Plastic zones at fatigue crack tips and at notch tips

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ABSTRACT

It was undertaken to determine whether or not it would be appropriate to use fatigue crack starter notched specimens instead of fatigue cracked specimens for testing the $K_{IC}$ of metallic materials. Experimental measurement of the plastic zone length at both types of notch confirmed that it is possible to conduct fracture toughness tests with fatigue crack starter notched specimens. The relevance of Irwin’s plastic zone correction was examined.

1. INTRODUCTION

Fatigue cracked specimens are used to determine the fracture toughness ($K_{IC}$) of metallic materials by means of a test [1]. However, it is not technologically easy to induce fatigue precracking. Depending on the type of material, it is difficult to induce a crack which does not develop in a slant. It would therefore be of great practical benefit if there were an easy way to test fracture toughness by means of a fatigue crack starter notch. One may be able to determine the possibility of testing with a fatigue crack starter notch by comparing the mechanical properties of a fatigue crack tip and a fatigue crack starter notch tip when they are subjected to a load. Such mechanical properties are closely related to the elastic-plastic state of the material.

It has been noted that when a low carbon steel sheet is pressed, visible bands of deformation appear on the product surface at the location where yield point elongation brought about a plastic deformation. These visible bands of deformation are called stretcher strains or Lueders’ lines [2]. Specimens subjected to compression or to tension exhibit Lueders’ lines to the naked eye. Finally, experiments have ascertained that Lueders’ lines also become visible in cases of bending and torsion [3]. Summarizing from these data, Lueders’ lines or stretcher strains will henceforth be referred to as Yield Stress Patterns [4].

It is easy to observe yield stress patterns with the naked eye. Yield
stress patterns enable one to recognize the presence of an elastic-plastic state immediately. Therefore it was undertaken to compare the plastic zones ensuing from tensile loads at the tips of a fatigue crack and of a fatigue crack starter notch by means of direct visual observation.

2. EXPERIMENTAL PROCEDURE

Figure 1 displays the dimensions of the test specimen [1]. The fatigue crack starter notch tip had a radius of 0.05 mm. The fatigue crack starter notch will henceforth be referred to as the notch. The magnification given in Figure 2 exhibits the surface state of the zone at the fatigue crack. Table 1 presents the chemical components of the test material. The object of a fracture toughness test is brittle material. Test specimens made of brittle material should therefore be used when investigating plastic zones. However, yield stress patterns do not appear in brittle materials, whereas they do develop in ductile materials. The material presented in Table 1 was used because it is possible to make direct observation of phenomena related to plastic zones if a specimen of ductile material, in particular carbon steel, is used in the experiment.

A tensile test of the material in Table 1 indicated that it had a yield stress of 335 MPa. The test specimen was vacuum annealed subsequently to machine processing. The surface of the test specimen was then finished off to a fine smoothness with emery paper, so as to make the striped patterns that would appear on the surface easily observable. The surface was measured to have an Rmax value of less than 0.5μm. A material testing machine was used to apply a tensile load to the test specimen. A crosshead speed of 0.1 mm/min was chosen. As the tensile load increased, a video camera with a magnification of 15 was used to observe the yield stress patterns in the zones of the fatigue crack tip and notch tip.

3. RESULTS AND DISCUSSION

3.1 The Shape of the Plastic Zone at the Fatigue Crack Tip and At the Notch Tip

Figure 3 illustrates the yield stress patterns as they appear in the vicinity of the fatigue crack tip. The recorded data of (1) to (3) in Figure 3 changed in direct proportion to the increase of tensile load. By inference from the mechanical properties of yield stress patterns, the patterns seen in Figure 3 indicate that there is a plastic zone at the fatigue crack tip.

The plastic zone in Figure 3 appears when the material reaches its yield point. The zone at the tip of crack satisfies the von Mises yield criterion, and is expressible as the following equations [5]:

Plane stress condition:

\[ r_y(\theta) = \left( K_I / 4\pi \sigma_y^2 \right) \left[ 1 + \cos\theta + 3/2 \cdot \sin^2\theta \right] \]  \hspace{1cm} (1)

Plane strain condition:
\[ r_r(\theta) = \left( \frac{K_1^2}{4 \pi \sigma_y^2} \right) \left[ (1 - 2\nu)^2 (1 + \cos \theta) + \frac{3}{2} \cdot \sin^2 \theta \right] \] (2)

Figure 4 presents a comparison of the experimentally measured shape of the plastic zone with the shape suggested by the loci of the points that satisfy the von Mises yield criterion. The curved lines were obtained by drawing the loci of the points of the equations (1) and (2) for the plastic zone given in Figure 3. Notation (1) in Figure 4 represents equation (1), and notation (2) represents equation (2).

