the two to unity at crack initiation: \( D_f + D_c = 1 \). Endurance is defined as the number of cycles to initiate a crack of specified depth \( a_0 \). For thick-section components, \( a_0 \) may be assumed to be the diameter, \( a_r \) say, of conventional uniaxial endurance specimens used to derive the baseline endurance data. For thin-section components, however, \( a_0 \) may be a small fraction of the cross section, say 10\%, and less than \( a_r \). A size correction is applied by R5 to the baseline endurance data in the case \( a_0 < a_r \). The fatigue damage per cycle, \( d_f \), is the reciprocal of this modified endurance. Total fatigue damage, \( D_f \), then follows by summation over the cycle history.

Creep damage per cycle, \( d_c \), is defined by the ductility exhaustion integral over the dwell period \( t_h \):

\[
d_c = \int_{0}^{t_h} \frac{\dot{\varepsilon}_c}{\dot{\varepsilon}_f(\dot{\varepsilon}_c)} \, dt
\]

where the creep ductility, \( \varepsilon_c \), is a function of creep strain rate, \( \dot{\varepsilon}_c \), in general. For a stress drop, \( \Delta \sigma' \), in the dwell with elastic follow-up factor \( Z \), this equation may be conservatively simplified to

\[
d_c = \frac{Z \Delta \sigma'}{E/\varepsilon_L}
\]

where \( \varepsilon_L \) is a lower bound creep ductility which is assumed to be independent of strain rate and \( E' = 3E/2(1 + \nu) \) is Young's Modulus modified for the difference in Poisson's ratios between elastic and inelastic response. The total creep damage, \( D_c \), is then simply the sum of \( d_c \) over the various cycles.

3. **COMPONENTS ASSESSED**

3.1 **Cast 1CrMoV SENB test specimens**

The SENB test specimen design used in the series of tests at GEC ALSTHOM as part of the Brite-Euram C-FAT project is shown schematically (Fig.1) together with the loading arrangement. The specimens contain a semi-circular notch on the lower face. Testing was by fully-reversed displacement-controlled bending applied by means of eight steel rollers, the outer four being fixed and the inner four connected to servo-hydraulic rams. Three types of load cycle were applied during the test programme,
namely: (i) pure fatigue; (ii) creep-fatigue with creep dwell periods of either 30 minutes or 16 hours at the maximum displacement of the cycle with the notch in tension; and (iii) a single case of a creep-fatigue cycle with a superimposed mean displacement increasing slowly with time.

3.2 "Thermina" test specimen

The "Thermina" series of tests were carried out at CEA [6, 7]. The design of the experimental test rig is shown in Figure 2 together with the main features of the particular specimen analysed here. This stainless steel (316) specimen consisted of an internal flange within a thin-walled cylinder. It was subjected to periodic cooling during which thermal shock was induced by forcing liquid sodium at a temperature of 400°C onto the outside surface of the specimen. Subsequently, an internal heater caused the temperature to return to 600°C for a hold period of 6 hours during which creep occurred. The specimen was subjected to repeated cycles. Because the flange responds more slowly to thermal shock than the much thinner cylinder wall, high thermal stresses were generated in the transition region between the flange and the cylinder wall.

Assessment of the "Thermina" tests has been fully reported in [5]; only brief details are therefore given here.

4. MATERIAL PROPERTIES

The material properties used in the assessments are listed in Table 1, where \( E \) is Young's modulus, \( \nu \) is Poisson's ratio, \( S_y \) is the minimum 0.2% yield stress and \( K_s \) is a factor applied to \( S_y \) to obtain the material ratchet limit \((K_sS_y)\). Cyclic-stress strain data are given in the Ramberg-Osgood form

\[
\Delta \varepsilon = \frac{\Delta \sigma}{E} + \left[ \frac{\Delta \sigma}{A} \right]^{1/\beta}
\]  

where \( \Delta \sigma, \Delta \varepsilon \) are the cyclic stress and strain and \( A, \beta \) are material constants.

