Modified solution of radial nozzle with thick reinforcement in spherical vessel head subjected to moment

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ABSTRACT: The application of thin shell theory for the thick reinforcement may not be appropriate in the shell assembly problems without any modification of thin shell theory. To obtain stresses or displacements of the thick shell reinforcement in the vicinity of radial nozzle of the spherical vessel head subjected to moment, solutions of the thick reinforcement have been derived from the modification of thin shell theory which introduces the nature of the thick shell. These solutions are found to be reasonable and be able to use for the design of nozzle with the thick shell reinforcement in spherical vessel head subjected to moment.

1. INTRODUCTION

This paper is one of a series investigating the elastic stresses radial nozzles with thick shell reinforcement in spherical vessel head subjected to different types of loading. A theoretical analysis for an external radial load applied to the nozzle has been presented previously. This paper reports a theoretical study of the modified solution in a radial nozzle with thick shell reinforcement subjected to an applied moment.

Nozzles in the spherical vessel are designed by area replacement method in the majority of pressure vessel code. The solutions for nozzles in spherical shells were presented by Bijlaard and Leckie. Their works are based on the asymptotic integration of the differential equations for the thin spherical shell. In both analyses, as is usual with shell assembly problems the shells are assumed to join at the intersections of the median lines of their thickness at the junctions which are the junction of cylinder and sphere (reinforcement) and the junction of two spheres with different thickness (reinforcement and main shell).

Limit analysis of radial nozzles in spherical shells was reported by Cloud. In this analysis the reinforcement was considered as a solid ring (rigid body) instead of a part of shell.

As recent works, the theoretical analysis expanded the Leckie solution to the pad reinforced radial nozzle in spherical vessel heads were reported by Gill and a theoretical analysis for the reinforced radial nozzles in ellipsoidal vessel head was reported by Chang.

The purpose of the present paper is to provide the modified solutions of radial nozzle with thick shell reinforcement in spherical vessel head subjected to moment to obtain stresses and displacement of the thick shell reinforcement.
2. THEORETICAL ANALYSIS

The structure being analyzed is shown in Fig. 1. It consists for the purposes of analysis of a spherical vessel (main shell), integral reinforcement (referred to as reinforcement), and a semi-infinite cylindrical shell (the flush branch connected to the spherical vessel).

In the analysis of the structure under the action of a bending moment applied to the branch, the geometric assumptions are as follows: (1) intersection point of reinforcement and cylinder is the intersection of the middle plane of reinforcement and middle plane of cylinder, (2) intersection point of reinforcement and main shell is the normal plane of reinforcement and the middle plane of main shell at \( \phi_2 \) (Fig. 2).

For each part of the structure, the complete solution is found by superimposing an edge solution on a membrane solution. The solutions for cylinder are given by Kraus\(^8\). For the sphere the solutions given by Leckie\(^4\) are used with some modification according to the each analysis of case. In both solutions there is no limitations on the geometrical parameters.

According to the different analyses for the thick reinforcement, the detail analysis is distinguished by the three cases.

Case 1: No correction for geometry and no modification of solution (referred to as current method); even though the middle planes of the reinforcement and the main shell are not coincide and the reinforcement is beyond the range of the thin shell, it is assumed that their middle planes are coincide and the reinforcement is the thin shell.

Case 2: Correction for geometry only (referred to as moment modified method); even though the reinforcement is beyond the range of the thin shell, it is assumed that the reinforcement is the thin shell. Only the effect due to the difference of distance, \( \delta_t \) (Fig. 1), between middle planes of reinforcement and main shell is considered in the calculation of moments of the reinforcement at junction of the reinforcement and main shell, \( \phi_2 \).

Case 3: Correction for geometry and modification of rotation function in solution (referred to as rotation modified method); derivative of rotation, \( \beta \), is neglected in the thin shell solution which means that the rotation of the thick reinforcement may be constant through all reinforcement section, to reflect the characteristics of the thick shell on the solution of the thin shell and the effect due to the difference of distance between middle planes of reinforcement and main shell is considered the same as Case 2.

Fig. 2 shows membrane and edge forces and moments acting on the junctions of the components of the structure. The general expression used in the analysis for each of the components parts of the structure are given below.

