A new and simplified method for estimating residual stress in welded structures with complicated shape using inherent strain

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ABSTRACT: We present a new and simplified method of estimating residual stress in welded structures using inherent strain. The method makes use of elastic analysis with the finite element method, and can be used to efficiently compute the residual stress remaining in complex welded structures. The estimated residual stress distributions agree well with the results of strain-gauge measurements. The proposed method in this paper can estimate welding residual stress in three-dimensional structures with complicated shape in a simple way.

1. INTRODUCTION

In nuclear power plants, great reliability is demanded of components so that they remain sound in long use [1]-[3]. Residual stress due to welding affects the performance of structures in various ways. It is important to know precisely the distribution and magnitude of such welding residual stress, both for the fatigue strength design and for the evaluation of crack propagation. Tensile residual stress has a tendency to shorten the fatigue life of structures, whereas compressive stress extends the fatigue life. The distribution of welding residual stress strongly affects fatigue and stress-corrosion crack growth. The stress distribution is essential for predicting the crack growth rate and the span of life. In particular, it is important to evaluate the welding residual stress in the welded joint when a pipe penetrates the spherical head of a pressure vessel in a nuclear plant, to prevent against any possibility of stress-corrosion cracking or fatigue failure during extended use [4][5].

It is very useful for evaluating the reliability of components to measure residual stress in welded structures directly. A lot of measuring methods have been developed and applied for the components, using strain gauges, hole-drilling, X-ray diffraction, neutron diffraction, or other methods. But it is well known that analytical methods are much more general and widely applicable.

Residual stresses in welded structures are generally determined by using thermal elastic-plastic analysis with the finite element method [4]-[8]. Such analysis is effective for two-dimensional structures, but is impossible to use to obtain precise values in three-dimensional structures. This is because the calculations required are beyond the capacity of even the latest high-speed computers.

Another method that can be used is to estimate welding residual stress using inherent strain, or "eigen strain," which is a source for generating the residual stress [9][10]. The method of estimation using inherent strain can be applied within other estimation techniques that use both the released strain measurement and the elastic finite element method [11][12]. Three-dimensional residual stress can be obtained by the elastic calculations using the inherent strain, which is calculated from data for released elastic strain measured directly from the welded joint.
of the object being evaluated. However, a great deal of time and cost is required when using the conventional inherent strain method, to prepare the inherent strain value that will serve as the input data for the elastic analysis. This takes so long because numerous released elastic strain measurements must be taken, in order to calculate the inherent strain distribution. The residual stress cannot be analyzed without experiment for strain measurement of the object to be evaluated in the conventional method.

In this paper, we present a new and simplified method for estimating residual stress in welded structures with complicated shape. The method is then validated by calculating the residual stress at the welded juncture of a small diameter pipe penetrating the spherical head of a pressure vessel, as an example. This calculation is conducted based on the inherent strain inferred from another simple welded joint. The inferred inherent strain distribution at the welded-pipe vessel-penetration juncture is assumed to be a simple distribution, and the residual stress is calculated by elastic analysis using this assumed inherent strain. Finally, the residual stress distribution obtained using the proposed method is compared with the results of direct measurement of a mock-up specimen, in order to study the applicability and accuracy of the new method. The merit of this method is that actual experiments are not necessary.

2. THEORY OF INHERENT STRAIN ANALYSIS

The elastic response equations are generally derived as the relationships between the vectors of inherent strain \( \{e^*\} \), elastic strain \( \{e^e\} \), and stress \( \{\sigma\} \) which occur at any location in an elastic body owing to the inherent strain,

\[
\{e^e\} = [H^*] \{e^*\} \tag{1}
\]

\[
\{\sigma\} = [D]\{e^e\} = [D][H^*]\{e^*\} \tag{2}
\]

where \([H^*]\) is the elastic strain and inherent strain matrix, and \([D]\) is the elastic stress and strain matrix.

The distribution zone and the magnitude of inherent strain do not change as long as no new inherent strains are added by plastic deformation, such as that resulting from mechanical cutting, external loads, and so on. Therefore, the inherent strain can be estimated from the observed value of elastic strain as follows, even if the geometry of the object changes as a result of cutting \([9]\[10]\).