<table>
<thead>
<tr>
<th>TEST CASE</th>
<th>ELASTIC</th>
<th>CYCLIC</th>
<th>RELAXATION</th>
<th>CREEP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E ) (MPa) ( \nu ) ( K_s ) ( S_y ) (MPa)</td>
<td>( A ) (MPa) ( \beta )</td>
<td>( B^\gamma ) b (h(^{-1}))</td>
<td>( B ) ( n )</td>
</tr>
<tr>
<td>1CrMoV SENB Specimen</td>
<td>166,600 0.3 0.9 373.2</td>
<td>1658 0.153</td>
<td>0.0484 8819.4</td>
<td>not used ( Z=1.84-2.52 ) [8]</td>
</tr>
<tr>
<td>316SS Thermina Specimen</td>
<td>154,700 0.29 1.0 227.0</td>
<td>1577 0.160</td>
<td>0.0615 19.775</td>
<td>( 1x10^{-18} ) 5</td>
</tr>
</tbody>
</table>

TABLE 1. MATERIAL PROPERTIES

Creep deformation follows the simple power law relationship defined by
\[ \varepsilon_{cr} = B \sigma^n t \]

where \( \varepsilon_{cr} \), \( \sigma \) and \( t \) are creep strain, stress and time, respectively, and \( B \) and \( n \) are material constants.

Stress Relaxation \( \Delta \sigma' \) in time \( t \) is calculated using the Feltham equation

\[ \frac{\Delta \sigma'}{\sigma_0} = B'' \ln \left( \frac{bt}{Z} + 1 \right) \]

where \( \sigma_0 \) is the initial stress, \( Z \) is the elastic follow-up factor, and \( b, B'' \) are material constants (Table 1).

Creep ductility values are required for Equations 1, 2. For the 1CrMoV SENB specimens, cast specific data were available from the C-FAT project, which meant that allowance could be made for the effect of strain rate. This gives a significantly higher ductility than the material lower bound value. The effect of stress state on creep ductility is allowed for by using the multiaxial ductility, \( \bar{\varepsilon}_f \), in place of \( \varepsilon_0 \) in Equation 2, following R5 Volume 3, where for principal stress ratio \( \sigma_2/\sigma_1 < 0.5 \):

\[ \bar{\varepsilon}_f = \left( 1 - \frac{\sigma_2^2}{\sigma_1} \right) \varepsilon_f \]

For the 'Thermina' case, cast specific data were available both from tests on specimens made from the material batch and from tests on actual components which indicated a ductility of 56%.

Fatigue data in the form of number of cycles to failure plotted against total strain range were chosen from a Nuclear Electric database for 316 steel for the 'Thermina' analyses and from [8] for the 1CrMoV SENB specimens.

5. **FINITE ELEMENT ANALYSES**

5.1 **Thermal analyses**

These were carried out for the 'Thermina' geometry using the finite element thermal analysis program FLHE, part of the BERSAFE [9] suite, to calculate the temperature profile within the component at each stage of the applied thermal transient.

5.2 **Elastic Analysis**

Linear elastic stress analyses for the 'Thermina' specimen included the primary (axial) loading and were performed at several steps during the thermal transient, using BERSAFE [9], to enable the extremes of the loading cycle to be calculated. For the 1CrMoV SENB specimens, the analyses modelled the loading cases with the notch in compression and in tension, both with and without a superimposed axial primary stress of 200MPa simulating the experimental loading.

5.3 **Elastic Follow-up Analysis**

A monotonic creep calculation was carried out for the 'Thermina' geometries using BERSAFE [9]. This enabled the elastic follow-up factor, \( Z \), to be estimated as
\(\Delta = 3.57\), the starting condition being the elastic stress distribution calculated at the beginning of the creep dwell. For the SENB analyses values of between \(\Delta = 1.84\) and \(2.52\) were taken from a non-linear analysis of the component [8].