**Cylinder**

At a distance \( x \) from the edge of the cylinder (see Fig. 1) the horizontal displacement, the rotation angle, and the stress resultant due to membrane and edge forces as follows, the shell being assumed semi-infinite\(^8\):

\[
w = \delta_h = -\frac{M}{AE}r_c \cos \theta + \frac{e^{-\lambda_c x}}{\lambda_c} \left[ H_1 \cos \lambda_c x + \lambda_c M_1 (\cos \lambda_c x - \sin \lambda_c x) \right] \cos \theta \quad (1)
\]

\[
\beta = -\frac{e^{-\lambda_c x}}{2K_c \lambda_c^2} \left[ H_1 (\sin \lambda_c x - \cos \lambda_c x) - 2M_1 cos \lambda_c x \right] \cos \theta \quad (2)
\]
\[ N_x = \frac{M}{\pi r_c^3} \cos \theta + \frac{e^{-\lambda_c x}}{r_c^3 \lambda_c} \left[ H_1 \sin \lambda_c x + 2 \lambda_c M_1 \cos \lambda_c x \right] \cos \theta \]  
\[ M_x = \frac{M}{\pi r_c^3} \cos \theta + \frac{e^{-\lambda_c x}}{\lambda_c} \left[ H_1 \sin \lambda_c x + 2 \lambda_c M_1 \cos \lambda_c x \right] \cos \theta \] 

where, \[ \lambda_c^2 = \frac{3(1-\nu^2)}{r_c^2 \epsilon_c^2}, \quad K_c = \frac{E \epsilon_c^2}{12(1-\nu^2)} \]

All these equations at the juncture 1 may be written in terms of unknown \( H_1 \) and \( M_1 \).

**Reinforcement**

This has two boundaries and is subjected to external load on the inner and outer edges. The horizontal displacement, the rotation angle, and the stress resultants due to membrane and edge forces at any meridional position defined by \( \phi \) are given by Leckie\(^a\).

\[ u = \frac{P_2 R \sin^2 \phi}{4(1-\nu)D} \left( \frac{2 \cos \phi}{\sin^2 \phi} + 2 \cos \phi \ln \frac{1-\cos \phi}{1+\cos \phi} \right) - \frac{1}{(1-\nu) \sin \phi} \left( A_1 \tilde{T}_1 + A_2 \tilde{T}_2 + B_1 \tilde{T}_3 + B_2 \tilde{T}_4 \right) \cos \theta \]  
\[ v = \frac{P_2 R \sin^2 \phi}{4(1-\nu)D} \left( \cos \phi \ln \frac{1-\cos \phi}{1+\cos \phi} - \frac{2 \cos \phi}{\sin^2 \phi} + \frac{1}{(1-\nu) \sin \phi} \left( A_1 \tilde{T}_1 + A_2 \tilde{T}_2 + B_1 \tilde{T}_3 + B_2 \tilde{T}_4 \right) \right) \cos \theta \]  
\[ w = \frac{1}{(1-\nu^2)} \left( A_1 (\tilde{T}_1 + 2 \kappa^2 \tilde{T}_2) + A_2 (\tilde{T}_2 - 2 \kappa^2 \tilde{T}_1) + B_1 (\tilde{T}_1 + 2 \kappa^2 \tilde{T}_3) + B_2 (\tilde{T}_2 - 2 \kappa^2 \tilde{T}_3) \right) \cos \theta \] 

\[ \delta_n = v \cos \phi + w \sin \phi \]  
\[ \delta_s = v \cos \phi - w \sin \phi \]  
\[ \beta = \frac{P_2 R \sin^2 \phi}{4(1-\nu)D} \left( \cos \phi \ln \frac{1-\cos \phi}{1+\cos \phi} - \frac{2 \cos \phi}{\sin^2 \phi} \right) + \frac{1}{R(1-\nu)} \left( A_1 \left( 2 \kappa^2 \tilde{T}_2 - v \tilde{T}_1 \right) - A_2 \left( 2 \kappa^2 \tilde{T}_1 + v \tilde{T}_2 \right) \right) \]

\[ \frac{1}{R(1-\nu)} \left( B_1 \left( 2 \kappa^2 \tilde{T}_4 - v \tilde{T}_1 \right) - B_2 \left( 2 \kappa^2 \tilde{T}_1 + v \tilde{T}_4 \right) \right) \]  
\[ N_\phi = \frac{P_2 R \sin^2 \phi}{R^2 (1-\nu)} \left( \frac{1}{\sin^2 \phi} \right) \left\{ A_1 \left( \tilde{T}_1 \cot \phi - \frac{\tilde{T}_1}{\sin^2 \phi} \right) + A_2 \left( \tilde{T}_2 \cot \phi - \frac{\tilde{T}_2}{\sin^2 \phi} \right) + \right. \]