The shape of the experimentally measured plastic zone of Figure 4 resembles the shape calculated from the loci of the von Mises yield criterion. The experimentally measured length of the plastic zone is the same as locus (1) of the theoretical values of the plane stress condition. The shape of the plastic zone given in Figure 3 resembles the shape of the plastic zone as derived from the etching method [6]. The shape of the plastic zone at the fatigue crack tip in Figure 4 is the same as the locus of formula (1) of the plane stress condition. Accordingly, the plastic zone will be examined when it expresses the plane stress condition.

Figure 5 exhibits the yield stress patterns that develop at the notch tip from tensile load. The plastic zone which develops at the notch tip is similar to the plastic zone which developed at the fatigue crack tip in Figure 3.

3.2 Plastic Zone Length

3.2.1 The Experimental Results

An experimental measurement of the plastic zone length can be obtained from Figure 3. Calculation of the displacement relating to tensile load with the data of Figure 3 yields the relation between plastic zone length and tensile load. The plastic zone length is adjusted for the stress intensity factor \( K_1 \). \( K_1 \) of the arc shaped specimen is expressed by equation (3) [1]:

\[ K_1 = \left( \frac{P}{BW^{1/2}} \right) \left[ 3x/W + 1.9 + 1.1 \frac{a}{W} \right] \times \left[ 1 + 0.25(1 - \frac{a}{W})^2(1 - r_1/r_2) \right] f(a/W) \] (3)

Figure 6 presents the results of adjusting the experimentally measured values of the plastic zone length at the fatigue crack tip and at the notch tip for \( K_1 \). The vertical axis represents the plastic zone length \( r_p \), while the horizontal axis represents \( K_1 \). In Figure 6, regardless of whether it is a fatigue crack tip or a notch tip, the experimentally measured values of \( r_p \) are located to the upper right of the vertical line. At this moment, they have almost the same value in relation to any given \( K_1 \). However, taking the vicinity of \( r_p \): 1.2 mm as a boundary, the locations inclining toward this boundary undergo a slight change. The experimental measurement for \( r_p \) in relation to \( K_1 \) was thus obtained. It will be attempted to verify the formula for the plastic zone length.
3.2.2 The Formula for the Length of the Plastic Zone

The length of the plastic zone at the tip of the crack for the plane stress condition is given in the following equation \( r_d \) by substituting in the monaxial yield stress value \( \sigma_y \) for the equation of the elastic stress distribution.

\[
    r_d = \left( \frac{1}{2} \pi \right) \left( \frac{K_1}{\sigma_y} \right)^2
\]

The value given in Equation (4) is called the first approximation. The following equation results by computing the redistribution of the stress of the part extending beyond the elastic stress distribution of \( \sigma_y \):

\[
    a_e = a + r_d
\]

Irwin considered \( a_e \) to be the length of the imaginary crack. This value \( a_e \) is therefore called the crack length of Irwin's plastic zone correction. Application of Irwin's plastic zone correction renders the length of the plastic zone \( r_p \) to be:

\[
    r_p = 2 r_d
\]

3.2.3 The Experimentally Measured Length of the Plastic Zone and the Theoretical Value

In order to compare the experimental value and theoretical value, in Figure 6 the straight lines (a) and (b) were obtained by plotting the lines of equation (4) and equation (6). The points in Figure 6 exhibit the values that were obtained by measuring the plastic zone length from the plastic zone shape according to FEM. The experimental values and the values given from FEM are located in the vicinity of the straight line (a) that represents equation (4). In Figure 6, assuming that the experimental measurements of \( r_p \) are less than 1.2 mm, they will be located in the vicinity of line (a).

The following equation (7) is one of the equations that express the plastic zone lengths that satisfy the conditions for small scale yielding:

\[
    \omega < \left( \frac{0.1 \sim 0.2}{(2.5) \left( \frac{K_1}{\sigma_y} \right)^2} \right)
\]

The value \( K_1 \) of the specimen in the present experiment was 25 MPa m\(^{1/2}\) and its yield stress was 335 MPa. The scope of small scale yielding was ascertained to be 1.39 mm by calculating according to formula (7) under these conditions. This value approximates the aforementioned value of 1.2 mm. In Figure 6, all of the values of \( r_p \), which are less than 1.2 mm are located above line (a). The results of Figure 6 experimentally confirm that formula (7) is an equation for the scope of small scale yielding. Under the condition of small scale yielding the experimentally measured value for the length of a plastic zone coincides with the value of the first approximation.
The positions of the experimentally measured values of a small scale yielding state are located in Figure 6 along line (a), which forms their first approximation. However, the values of those positions obviously run counter to the values as calculated according to Irwin's plastic zone correction given by line (b) in Figure 6. As Irwin's plastic zone correction is generally acknowledged to be true, the inconsistency in Figure 6 is a problem which cannot be evaded.