5.4 Shakedown Analysis

The R5 procedure requires a shakedown analysis to be performed. This is the superposition of a self-equilibrating residual stress field, constant at all times in the cycle, onto the extremes of the loading cycles described in Section 5.2. If the resultant shakedown stress distributions are below yield at all positions in the structure, then 'strict shakedown' is achieved. A less rigorous condition known as 'global shakedown' occurs if no more than 20% of any cross section is outside yield.

The residual stress field chosen for the 'Thermia' geometries was a combination of a factored thermal stress field, generated as in Section 5.2 in the absence of primary loading, and a stress field caused by a localised region of high temperature on the outer surface of the specimen opposite to the transition region between flange and cylinder. For the SENB specimens, because the extremes of the loading cycle are virtually equal and opposite in the absence of primary loading, a residual stress is only required for the case with a superimposed axial loading. The residual stress is generated in this case by postulating a through thickness temperature distribution. The scaling factors on the residual stress are varied to minimise the start-of-dwell stress.

The geometries and test cases considered lead to a variety of creep-fatigue load histories. The 'Thermia' specimen was, strictly, outside the definition of global shakedown, with up to 40% of the critical cross-section being outside yield in the cyclic state. The SENB specimen test cases covered conditions from up to the limit of global shakedown to within strict shakedown. The analyses therefore provide a demanding test of the R5 methodology over a wide range of loading conditions.

6. ENDURANCE ASSESSMENTS

6.1 Fatigue damage

The fatigue strain range is calculated from the equivalent elastic stress range \(\Delta \sigma_{el}\) together with an allowance for enhancement of the strain range due to plasticity and creep. Because equivalent stresses cannot be added directly, \(\Delta \sigma_{el}\) is calculated using the differences in component stresses between the peak combined loading case and the steady state case. The effect of creep on the fatigue strain range is allowed for by adding \(2(1+\nu)\Delta \sigma_{ref}/3E\) to the elastic strain range, where \(\Delta \sigma_{ref} = \sigma_0 B'/\ln(bt+1)\) is the uniaxial stress relaxation stress in time \(t\) from stress \(\sigma_0\), the creep reference stress, i.e. the maximum stress in the steady state case, and \(B'/b\) are constants (Table 1). The enhancement of the fatigue strain range due to plasticity is calculated by the use of the Neuber construction as shown in Figure 3 where the total strain range is defined by the intersection of the hyperbola \(\Delta \sigma \Delta \varepsilon = \text{const.}\), through the modified elastic point, with the cyclic stress-strain curve. A further correction factor \(\Delta \varepsilon_{vol} [1]\) is made to allow for the fact that elastic straining takes places with a Poisson's ratio of typically \(\nu = 0.3\), whereas for inelastic straining \(\nu = 0.5\).

The total fatigue strain range calculated above is used in conjunction with fatigue endurance data to enable an estimate of the number, \(N_0\), of fatigue cycles to grow a crack to size \(a_0\) to be made. Such data are generally based on failure of test specimens of cross section size typically 6-12mm. For thin-walled components like the
'Thermina' specimen, it is necessary to partition the cyclic endurance into initiation and growth components. The method of R5 Volume 3 is then used to estimate a reduced fatigue life to allow for smaller failure crack sizes. A crack size \( a_0 = 0.2\text{mm} \) was taken for the 'Thermina' analysis. A size correction was not applied in the case of the thick-section SENB specimens.

6.2 Creep damage

The stress relaxation \( \Delta \sigma' \) during each creep dwell of duration \( t \) is calculated using Equation 5. The creep damage per cycle \( d_c \) is then estimated using Equation 2 together with the creep ductility value as described in Section 4.

6.3 Total damage

The total damage per cycle is calculated from \( d_t = 1/N_0 + d_c \) and hence the creep-fatigue endurance is \( N_0^* = 1/d_t \).