At first, to measure the elastic strains induced in the structural member, as many changes of strain as possible are observed by cutting, or the like. The measurement equation is written as follows, to take into account various kinds of measurement errors that may be included in the observed strains.

\[
\{e^e\} - [H^*]\{e^*\} = \{V\} \tag{3}
\]

where \(\{e^*\}\) is the most probable value for inherent strain, and \(\{V\}\) is the residual. The former can be determined from the condition that minimizes the sum of the squares of the residual \(\{V\}\), as follows.

\[
\{e^*\} = ([H^*]^T[H^*])^{-1}[H^*]^T\{e^e\} \tag{4}
\]

The most probable value \(\{\hat{e}\}\) for the welding residual stress at any location can be determined by substituting the most probable value \(\{e^*\}\) obtained for the inherent strain into \(\{e^*\}\) in Eq. (2).

When the vectors of inherent strain \(\{e^*\}\) are already known, residual stress can be obtained by elastic analysis without calculating \([H^*]\).

\[
\{\sigma\} = [D](\{e\} - \{e^*\}) \tag{5}
\]

where \(\{e\}\) is total strain, and can be expressed as the sum of the inherent strain \(\{e^*\}\) and the elastic strain \(\{e^e\}\), as can be seen by comparison of Eqs. (2) and (5).
3. A NEW METHOD FOR ESTIMATING RESIDUAL STRESS

With conventional inherent strain analysis, it is necessary to measure residual strains from a mock-up specimen to be evaluated and then to calculate inherent strains by solving an inverse problem using the measured strain data. Producing mock-up specimens and measuring take a lot of time and are expensive, particularly for complicated welded structures. It also takes great deal of time even to measure the strains on the surface alone.

The inherent strain distributions of complicated welded structures should be assumed to be simple, in order to make solving this difficult problem much easier. This is helpful because welding residual stress can be obtained by the elastic calculation without conducting actual experiments, if the inherent strain distribution is already known. The inherent strain in complicated welded structures should be inferred from the inherent strains of welded joints having simple shapes.

4. RESIDUAL STRESS ANALYSIS FOR A COMPLEX WELDED STRUCTURE

4.1 Configuration of Welded Pipe Joint

The distribution of inherent strain at the welded joint of a pipe penetrating a pressure vessel can be assumed to be simple in order to make it easier to calculate the residual stress. The inherent strain distribution in the object was inferred from the inherent strains in welded joints of simple shape, such as bead-on plate, plate butt-joint, pipe butt-joint or plate T-joint structures.

The example configuration was the welded juncture of a small diameter pipe where it penetrates the spherical head of a pressure vessel as shown in Figs. 1 and 2. The pressure vessel is made of a low carbon alloy, and is clad on the inner surface layer with high nickel alloy. The pipe is austenitic stainless steel, and the weld metal is also a high nickel alloy. The angle 0 between the pipe axis and the tangent plane of the pressure vessel varies from perpendicular to 45 degrees, according to the positions of the pipe on the spherical head of the vessel. The angles 0 which were picked up as the particular example for our calculation model were 90 degrees as shown in Fig. 1 and 60 degrees in Fig. 2.

4.2 Assumed Inherent Strain

The shapes of the inherent strain distributions of three vertical paths through a bead-on plate structure indicate that the maximum value is uniformly located in the weld metal part, and drop steeply from there, and finally reach zero at a certain point well removed from the weld metal part [13]. This can be approximated as a trapezoidal distribution. Longitudinal inherent strains are of the same magnitude on the weld metal as its two transverse inherent strains, and the zone of distribution of the longitudinal inherent strain is 140 % of the width of the two transverse inherent strains. The shear components of inherent strain are nearly zero. The inherent strain distributions of three welded plate T-joints, with different plate widths, which were welded symmetrically and simultaneously in single welding passes also resemble the trapezoidal distributions as shown in Figs. 3 and 4 [14]. The inherent strain distributions are nearly same although the plate width is changed. A multi-pass welded plate T-joint also has this same tendency of those with the single-pass T-joints [15]. Furthermore, a multi-pass welded plate butt-joint and a multi-pass welded pipe butt-joint also have inherent strain distributions that are nearly trapezoidal, and these two distributions are almost identical when the shape of cross-section and welding conditions are same, as can be seen from the through-thickness distribution at the weld metal [16].

We can generalize from this, to assume that the inherent strains of various welded joints with simple shapes all have the same tendency to show trapezoidal distributions in three
vertical paths around the weld metal, and to have shear components near zero. The width and magnitude of the inherent strain distribution depend on the kinds of materials, heat input, and the configuration of the joint.

