Analysis of a cruciform bending specimen under biaxial loading

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ABSTRACT
A co-operative research programme in the field of the local approach to cleavage fracture applied to reactor pressure vessels is conducted between EDF, CEA and Framatome. The purpose of this programme is to improve knowledge of this kind of approach for reactor pressure vessel fracture mechanics integrity assessments regarding the risk of brittle fracture. Within the framework of this programme, an examination of the influence of biaxial loading on cleavage fracture toughness are made. With this purpose a cruciform bend specimen test, performed at Oak Ridge National Laboratory, is analysed using Beremin model and a beneficial effect on the crack behaviour is shown.

LIST OF NOTATIONS
E Young’s Modulus.
v Poisson ratio.

\[ \sigma_y \] Yield stress.

CMOD Crack Mouth Opening Displacement.
LLD Longitudinal Line Displacement

\[ K_f = \sqrt{GE'} \] Equivalent stress intensity factor with \[ E' = \frac{E}{1 - v^2} \]

G Energy release rate.

\[ \sigma_w \] Weibull stress

\[ \sigma_u \] Ultimate stress

\[ V_0 \] Size of the microstructure of the material

m Weibull parameter
1. **Introduction**

Several tests have been performed at Oak Ridge National Laboratory (USA), in order to examine the influence of biaxial load on the cleavage fracture toughness of shallow flaw specimens under conditions prototypic of a reactor vessel. Among them, BB-4 [1] reproduces a configuration of biaxial bend specimen. The specimen has a cruciform-shaped geometry with a straight through-crack with a uniform depth. 2D and 3D finite element elastic-plastic calculations using CASTEM2000 finite element code [2] have been performed with both biaxial and uniaxial loadings, in order to evaluate the influence of loading on the crack behavior and to compare with test results. Moreover the Beremin model [4] is applied to analyse the failure probability.

2. **Local approach to cleavage fracture**

The local approach to cleavage fracture (Beremin model) has been developed by F. Mudry [5]. This approach is based on the weakest link theory and the use of a Weibull statistics. The model is based on the use of damage criterion which is computed from the history of mean stress and strain in an elementary material cell. The size of this elementary cell is a material parameter, related to the material microstructure. The probability of cleavage failure is evaluated by the following expression, taking into account the stress and strain field at the crack tip:

$$ P = 1 - \exp\left(-\frac{\sigma_w}{\sigma_u}\right)^m $$

There is no coupling between damage and plasticity. The failure probability is evaluated by a specific post processor of classical elastic-plastic computation. $m$ and $\sigma_u$ are two parameters characteristic of the material resistance, characterising the scatter (Weibull parameter $m$) in cleavage fracture and the resistance of material to rupture (critical stress $\sigma_u$) and are identified using mechanical tests on axisymmetrically notched tensile specimens. $\sigma_w$ is the 'Weibull stress' defined by:

$$ \sigma_w = \left(\sum_i (\sigma_{\text{ni}})^m dV_i / dV_0\right)^{1/m} $$

with:

- $\sigma_{\text{ni}}$ the maximum principal stress in the finite element $i$
- $V_i$ the volume of the mesh element $i$
- $V_0$ the elementary volume related to the material microstructure

The numerical integration of $\sigma_w$ is made only in the elements which are plastically deformed.

3. **Problem statement**

3.1 **Geometrical data**

The structure is a cruciform bend specimen for biaxial fracture toughness testing. The total length of this specimen in longitudinal or transverse direction is 61 cm. Three slots are machined into each arm to minimize diffusion of the load around the test section containing the through-crack which is 10 mm deep. The crack is cut between two opposite central load diffusion control slots in order to produce a two-dimensional shallow crack with no singularity on the surface. Figure 1 summarizes this geometrical description. Actually, in uniaxial load configuration, the design of slot in the test is not as shown on figure 2. The two slots on both sides of the crack on transversal arm are in fact 8.9 mm shorter. It could explain some differences between calculation and experimental results in this particular case.

3.2 **Material**

The specimen is made of A553 grade B class 1 steel. The material properties at -46°C are the following:

- Young's Modulus: $E = 205170$ MPa.
- Poisson ratio: $\nu = 0.3$. 

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Yield stress : \( \sigma_y = 452 \text{ MPa} \).

Strain Hardening curve is provided on figure 2.
The Weibull parameters used for Beremin model application are :
\[ m = 22 \quad \sigma_u = 2560 \text{ MPa} \quad \nu = 50 \mu m^3 \]
These values correspond to classical values for French RPV A508 cl 3 material. The material used by ORNL has not been characterized for this type of approach.

3.3 Loading
Computations are made with two loadings (see figure 1):
- uniaxial load noted as 0 : 1
- biaxial load noted as 0,6 : 1
Experimental measurements show a uniaxial failure load reached at 784 kN and a biaxial failure load at 818 kN.

3.4 Boundary conditions
The center of the cruciform bending specimen is sitting on a rigid square. Thus, in the case of uniaxial load, a 2D model of the four point bending specimen is made. Moreover, because of symmetry only one quarter of the specimen is represented with symmetry conditions.

3.5 Mesh
Slots are represented with a double node surface. The 3D mesh contains approximatively 14000 nodes made of 20 node isoparametric brick elements (CUB20 in CASTEM2000 [2]). A very refined mesh is used along the crack front. To apply the Beremin model, brick elements of 50 \( \mu m \) by 50 \( \mu m \) are taken along the crack front on the expected direction of crack growth.