\[ B_1 \left( \tilde{T}_3 \cot \phi - \frac{\tilde{T}_3}{\sin^2 \phi} \right) + B_2 \left( \tilde{T}_4 \cot \phi - \frac{\tilde{T}_4}{\sin^2 \phi} \right) \} \] \cos \theta \]  
\[ M_\phi = \left[ \frac{P_2 R \sin^2 \phi}{12 R^2 (1-\nu)} \right] \left\{ A_1 \left( \frac{1+\nu}{a} \right) (\tilde{T}_2 - 2 \kappa^2 \cos \phi \left( \tilde{T}_2 - \tilde{T}_3 \cot \phi \right) + v \cos \phi \left( \tilde{T}_1 - \tilde{T}_2 \cot \phi \right) \right) + \right. \]

\[ A_2 \left( \frac{1+\nu}{a} \right) (\tilde{T}_2 - 2 \kappa^2 \cos \phi \left( \tilde{T}_1 - \tilde{T}_2 \cot \phi \right) + v \cos \phi \left( \tilde{T}_1 - \tilde{T}_2 \cot \phi \right) \right) + \]

\[ B_1 \left( \frac{1+\nu}{a} \right) (\tilde{T}_3 - 2 \kappa^2 \cos \phi \left( \tilde{T}_2 - \tilde{T}_3 \cot \phi \right) + v \cos \phi \left( \tilde{T}_3 - \tilde{T}_3 \cot \phi \right) \right) + \]

\[ B_2 \left( \frac{1+\nu}{a} \right) (\tilde{T}_4 - 2 \kappa^2 \cos \phi \left( \tilde{T}_3 - \tilde{T}_4 \cot \phi \right) + v \cos \phi \left( \tilde{T}_4 - \tilde{T}_4 \cot \phi \right) \} \] \cos \theta \] 

where, \[ \alpha = \frac{t^2}{12 R^2}, \quad K = \frac{E \epsilon_c^2}{12(1-\nu^2)}, \quad \kappa^4 = \frac{1}{\sqrt{2}} \left[ \frac{12(1-\nu^2) R^2}{t^2} - \nu^2 \right] \]
\( A_1, A_2, B_1, B_2 \) are arbitrary constants and the functions \( T_i \) are:

\[
T_1 = \frac{\phi}{\sin \phi} \text{ber}'z, \quad T_2 = \frac{\phi}{\sin \phi} \text{bei}'z, \quad T_3 = \frac{\phi}{\sin \phi} \text{ker}'z, \quad T_4 = \frac{\phi}{\sin \phi} \text{kei}'z
\]  

(18)

where,

\[ z = \sqrt{2k \phi} \]  

(19)

All these equations at the junction 1 and 2 are expressed in terms of the unknown constants \( A_1, A_2, B_1, B_2 \) and external load \( M \) for a given geometry and material.

In thick shell the rotation can be assumed to be constant or the rotation derivative be very small and negligible along the meridional direction. In the analysis of case 3, the rotation derivative, \( \beta \), is neglected from the equation (18).

**Main Shell**

Expressions are calculated from those of reinforcement by replacing \( R \) by \( R_S \), \( t \) by \( t_s \) and \( P_2 \) by \( P_4 \) with the constants \( A_1, A_2, B_1, B_2 \) by \( A'_1, A'_2, B'_1, B'_2 \). Since this shell has only one boundary:

\[ B'_1 = B'_2 = 0 \]  

(20)

Unknown in these expressions are now \( B'_1, B'_2 \), and external load \( M \).

**Boundary Conditions**

It is assumed that there is no relative displacement and rotation between the component parts of the structure at each junction.

\[
u_1 + \delta_{h_1} = u_2 + \delta_{h_2}
\]

(21)

\[
\beta_{h_1} + \delta_{\alpha_1}/t_2 = \beta_{h_2} + \delta_{\alpha_2}/R
\]

(22)

\[
u_3 + \delta_{h_3} = u_4 + \delta_{h_4}
\]

(23)

\[
\beta_{h_3} + \delta_{\alpha_3}/R = \beta_{h_4} + \delta_{\alpha_4}/R
\]

(24)

Horizontal force and moment equilibrium at each junction:

\[
H_1 = H_2
\]

(25)

\[
M_1 = M_2
\]

(26)

\[
H_3 = H_4
\]

(27)

\[
CF \cdot M_3 = M_4
\]

(28)

\( CF \) is correction factor due to difference of distance between middle plane of reinforcement and middle plane of main shell. For the analysis of case 1 the value is one and for the other analysis of cases the values are calculated as follows:

\[
CF = 3(t_s/t_R^2) - 3(t_s/t_R)^2 + (t_s/t_R)^3
\]  

(29)

