A discussion of the use of finite elements to analyse safety-critical concrete structures

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ABSTRACT.

The use of non-linear finite element analysis to analyse concrete structures can provide an excellent technique for the engineering analyst, modelling the behaviour of reinforced and prestressed concrete pressure vessels and containment designs. However, the technique was not perceived to be developed adequately in order to demonstrate the concrete's response and structure's behaviour in relation to safety-critical situations, as well as the concept of showing a reserve strength of the design. The UK nuclear industry realised the potential to be gained from the non-linear finite element analysis technique. Therefore, a research and development project was inaugurated in 1993 to strengthen the understanding, knowledge, methods and future application of this promising analysis technique. This project was sponsored, supported and monitored by the UK nuclear industry.

This Conference paper summarises this interesting work together with the significant results and conclusions obtained during the study period. In addition, a framework is developed to enable engineering analysts to assess the integrity of vessels and containments using the non-linear Finite Element Method (FEM). Applying the basic design logic of the early concrete pressure vessel designers, it is possible to formulate an analysis strategy from the design pressure up to the hypothetical ultimate load, while assessing the reserve strength of the structure.

Another SMIRT Conference paper by Prof. Owen and Prof. Bicanic looks at the considerations involved in the pure numerical methods used for the FEM solution, which again, was an integral part of this research and development project. Meanwhile, this Conference paper will concentrate on the generic issue of modelling safety-critical concrete structures by the FEM in terms of an applied tool.
Many Finite Element Method (FEM) code developers can supply very capable software packages, all of which need to have progressed through sophisticated verification and validation procedures. With exponential increases in computing power over the past half-decade, both linear and non-linear structures can be analysed with relative ease, even through the use of smaller desktop workstations. This trend is obviously set to continue. For many engineers, there is a general belief that computer modelling of engineering products using the FEM is so advanced now that experimental testing is unnecessary. This may be true for the majority, but some designs are extremely difficult to model. A safety-critical concrete structure loaded up to its ultimate load is a classic example. That said, the use of non-linear FEM analysis to study concrete structures can provide an excellent technique for the engineering analyst in support of the operating utility and a safety authority. The UK nuclear industry realised the potential to be gained from the non-linear FEM analysis technique, but, after technical review, the method was not perceived to be adequately developed for safety justification purposes. For PCPVs and Primary Containments which are large, complex in design and possibly inaccessible in certain regions, there is a need to model their behaviour accurately under loading which extends into the non-linear regime up to final failure. While for safety case justification, there is a need to ascertain the structure's “failure mode”. To address these issues, a 3-year research and development project was inaugurated in 1993 to strengthen the understanding, knowledge and methods, together with assessing the future application of the non-linear FEM analysis technique. This project was sponsored, supported and monitored by the UK nuclear industry, with coordination through the Health and Safety Executive (HSE). Another Conference paper by Prof. Owen and Prof. Bicanic looks at the considerations involved in the pure numerical methods used for the FEM solution, which was an integral part of this research and development project. Meanwhile, this Conference paper will concentrate on the generic issue of modelling safety-critical concrete structures by the FEM in terms of an applied tool.