7. RESULTS

Results obtained using simplified methods are shown in Table 2. Comparisons of the predicted number of cycles to failure with test data are given in Table 3.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Dwell Time (hours)</th>
<th>Applied Loading</th>
<th>Calculated Strain Range (%)</th>
<th>Fatigue Life (cycles) ( N_0 )</th>
<th>Creep Damage per cycle ( d_c )</th>
<th>Life Prediction (cycles) ( N_0^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SENB</td>
<td>0.5</td>
<td>±0.5mm</td>
<td>1.289</td>
<td>345</td>
<td>2.61\times10^3</td>
<td>183</td>
</tr>
<tr>
<td>SENB</td>
<td>16</td>
<td>±0.5mm</td>
<td>1.699</td>
<td>250</td>
<td>5.28\times10^3</td>
<td>108</td>
</tr>
<tr>
<td>SENB</td>
<td>16</td>
<td>±0.4mm</td>
<td>1.267</td>
<td>357</td>
<td>4.43\times10^3</td>
<td>138</td>
</tr>
<tr>
<td>SENB</td>
<td>0.5</td>
<td>±0.3mm</td>
<td>0.524</td>
<td>1417</td>
<td>1.28\times10^3</td>
<td>510</td>
</tr>
<tr>
<td>SENB (plus primary load)</td>
<td>0.5</td>
<td>±0.4mm +200MPa</td>
<td>0.843</td>
<td>625</td>
<td>3.23\times10^3</td>
<td>311</td>
</tr>
<tr>
<td>Thermina</td>
<td>6</td>
<td>thermal + end load</td>
<td>0.893</td>
<td>1197</td>
<td>3.00\times10^3</td>
<td>260</td>
</tr>
</tbody>
</table>

**TABLE 2. LIFE PREDICTIONS USING SIMPLIFIED METHODS**

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Dwell (hrs)</th>
<th>Applied Loading</th>
<th>Test endurance</th>
<th>Non-linear methods</th>
<th>Simplified methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>SENB</td>
<td>0.5</td>
<td>±0.5mm</td>
<td>436</td>
<td>385 [8]</td>
<td>183</td>
</tr>
<tr>
<td>SENB</td>
<td>16</td>
<td>±0.5mm</td>
<td>177</td>
<td>81 [8]</td>
<td>108</td>
</tr>
<tr>
<td>SENB</td>
<td>16</td>
<td>±0.4mm</td>
<td>272</td>
<td>108 [8]</td>
<td>138</td>
</tr>
<tr>
<td>SENB</td>
<td>0.5</td>
<td>±0.3mm</td>
<td>4000</td>
<td>1111 [8]</td>
<td>510</td>
</tr>
<tr>
<td>SENB (plus primary load)</td>
<td>0.5</td>
<td>±0.4mm +200MPa</td>
<td>576</td>
<td>526 [8]</td>
<td>311</td>
</tr>
</tbody>
</table>

**TABLE 3. COMPARISON WITH TEST DATA**

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REFERENCES

9. ACKNOWLEDGEMENTS

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8. CONCLUDING REMARKS

The R5 procedure has been applied to estimate the creep-fatigue endurance of a variety of test specimens subjected to cyclic mechanical and thermal loadings. The simplified route based on shakedown analysis and using best estimate material data in most cases of estimated cycles to crack initiation of between 40% and 80% of results in most cases. The results are closer than existing design codes based on elastic analysis but retain a degree of conservatism.

In general, the CREP-MVINB SENB specimens show a reasonable agreement between predicted strain ranges when compared with parallel inelastic analysis [9]. Similarly, the Thermina' analysis shows good agreement for the predicted strain range when compared with inelastic analysis [3], although stresses at the beginning of the dwell are again higher. In all but two cases, the simplified route gives a more conservative life with the test results. In the majority of cases, the simplified route predicts lives of between 40% and 80% of those obtained on test, demonstrating the inherent conservative nature of the procedure.
FIG. 1 Schematic Drawing of SENB Specimen and Loading Arrangement

FIG. 2. Drawing of 'Thermina' Test Rig and Detail of Specimen

FIG. 3. Neuber Construction