4. Uniaxial load configuration
In 2D calculation, we assume plane strain conditions. In the 3D case we have done two different calculations : one with slots (called 3D) and one without slot (called 3D NO-SLOT). Figures 3 shows the longitudinal load versus CMOD curve for 2D and 3D configurations and compares these values with the test results (called ORNL_EXP). One can notice that CMOD of 2D configuration is close to 3D calculation and experimental value.
Furthermore on figure 4, for a longitudinal load, the LLD-2D is lower than LLD-3D. In fact in the 2D calculation, we have not represented the slots and it seems that they make structure less rigid. The 3D computation without slot mode shows the same tendency if we compare the results with 3D calculation or test data (called ORNL-EXP). In general, we find a good agreement between 3D calculations and experiment for CMOD and LLD.
The plot of \( K_f = \sqrt{GE} \) (where G is calculated by the G_theta method [3]) along the crack front shows good correlation for the mid-points of the crack (30 mm < s < 56 mm) between CEA analysis and ORNL finite element analysis (called ORNL-FEA) although the load in CEA analysis is 652 kN and in ORNL is 665 kN. One can also notice that the slot tends to minimize the decrease of \( K_f \) on the point at the edge of the crack.
To sum up the results we have : \( K_f \) (without slot) < \( K_f \) (with slot) along the crack front.
One can notice on figure 10 that the load corresponding to a strong increase of failure probability computed in 3D configuration is close to 784 kN (critical value in the test).

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5. Biaxial load configuration

As for the case of uniaxial load, two 3D calculations are made: one with slot model and one without slots. Figure 8 shows the CMOD evolution as a function of longitudinal load: the computation results including slots are closer to test data than the results without slots. We can assume that the lack of slot tends to rigidify the structure and lead to a smaller crack opening. One can also notice on figure 7 a good agreement between \( K_f \) values obtained by ORNL and CEA finite elements analysis with slots. As for the case of uniaxial load, here also:

\[
K_f \text{ (without slots)} < K_f \text{ (with slots)} \text{ along the crack front.}
\]

A decrease of \( K_f \) can be observed on the point on the edge of the crack. Moreover the maximal value of \( K_f \) is reached at the mid-crack point. The load corresponding to an abrupt increase of failure probability (figure 10) is close to 818 kN in 3D configuration (critical value in the test).

6. Comparison between uniaxial load configuration and biaxial load configuration

The crack opens less under biaxial load than under uniaxial load. One can notice indeed on figure 8 that CMOD (biaxial) < CMOD (uniaxial) and that the same tendency can be observed with and without slot. Moreover the biaxial load gives the smallest yielded zone at the crack front.

The plot of \( K_f \) along the crack front on figure 11 shows that the difference between biaxial and uniaxial stress becomes important with a threshold value of 20 mm. Furthermore the \( K_f \) value is higher for uniaxial load than for biaxial load.

One can see on figure 10 that the failure probability is higher for uniaxial loading than for biaxial loading for a load about 800 kN. The ‘\( \sigma_n \) versus \( K_f \)’ curve (figure 11) shows a small influence of loading.

Thus the beneficial effect of biaxial load on the crack behaviour appears on CMOD but also on failure probability. We must add however that the Weibull model doesn’t take into account the small amount of ductile tearing before cleavage.

7. Conclusion

2D and 3D calculations under biaxial and uniaxial loadings has been performed and show that:

- 2D configuration is too rigid because it doesn’t represent the effect of slots,
- the slots have a large influence not only on the global deformation behaviour but also on \( K_f \) factor,
- the correlation between 3D finite element model with slots and experimental data is good,
- Beremin model application in 3D permits to find a failure load close to test failure load,
- biaxial loading has a beneficial effect on crack behaviour not only on CMOD but also on failure load.

REFERENCES

[1] - FALSIREF II WORKSHOP
   November 8-10, 1994
   Atlanta, Georgia

[3] - X-Z SUO and A. COMBESCURE
Energy release rate—and J—integral for cracks propagating in non-homogeneous media
problem formulation. Part II: finite element applications.
Revue européenne des éléments finis - Volume 2 - N°2/1993 - Hermès

A local criterion for cleavage fracture of a nuclear pressure vessel steel.
Metallurgical Transactions A
Vol. 14A, p. 2277-2287

[5] - F. MUDRY
Etude de la rupture ductile et de la rupture par clivage d'aciérs faiblement alliés
Thèse d'Etat, Université de Technologie de Compiègne, 1982

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**Figure 1**

Geometrical data - loading
A533 grade B classe 1 steel (T = -46 °C) 
Strain hardening curve 
**Figure 2**

LLD as a function of longitudinal load 
(Uniaxial load) 
**Figure 4**

CMOD as a function of longitudinal load 
(Uniaxial load) 
**Figure 3**

Variation of $K_I$ along the crack front 
(Uniaxial load) 
**Figure 5**
CMOD as a function of longitudinal load (Biaxial load)  
**Figure 6**

Comparison between biaxial and uniaxial loads CMOD  
**Figure 8**

Variation of $K_f$ along the crack front (Biaxial load)  
**Figure 7**

Comparison between biaxial and uniaxial loads - Variation of $K_f$ along the crack front  
**Figure 9**
Failure probability as a function of longitudinal load

Figure 10

σ_w as a function of $K_f$

Figure 11