For the analysis of loadings on a prestressed concrete structure that go above the design / test pressure, it is necessary to implement different analysis logic with added care in the interpretation of solution results. This is because the concrete material acts in a highly non-linear manner and exhibits a complex response in relation to the failure envelope and the numerical solution algorithm that “solves” the problem. The FEM structural idealisation that makes up the model must be valid in terms of the geometry, material response, applied load and the specific numerical process to achieve the solution’s results. In the case of concrete as a structural material, the material response changes will not necessarily be 'progressive', but can be severely non-linear when cracking abruptly occurs with a resultant loss in effective tension strength. This can be especially significant in a massive structure with considerable redundancy, but having various potential load paths and failure mechanisms at play as the loading is increased. The significance of the path-dependency of the load bearing in the concrete structure becomes greater for an increasing inelastic response. Load re-distribution will occur in the structure's mass as cracks initiate and propagate for rising pressures above the design pressure. Not only must the FEM solution procedure detect individual inelastic events in the solid structure as a whole, but also, the correct sequence of inelastic events combined together must eventually produce a “unique-converged solution” which adequately represents the behaviour under load, with the final failure mode(s) and ultimate load magnitude confirmed.
Most material models which simulate the concrete (whether reinforced or prestressed) will include some form of factor to account for the gradual loss of tension strength at a point due to cracking, usually defined as tension stiffening, or a softening slope. Such a condition is often referred to generally as the "post-peak" response. It has been frequently recognised that such softening models should not be included only as a numerical feature (ie, smooth the effects of a sudden loss of tension strength and to facilitate convergence), but that a proper physical interpretation of these phenomena and a corresponding enhancement of traditional concrete models is necessary. The major problem here is the effect of mesh-sensitivity on the final solution results; on a simple basis, even though a numerical solution may have been achieved, the resulting behaviour predicted could be totally invalid. The degree of mesh-sensitivity varies in terms of whether modelling plain concrete, reinforced concrete or prestressed concrete with different percentages of reinforcement etc. Some FEM concrete material definitions attempt to minimise or avoid this mesh-sensitivity by enhancements based on "Fracture Energy", $(G_f)$ considerations. Correct mathematical treatment of such softening models is a subject of considerable research and debate at present, however, Fracture Energy based continuum formulations (and non-local micro plane models) have been shown to be effective. For the practicing engineering analyst, it is important that the dominance of a shear or flexural failure mode be correctly detected and analysed in the final "unique-converged solution". As stated previously, another Conference paper by Prof. Owen and Prof. Bicanic looks at these considerations in specific detail.

The particular concrete constitutive formulation used to model the concrete should be very carefully appraised. The procedure used to produce a FEM structural idealisation for non-linear concrete analysis should include a test sequence to ensure that the mesh is correctly configured to detect and analyse the progressive inelastic response of the concrete. Making the specific configuration of meshing of the finite elements to represent the structural idealisation therefore needs careful validation. Meshes that were made for the analysis of linear elastic behaviour may well be in-effective and possibly seriously in error if re-used as a finite element mesh for non-linear loading regimes in a concrete structure. In addition, unless a valid post-peak technique is applied then there is a danger of the solution being un-representative of the actual concrete vessel's non-linear behaviour at higher loads. Any small variations from the "valid" response at low structural loads (or pressures) will give rise to considerable errors in the solution of behaviour at higher loads. It is believed that this problem has occurred in various circumstances, but to clearly identify the root-cause of differences between standard FE-codes for the same non-linear loading can be very confusing.

From various analyses performed in this study with a constant concrete-bounding geometry, but with a varying finite element mesh density, it has been confirmed that the Strain-Softening post-peak method of non-linear solution can introduce mesh-sensitive results in the solution. The Fracture Energy post-peak method was shown to prevent this problem. Therefore, to correctly represent the concrete structure's behaviour up to over-pressure conditions using the FEM requires the use of valid techniques combined with carefully judged modelling assumptions; a major priority being to determine if mesh-sensitivity occurs with a given FEM model idealisation that is numerically representing the concrete structure. Essentially, the evil to watch out for is Strain-Localisation in the FEM model that can cause mesh-sensitivity problems. On a practical basis, this means that unless the FEM solution process computes the correct non-linear post-peak cracking behaviour within a thick-section concrete structure, even
at lower applied loads, the potential errors introduced by mesh-sensitivity of the solution results will be grossly inaccurate compared to the actual structural behaviour. This introduces the theoretical danger that the concrete structure's failure mode indicated from the FEM results is computed, but could well be totally invalid. Consequently, the non-linear response of the massive concrete material to over-pressure loading must be accurately calculated by the numerical finite element (continuum) model idealisation in the non-linear regime. This discussion is especially relevant to the PCPV design which is characterised by its thick-section cylindrical walls and end slabs. It also applies to containment structures which have thick-section basemats. Of special importance here is the possibility that within a thick-section concrete design, the mode of failure of the concrete could be triggered into a shear-mode as a result of a localised shear mechanism being produced in the structure for increasing pressure. This type of failure mode was known right at the outset of the conceptual phase of the early PCPV design studies, which was one of the main reasons why small scale experimental testing of scale-vessels was carried out to check that a shear failure mode was not of concern. The principal danger here is the possibility of a localised shear mechanism being produced in the structure for increasing pressure, but the FEM modelling may be unable to detect and indicate this failure mode. If a shear failure mode were to occur in the real structure, then a premature and sudden failure could be introduced which is contrary to the concept of the benefits of a "gradual failure mode". The pressure vessel designers aimed for a gradual mode of failure, always trying to design-out any possibility of shear failure conditions, while the experimental scale model tests would substantiate the failure mode to be progressive with no sudden catastrophic shearing.