Making inferences based on the inherent strain distributions of these simple-shaped joints, we arrived at the assumption that the distribution of inherent strain in the welded joint of a pipe penetrating a pressure vessel is also trapezoidal around the weld metal. In order to attribute the inherent strain distribution to the object, welding-pass direction \( \xi \) and the two perpendicular directions \( \zeta \) and \( \eta \) at any location are defined as shown in Fig. 5. The block of weld metal can mean the whole mass deposited in multiple passes. Assumed inherent strains and their zones of distribution in the welded joint are shown in Fig. 6. The weld-directional inherent strain \( \varepsilon_{\xi}^* \) is \(-2.5 \times 10^{-3}\) units on the weld metal itself and over the 50-mm distribution zone from the weld metal toe. Both transverse inherent strains \( \varepsilon_{\zeta}^* \) and \( \varepsilon_{\eta}^* \) are of the same magnitude as \( \varepsilon_{\xi}^* \) and extend across 36-mm distribution zones.

The inherent strain changes with differences in materials, particularly with differences in their Young’s moduli and in the yield stresses [14]. However, the Young’s moduli and yield stresses are almost the same for the three materials making up the welded pipe joint. It can thus be assumed that the inherent strain distribution of the welded joint does not depend on differences in materials. The heat input and welding method in the first three and subsequent ten welding passes are different, as we will see in the next section. The inherent strain distribution is considered to be determined by the heat input in the last ten welding passes, with no relation to that of the first three passes [5]. Therefore, the heat input and welding method of the final ten passes are used as the reference in the joint of the welded pipe penetrating the pressure vessel.

In consideration of the differences in materials and heat inputs, the magnitude of the weld-direction inherent strain on the weld metal and the width of its distribution were inferred from those of the infinitely thin plate butt- and T-joints with simple shapes, because the welded pipe joint object was treated as having infinite width. The two transverse inherent strains are almost zero in thin welded joints, and are assumed to have the same magnitude on the weld metal and to have distribution zones \( 100/140 = 71 \% \) as wide as that of the longitudinal inherent strain [13], by inference from the inherent strain distribution of a bead-on-thick plate structure.

The residual stresses of two models were calculated using this assumed inherent strain by the elastic analysis with the finite element method. The perpendicular-joint model was analyzed using an two-dimensional axi-symmetric mesh model, and the 60-degree model was analyzed using a three-dimensional model. Both models were given the inherent strain distributions at any location along the welding-pass direction and its transverse directions as shown in Fig. 5.

4.3 Residual Stress Measurement by Strain Gauge

The welding residual stresses in a mock-up specimen were also measured using strain gauges, for comparison with the results of our inherent strain analysis. The mock-up specimen consisted a small diameter pipe connected to a thick plate by thirteen passes of circumferential welding. The thick plate simulated the spherical head of a pressure vessel. The first three welding passes were made by tungsten inert gas arc welding, and the last ten passes were made by a shielded metal arc welder. Two types of mock-up specimens were produced, with angles corresponding to those show in Figs. 1 and 2.

Strain gauges were attached to the inner and outer surfaces of the pipe after connection-welding. The specimens were then cut into many small pieces around the strain gauges to release residual stress, and the circumferential released strain \( \varepsilon_{\theta} \) and the axial strain \( \varepsilon_{\zeta} \) were measured. Residual stresses \( \sigma_{\theta} \) and \( \sigma_{\zeta} \) in the circumferential and axial directions were obtained from these data using the plane stress condition equations, using the stress-strain relation in plane stress condition.
4.4 Comparison of Residual Stress Distribution

The comparison of the residual stress distributions on the inner surface of the perpendicular pipe penetrating pressure vessel is shown in Fig. 7. The stress distribution from our inherent strain analysis agrees completely with the direct measurement values of strain gauges in both circumferential and axial directions. Hoop stresses are all tensile and concentrate near the welding metal deposit at the pipe inner surface. The welding deposit causes a compressive axial stress on the inner surface near the deposit, and tensile axial stresses are distributed on both the sides of the inner surface.