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Solution

By the use of equation (1) to (4) for the cylinder equations, (7) to (14) for the reinforcement, with similar equations for the main shell, the eight equations (21) to (28), may be set up in terms of the eight unknowns \( H_1, M_1, A_1, A_2, B_1, B_2, B'_1 \) and \( B'_2 \) and external load, \( M \), in matrix form.

\[
\begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{18} \\
    a_{21} & \cdots & \cdots & a_{28} \\
    \cdots & \cdots & \cdots & \cdots \\
    \cdots & a_{y} & \cdots & \cdots \\
    \cdots & \cdots & \cdots & \cdots \\
    \cdots & \cdots & \cdots & \cdots \\
    \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
    H_1 \\
    M_1 \\
    A_1 \\
    A_2 \\
    B_1 \\
    B_2 \\
    B'_1 \\
    B'_2 \\
\end{bmatrix}
= \begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3 \\
    b_4 \\
    b_5 \\
    b_6 \\
    b_7 \\
    b_8 \\
\end{bmatrix}
\]

(30)

The solution for a given bending moment \( M \) was obtained by means of a computer analysis.

3 RESULTS OF ANALYSIS

The foregoing theoretical analyses have been employed to calculate the stress distributions in the reinforcement for the seven specimens as shown in Table 1.

The geometric parameters, material properties and bending moment used for these analyses are shown in Table 2. The required thickness of the reinforcement calculated by area replacement method is 188mm and the ratio of the thickness to radius is 0.14. In order to reduce the stresses induced in the reinforcement due to the external piping loads the thickness of reinforcement is increased up to 404mm (the ratio, 0.28). In order to investigate the effect due to the increase of the reinforcement thickness per bending moment, even though the reinforcement is beyond the range of the thin shell, Case 1, 2 and 3 analyses are carried out.

As a result of calculation, the theoretical stress distributions at the junction of the reinforcement for the seven specimens are shown in Fig. 3 and 4.

In case that the reinforcement is within the range of the thin shell (the ratio less than 0.1), the deviation between the moment modified method and the current method must be small since the difference of distance between middle planes of reinforcement and main shell is small. This phenomenon can be found in these figures.

It is shown beyond the range of thin shell that the deviation between two modified methods and current method is large. When the middle planes are not coincide the deviation is considerably large at the junction of the reinforcement and main shell, \( \phi_2 \). It is found out that the current method can not be used in case that the reinforcement is within the range of thick shell and the difference of distance between middle planes of reinforcement and main shell exists.

The deviations between moment modified method and rotation modified methods exist in the range of thick shell and change with thickness to radius ratio in the range of thick shell. It can be said that the deviations represent the characteristics of thick reinforcement for a bending moment.
4 CONCLUSIONS

For two modified methods (Cases 2 and 3) and current method (Case 1), elastic stress analyses have been carried out for seven different thickness of reinforcement of the radial nozzle in a spherical vessel head subjected to a moment.

The moment modified method (case 2) is applicable to the shell reinforcement which is in the range of thin shell, $t/R<0.1$ and whose middle plane do not coincide with the middle plane of main shell. The results of the rotation modified method are good agreement with those of FEM for the thick shell reinforcement with $t/R\geq 0.1$.

In the initial thick shell reinforcement design for the moment nozzle load, the rotation modified method is applicable.

REFERENCES

Table 1  Analysis Parameters for Seven Specimens

<table>
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<tr>
<th>No.</th>
<th>$tr/R_k$</th>
<th>$tr/ts$</th>
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<tr>
<td>1</td>
<td>0.08</td>
<td>1.09</td>
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<tr>
<td>2</td>
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<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>3.57</td>
</tr>
<tr>
<td>7</td>
<td>0.28</td>
<td>4.24</td>
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Table 2  Geometric Parameters and Material Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Cylinder Thickness, $tc$</td>
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<tr>
<td>Main Shell Thickness, $ts$</td>
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<tr>
<td>Cylinder Radius, $rc$</td>
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<td>Main Shell Radius, $Rs$</td>
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<td>Poisson's Ratio, $\nu$</td>
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<td>Young's Modulus, $E$</td>
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<td>Yield Strength, $\sigma_y$</td>
<td>297MPa</td>
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<td>Applied Moment, $M$</td>
<td>620kN-m</td>
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</table>

Fig. 1  Nozzle Details and Notation

(a) Membrane forces  (b) Edge forces

Fig. 2  Membrane and Edge Forces on Components of Structure
Fig. 3  Stress Intensities at $\phi_1$ on the Inside Reinforcement

Fig. 4  Stress Intensities at $\phi_2$ on the Outside Reinforcement