For the present study discussed here, three main aspects of structural idealisation were studied; discretisation sensitivity, element size sensitivity, technique used to model the 'post-peak' response and the recommended analysis procedure. It is interesting to note that a finite element mesh built to solve an elastic analysis does not necessarily provide guidance as to the 'design' of the most effective discretised mesh layout for a non-linear problem, even though tensile stress regions can be identified in terms of location and magnitude. Therefore, "automatic" mesh generators may not produce an effective finite element mesh for non-linear modelling of concrete without intuitive control of the valid parameters. Further, even though a finite element mesh may be able to accurately resolve concentrated stress regions with a linear elastic analysis, utilising a finer mesh density does not necessarily guarantee effectiveness for a non-linear concrete analysis. Unfortunately, application of an arbitrary mesh refinement technique may in fact reduce the mesh's capability of analysing localised inelastic events in the concrete matrix. Consequently, the following principle factors should be considered in the structural idealisation and analysis of safety margin conditions for PCPVs and Primary Containments:

- the discretisation of the FE mesh must account for the non-linear effects of the concrete, specifically the existence and progressive distribution of localised events (cracking) and the 'post-peak' response;
- the element size of the FE mesh should not grossly affect the solution, the analyst performing the computations needs to account for any "mesh-objectivity" of his model.
- the analyst should have knowledge and confidence of whether a given structural idealisation / mesh is capable of capturing localised events and failure mechanisms effectively;
- the FEM analysis must provide a valid "unique-converged solution".
Unless these simplified aspects of using the FEM for over-pressure modeling of concrete structures up to the ultimate load are accounted for, analysis results from different FE-codes and different analysts may vary considerably with no clear reasons for discrepancies. This then poses the question of the application of "benchmarks" that must be agreed, with the need for verification and validation procedures to be clearly defined. Based on the results of non-linear PCPV analyses performed in this project, it has been confirmed that the application of the Fracture Energy method as a post-peak response criterion is important in order to prevent mesh-sensitivity. The main concrete's behaviour in the non-linear regime is correctly modelled and predicts the final failure mode of the structure. Effects of localised failure conditions and their effect on the global response of the vessel is satisfactorily resolved. However, with the analysis work processed as part of this project, considerable difficulty was initially experienced in being able to apply the Fracture Energy post-peak method for the modelling of vessels and containments. One major difficulty with the vessel analyses was to actually keep the solution process going. Another difficulty was the effect of progressive load increment cutting during the pressurisation load step when using the 'arc-length' solution technique. These two major difficulties can be categorised into "Premature Solution Cessation" and "Solution Locking (or Choking)". Various sensitivity studies had to be carried out with appraisals of the "solution" results until a clearer understanding emerged of why these difficulties were occurring, (please refer to the sister paper from this work).

When the finite element idealisation starts to go into the non-linear regime of response then Premature Solution Cessation can occur. This difficulty regularly occurred with a vessel analysis when the theoretical pressurisation first caused a stress concentration region to impinge on the concrete failure surface envelope. At this stage of the analysis the stress state was equivalent to the transition point of the FE-code's failure surface, where a numerical singularity happens to exist. If the solution method cannot cope with the numerical singularity then the solution process simply stops (crashes). Several other analyses performed in the study experienced convergence difficulties, which almost invariably manifested themselves with the FEM Solution Locking. This Solution Locking can best be described as progressive load increment cutting; caused by the non-convergent solutions, as most programs are internally organised in such a way, as to automatically cut the load increment if solution does not converge in the pre-redefined number of iterations. In general, the purpose of load increment cutting is to decrease the level of nonlinearity within an increment and to enable the nonlinear solution algorithm to converge, so that the next load step can proceed from a valid and fully converged solution. Here, a fully convergent solution implies that both equilibrium and constitutive equations (stress-strain law) are satisfied within a pre-redefined tolerance limit. If convergence difficulties persist, this can lead to repeated load increment cutting, with solution locking corresponding to a situation when the load increment has been reduced to practically zero. It is interesting to note that up to the point of locking, the solution typically compares very well with experimental observations in terms of pressure displacement curves.