The residual stress on the outer surface of the pipe in the perpendicular welded joint is shown in Fig. 8. The stress distribution shown is for the outer pipe surface in the pressure vessel when it is soaked in water during use. The calculated distributions and measurement values have good agreements. Circumferential stress is tensile near the welding metal deposit at the pipe outer surface. Axial stress is compressive near the deposit except for the two values measured on the weld toe.

The comparisons between calculated and measured residual stresses in the circumferential and axial directions along the vessel center side of inner pipe surface for the 60-degree slanting welded joint are shown in Fig. 9. For the opposite vessel cylinder side, the inner surface residual stress is distributed as shown in Fig. 10. The residual stresses calculated using inherent strain agree well with the values of direct measurement using strain gauge. The distributions along the pipe outer surface were also confirmed to agree with each other. These results show that the calculated distributions using inherent strain agree well with measured values in slanting welded pipe joints, which are the typical example of the three-dimensional structures of complicated shape.

There is some level of mismatch between measured values and calculated distributions. One of the reasons for this is thought to be some unreliability of the measured values. The precise measurement of residual strain from the mock-up specimen is very difficult particularly at the pipe inner surface, because there is so little space inside the pipe. Another reason is thought to be that the measured values near the weld toe at the outer surface tend to be uncertain because these locations may be on the beginning and ending points of the circumferential welding passes, not on the steady welding part.

5. DISCUSSION

The estimated residual stress distributions using assumed simple inherent strain agree well with the value directly measured by strain gauges, both for perpendicular and slanted pipe joints. The time needed for computing these calculations is much less than that required for the thermal elastic-plastic analysis.

It takes a great deal of time to determine inherent strain distributions by the measurement from mock-up specimens, even for welded joints of simple shape. Once the measurement of inherent strain for simple-shaped welded joints has been saved in a database, residual stress can be estimated using the elastic finite element method, even if the welded structure to be evaluated has a complicated three-dimensional geometry such as that of the welded joint where a pipe penetrates the pressure vessel. This is done by using an assumed inherent strain inferred from the database of welded joints with simple shape.

The proposed method can be used to easily calculate welding residual stress in three-dimensional structures with complicated shapes, by using it as the basis for the elastic analysis. The longer the database of inherent strain distributions of simple-shaped welded joints under various welding conditions gather, the more the precisely residual stress of complicated three-dimensional welded structures can be predicted. It is considered that such database of the inherent strain can be effective tools to rapidly and precisely predict residual stress in the welded structure of complicated shape.
6. CONCLUDING REMARKS

A new and simplified method for estimating residual stress in complicated welded structures by inherent strain was studied, using the example of a welded juncture of a pipe penetrating the spherical head of a pressure vessel. The distribution of inherent strain at this welded pipe joint can be assumed to be a simple trapezoidal distribution. The inherent strain distribution of the structure was inferred from the distributions in welded joints with simple shapes. The residual stress distribution estimated using this inherent strain analysis agreed well with the results of direct measurements taken using strain gauges. The method proposed in this paper can be used to easily estimate the welding residual stresses in various three-dimensional structures with complicated shapes, by the application of the elastic finite element analysis.

REFERENCES

Fig. 1  Configuration of welded joint of pipe penetrating the pressure vessel (θ: 90-degree model).

Fig. 2  Configuration of welded joint of pipe penetrating the pressure vessel (θ: 60-degree model).

Fig. 4  Inherent strain distributions at three plate T-joints with different plate width of web and flange.

Fig. 5  Definition of direction with respect to weld metal part for assuming inherent strain distribution.

Fig. 3  Configurations of plate T-joints with different widths of web and flange.

Fig. 6  Assumed inherent strains and their distribution zones in welded joint of pipe penetrating pressure vessel.

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<tr>
<th>Type</th>
<th>Width of Web (mm)</th>
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**Fig. 7** Residual stress distribution on pipe inner surfaces of perpendicular welded joint where pipe penetrates pressure vessel ($\theta : 90$-degree model).

**Fig. 9** Residual stress distribution on pipe inner surface at vessel center side of slanting welded joint where pipe penetrates pressure vessel ($\theta : 60$-degree model).

**Fig. 8** Residual stress distribution on pipe outer surfaces of perpendicular welded joint where pipe penetrates pressure vessel ($\theta : 90$ degree model).

**Fig. 10** Residual stress distribution on pipe inner surface at vessel cylinder side of slanting welded joint where pipe penetrates pressure vessel ($\theta : 60$-degree model).