3.3 Plastic Zone Correction

3.3.1 The Process by which a Plastic Zone Develops at a Crack Tip

It would be useful to consider the process by which a plastic zone at a crack tip develops as a result of increasing static tensile load. As long as the load is small, line (1) in Figure 7 accurately expresses the elastic stress distribution at the crack tip. However, as the load increases the stress distribution in that area makes a parallel upward movement similar in form to that expressed by line (2). When the stress operating at the crack tip reaches yield stress, the stress in that location reaches its highest limit and assumes a stress distribution exhibited by line (3).

According to the explanation [5] for Irwin's plastic zone correction area A enclosed in the slanted lines of Figure 7 is an additional object of stress redistribution together with the crack tip. However, it is necessary to think of the plastic zone in the crack tip as having developed from this part of the energy. The energy of area A in Figure 7 is equal to the energy needed to produce the plastic zone. The energy diffused into stress redistribution is already used up in producing a plastic zone. It is therefore inconsistent to explain the plastic zone in terms of stress redistribution; it is incorrect to think that a plastic zone results from stress redistribution. It is more coherent to suppose that a plastic zone adjustment takes place when a plastic zone develops at a crack tip.

3.3.2 The Plastic Zone Correction of a Fatigue Crack Tip

When a crack starts to develop, a plastic zone develops together with the appearance of the crack at the location of its tip. Judging from the results of Figure 6, the length of the plastic zone that appears at the fatigue crack tip should be the value of the first approximation. The calculation of the plastic zone length of the present test specimen indicates that the value of the plastic zone length is 0.25 mm when a load of 3.3 kN is applied to the fatigue crack. This value can be deciphered from the upper portion of Figure 6; the length of the plastic zone at the fatigue crack tip is very short. This is also evident from the fact that the yield stress patterns at the fatigue crack tip of Figure 2 are difficult to discern. Putting it in practical terms, Irwin's plastic zone correction is not particularly necessary.

3.4 The Stress Intensity Factor in Relation to Crack Tip and Notch Tip
According to the experimental results of Figure 6, in relation to the same value of $\lambda$, the plastic zone lengths of a notch tip and of a fatigue crack tip are about the same. These results support the plausibility of assessing the value of fracture toughness by means of a notch tip specimen.

To test this possibility it was undertaken to compare the results of applying a stress intensity factor $K_0$ to a notch tip specimen with a radius of 0.05 mm and to a fatigue crack tip specimen. Figure 8 presents the values of $K_0$ for notched specimens and fatigue cracked specimens. The horizontal axis exhibits the ratio of the crack length or notch length "a" to ligaments "b", while the vertical axis exhibits $K_0$. For the same values of a/b in Figure 8, the experimental values of $K_0$ of the notch tip and fatigue crack tip are very close to each other. The results of this experiment suggest that, since the values of $K_0$ of a notch tip and a fatigue crack tip are almost the same, it is possible to use fatigue crack starter notched specimens instead of fatigue cracked specimens for a fracture toughness test. Using fatigue crack starter notched specimens would therefore probably be a convenient method for fracture toughness testing.

4. CONCLUSION

Direct observation of plastic zones brought about by tensile load was used to measure the plastic zone lengths of a fatigue crack tip and a fatigue crack starter notch tip of arc-shaped specimens made of carbon steel.

It was ascertained that for any value of $\lambda$, the plastic zone lengths of the fatigue crack tip and the notch tip were nearly equal. The values of $K_0$ for the fatigue cracked specimen and for the fatigue crack starter notched specimen are nearly the same. It was confirmed that it is feasible to conduct a fracture toughness test on a fatigue crack starter notched specimen without inducing a crack. It was also confirmed that it is unnecessary to concern oneself with plastic zone correction for stress redistribution.

REFERENCES

Fig. 5 The yield stress patterns in the notch tip zone

The energy of region A varies to the plastic zone P.

Fig. 6 The lengths of the plastic zones at the fatigue crack tip and at the notch tip to $K_I$

Fig. 7 The initiation and propagation process of the plastic zone at the crack tip

Fig. 8 $K_O$ in relation to the ratio of the crack length or notch length "a" to ligament "b"
Detailed diagram of A

Fig. 1 The shapes and dimensions of test specimens

Fig. 2 The surface state in the zone of the fatigue crack

Fig. 3 Propagation of the yield stress patterns in the fatigue crack tip zone

Table 1 The chemical compositions of the specimens (\%)  

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
</tr>
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<tr>
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<td>0.19</td>
<td>0.63</td>
<td>0.018</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Fig. 4 The collation of the experimental measurements of the plastic zone with the loci given by the formula satisfies the von Mises yield criterion