A model idealisation of a safety-critical concrete structure should be analysed by specifying an input data deck which uses the as-measured material properties with specified strengths, as-drawn geometry, technique of prestressing confinement (if applicable) and applied loading history. Any modifications to the FEM model should only be implemented in order to assess the sensitivity of variations / attenuation in the structure's condition, whether due to change of duty, ageing, damage or clarifying the presence of a potential cliff-edge effect. Any model
changes to achieve the "expected" failure mechanism for a given problem must be guarded against. Great care should also be exercised in simply trying to keep a finite element solution process "running" in terms of extending the computation process of a non-linear concrete idealisation further than is technically valid. This problem can be manifested when Strain-Softening criteria is utilised for characterisation of the post-peak response, which can be highly mesh-sensitive and produce totally artificial solutions unrepresentative of the structure being analysed. A number of FE-code developers and distributors have given invalid advice in this context, suggesting that the defining value of Strain-Softening be increased to achieve a "more-stable" numerical solution. The basic mistake here is that the numerical solution may be able to continue in terms of computation, but at the expense of the accuracy of solution results compared to an actual structure and consequently the danger of invalid assessment of the failure mode at higher loads approaching the reserve strength.

Sometimes, engineering analysts may not be fully familiar with the overall response regimes of a concrete pressure vessel / containment design. In addition, construction-project time constraints may introduce strict penalties if work is not finalised on schedule. There is therefore a natural tendency to try and get as much information out of one particular FEM idealisation as is possible. Unfortunately, this gives way to near-arbitrary modifications and change of variables to achieve a numerical solution, which is totally fictitious and lacks validity. Another inherent problem is the dependence on FE-code developers to supply a valid source of supporting information. This information can be polarised towards the questions asked and the type of problems previously investigated by the FE-code developer in support of their respective client's needs. It could be argued that to completely depend on the FE-code developer to supply an "ideal" finite element software package, and for the technical support information to always be perfect, is a naive and foolhardy stance to take. (This argument still applies even with the compilation of verification examples). From this present study it can be stated that the most troublesome problems which prevent a valid finite element solution from a commercial FE-code are:

- the numerical solution fails (or "crashes") as a result of Premature Solution Cessation;
- the numerical solution sticks at a certain load increment which basically "jams" the solution from proceeding up to a higher load level, this is known as Solution Locking, or simply Choking;
- the final load level is achieved, but the solution results are in fact invalid due to the problems of Strain-Localisation not being accounted for properly, and the existence of a Mesh-Sensitive solution which may indicate the wrong concrete failure mode(s).

The combination of these three major problems cause errors and (sometimes gross) inaccuracies in the finite element solution results. It is necessary for the FE-code developers to address these problems as best they can on an urgent basis.

On a general basis, one of the most difficult questions for all, is, how can the engineering analyst solve for the behaviour of the concrete structure accurately when such a wide range of loading needs to be addressed in the assessment of safety-critical concrete structures? From this fundamental question, two approaches could be argued. The first argument would be that the FE software-developer should provide a comprehensive and robust capability to solve structural concrete behaviour. This capability being given for all types of structural
configurations and load conditions. Alternatively, taking a more practical viewpoint, a counter argument is that given the complexity of modelling the response of the concrete material itself, then the added complication of numerous design configurations and diverse load conditions, how is it possible to achieve perfectly valid solutions with the present commercial FE-codes? It would be wise to take an approach which aims to first identify the limiting scope of the FE-code, outside of which it either will not solve or cannot produce a unique-converged solution, essentially clarifying a "Range of Applicability", or "RoA". There is a need for certain limiting controls be exercised in the assessment of integrity using analysis results produced from non-linear FE-codes. It is suggested that the principle control should be based on confirming the validity of the solution results for a given structure and the acceptable range of use. By following a RoA approach it will be possible to constrain the valid finite element solution to the specific behavioural regime of the structure being modelled. From application of this principle, it will be necessary to set the scope of the behavioural load range between normal and extreme conditions. In addition, a series of agreed benchmark analyses should be incorporated to enable substantiation of the RoA methodology. Essentially, the engineering analyst will be required to demonstrate that the FE-code being used is capable and can produce unique-converged solutions up to the load of concern. The best framework for a RoA approach stems from the behaviour "Phases" first advocated by I. Davidson for the PCPV design, published in 1972 in Nuclear Engineering and Design. The principal reason in choosing "Davidson's Phases" is that his logic was formed from the experience gained by the UKAEA in the testing of small-scale prestressed concrete models. These tests were the foundation to demonstrating the inelastic behaviour of a given design of vessel structure above its design pressure, of most importance was the knowledge that a vessel will gradually fail in a particular way at it's ultimate load. Therefore, the basis to this discussion can be summarised as follows:

- the Range of Applicability (RoA) of finite element methods should be based on the three behavioural regimes (or Phases), in a sequence of increasing applied load;
  - Phase I - linear to quasi-linear, continuum-mesh;
  - Phase II - highly non-linear, failure mode developing, continuum-mesh;
  - Phase III - failure mode established, acting as "mechanism", discrete-mesh;
- the FEM code and model idealisations should be able to solve for behaviour in all three behaviour regimes, but different finite element idealisations will probably be required in the three regimes.

To apply the Davidson's Phases is therefore an implied reference to actual experimental evidence and institutes a simple principle in order to set limits by which finite element idealisations can be "benchmarked." What is of most importance in the present context is the need to show the validity of the theoretical FEM model in terms of solution results within the identified behavioural "Phase" of concern. This logic can also be applied to other industries that have large and safety-critical concrete structures.

Assuming the problems of concrete failure surface singularities, Premature Solution Cessation, Solution Locking (or Choking) and Mesh-Sensitivity with Strain Localisation have been overcome, then the non-linear FEM could be applied in order to analyse the structural response at loads within Davidson's Phase II regime. In the definition of the finite element analysis model, specific concrete properties with valid solution techniques would be utilised as
a basis to the modelling. This could pave the way to "reducing the uncertainties" about the behaviour of vessels between the proof pressurisation and the theoretical ultimate pressure. With a reduced uncertainty about the inelastic response at higher loadings, a more manageable set of criteria could be formulated in order to determine the integrity and safety of the vessel's design. Further analyses could also be performed to assess the sensitivity of attenuation of the prestressing system forces, variations in material property values and the effect of smaller geometric discontinuities. The practical analysis limit of a finite element model which has a "continuum-mesh" should be adequate in order to assess the non-linear response of the vessel in Davidson's Phase II (and Phase I) regime; up to the point where the structure can no longer be modelled as a single-continuum structure, but, eventually becomes a mechanism with a segmented and sectored crack state. Therefore, modelling using non-linear continuum finite element analyses can be used to represent behaviour at pressures considerably higher than the proof pressurisation, towards the ultimate load bearing capacity of the structure. A continuum adaptive mesh, or alternatively, a discrete "mechanism" mesh could potentially be used to analyse the Phase II to Phase III regimes. It is recommended that practically based assessment criteria should be specifically compiled to assess the acceptability of the predicted response of the structure using computer-based modelling, as opposed to, an abstract condition of the structure which is based on simplified speculation and which may involve considerable un-quantified conservatism that is unnecessary. From this study, certain basic requirements have been established for an "ideal" finite element software package from the work performed during the present study.

From practical experience of modelling many types of vessel and containment designs, linked with a high level of academic support from active experts in the field, the following "ideal" FE-code is recommended to be used for the analysis of concrete safety-critical concrete structures:

- A concrete constitutive material model which is based on a smooth triaxial failure envelope.
- A specially developed solution procedure for use with the complex response of concrete, the solution procedure should be able to retain a robust capability to produce a unique-converged solution of the finite element idealisation being modelled.
- Tensile post-peak options may also need to "switched" for failure assessments that define the structure's mode of failure:
  (a) σ–ε option, or "Strain-Softening", suitable for diffused failure predictions;
  and (b) Gij option, or "Fracture Energy," suitable for both diffused failure and localised failure predictions, (which should be available anyway).
- Distinguishing between diffused or localised failure of the concrete material, (possible application of the Acoustic Tensor indicator in post-processing).
- An automatic or semi-automatic re-meshing capability could be developed in order to optimise the finite element mesh idealisation, based on the Acoustic Tensor failure indicator information. This re-meshing capability will allow the failure mode to be more precisely resolved by the finite element mesh idealisation.