A two dimensional model for cooling towers with geometrical imperfections and cracks

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ABSTRACT: This paper presents a 2D modelling based on Fourier series for cooling towers taking into account reinforced concrete behaviour, initial cracking and geometrical imperfections. The model can also compute the stability with the tangent stiffness matrix. A numerical analysis presents the influences of initial damages on the strength of the structure. The effects of creep under self weight and the ultimate strength under the wind pressure are analysed.

I INTRODUCTION

For better understanding of the mechanical behavior of cooling towers, it is necessary to determine the origin state and the evolution of the typical damages of these kind of structures. More than that, the model that should be used has to be able to take into account these damages and their evolution in order to predict the critical load of these structures. The measures which have been performed on many cooling towers have shown the same pathology. This pathology is characterised by a quasi systematic great number of meridional cracks and by geometrical imperfections in shape of waves in the circumferential direction, sometimes with a significant amplitude (1), (2). To quantify the effect of the observed damages and their evolution, parametric analyses are necessary. These analyses must take into account the mechanical loading such as the self weight or the wind, as well as the thermomechanical loading. The progressive aspect of damage leads to consider also the time factor and the creep effect.

The design and the definition of the safety factor requires to diagnose the observed damage. For this purpose, an extensive study of the structure and its environment history is needed. These are, for example, the process of construction with the effects of the early age concrete shrinkage and the displacement of the soil which in fact represents the boundary condition.

The purpose of this work is to present a 2D model enabling accurate parametrical analysis (3). This paper includes two parts: the first part presents an adaptation of the finite element code INCA which has been developed. This code is based on COMU element (4) and on a concrete model (5). This approach enables us then to make creep analysis and to calculate buckling loads with the tangent stiffness matrix (Trefftz criterion and incremental theory).

The second one is concerned with the determination of the evolution of stability coefficient in a creep analysis under self weight and of the ultimate strength under the wind pressure. In each case, effects of initials damages are analysed.
2 THE NUMERICAL MODEL

2.1 Shell element (COMU)

An axisymmetric modelling with Fourier series is able to describe shell kinematic submitted to non-axisymmetric loads or with geometrical imperfections. This element takes into account only the small rotations The displacement field of the quasi-axisymmetric structure decomposed in Fourier series is:

\[
\{q\} = \sum_n \begin{cases} 
W_n^{\theta} \cdot \cos(i_n \cdot \theta) + W_n^{\phi} \cdot \sin(i_n \cdot \theta) \\
U_n^{\theta} \cdot \cos(i_n \cdot \theta) + U_n^{\phi} \cdot \sin(i_n \cdot \theta) \\
V_n^{\theta} \cdot \sin(i_n \cdot \theta) + V_n^{\phi} \cdot \cos(i_n \cdot \theta) \\
\beta_n^{\theta} \cdot \cos(i_n \cdot \theta) + \beta_n^{\phi} \cdot \sin(i_n \cdot \theta) \end{cases}
\]

where \( W \) is the normal displacement to the shell
\( U \) is the axial displacement
\( V \) is the circumferential displacement
\( \beta \) is the rotation around tangent circumferential axis.

The thickness of the shell is divided into layers representing either steel or concrete. For the reinforcement, an equivalent orthotropic layer is adopted. It can be noted that for this problem, it is not necessary to consider a lot of layers to represent the concrete because these structures carry the external load mainly through membrane action (6). For the same reasons, transversal shear stresses are disregarded. Each of these layers works in plane stress.

2.2 Concrete model

2.2.1 Constitutive law

The choice of a concrete model is very important in the analysis of cooling towers. Former studies have shown the most important parameters of the model on the example of a structure submitted to its selfweight and to the wind pressure (7), (8). For this loading, the structure has nonlinear behaviour that can be divided in three parts:

1) Linear behaviour.
2) Crack plateau. The load level of this plateau and his length depend on the elastic limit in tension of the concrete and of the tension stiffening phenomenon
3) Hardening part and failure, which depend on the reinforcement.

From these studies it is clear that only two parameters are important: cracks limit and tension stiffening. Other parameters like nonlinear behaviour before cracking or crushing have no influence on the results (9).

We have adopted a model based on the theory of plasticity with a multicriteria loading surface. The biaxial behaviour of the concrete is assumed to be linear in compression and tension.
The load surface of this concrete model is classically delimited by two RANKINE criteria in tension written in principal directions:

\[
\begin{align*}
\sigma_1 & \leq R_1 \\
\sigma_2 & \leq R_2
\end{align*}
\]

and a DRUCKER-PRAGER criterion in compression:

\[
f(\sigma) = \sqrt{3J_2} + AI_1
\]

with \( I_1 \) = first stress invariant

\( J_2 \) = second invariant of the deviatoric stress tensor

The flow rule of this model is multi-linear and takes into account the tension stiffening and a softening behaviour in compression over the elastic limits.

### 2.2.2 Initially cracked concrete

On most of the cooling towers, cracks can be observed. These cracks are generally vertical and regularly distributed all over the shell. In some other cases, there are large local vertical cracks.

It is possible to take into account these initial cracks in our model by applying a specific flow rule which has a zero plastic strain in the perpendicular direction of the crack.

The characteristics of a crack are its length, its opening and its depth. To introduce these characteristics in the model, we can consider two possibilities:

1- there are lots of vertical cracks regularly distributed all over the shell (most of the analysed cases) and the cumulated opening of all the cracks \( OC \) on the circumference is linked with the zero plastic strain \( \varepsilon_p \) by the relation:

\[
OC = 2\pi R \varepsilon_p
\]

2- there are few vertical cracks in an angular sector \( \theta \) and we have:

\[
OC = \theta R \varepsilon_p
\]

The depth of the crack is defined by the number of layers affected by the plastic strain.

It can be noted that this is not a very accurate model (smeared technics) of the crack but the goal is to take into account the induced flexibility in the global behaviour of the structure. All characteristics of this concrete model are shown on figure 1.

### 2.3 Reinforcement

For steel, we assume a perfect elasto-plastic behaviour in orthotropic equivalent layer, working in plane stress.
2.4 Creep analysis

These RC shells are submitted to external loads during all their life time. Consequently, it is important to analyse the effect of the creep in the development of damages (geometrical imperfections and cracks) generally observed. The creep calculation is performed with a classical three point Simpson integration method. For the creep law, we choose the law given by the French rule BPEL91 generally used in the cases of structures submitted to constant load. The visco-plastic calculation is carried out by an alternative procedure : plastic calculation and visco-elastic calculation.

2.5 Buckling analysis

The following system gives us, if $\lambda = 1$, the loss of the shell stability (bifurcation):

$$[K_t + \lambda (K_0)] = 0$$

In this application, large displacements induced by the wind are disregarded. The stiffness of the structure can be strongly disturbed by cracking and damaging of the concrete. These phenomenon are taken into account in the tangent stiffness matrix $K_t$. Generally, the choice of a tangent stiffness in calculations is determined by comparison with experimental results. For RC cooling towers, because of the lack of experimental data, we have chosen to compute the tangent stiffness from $[D']$ matrix which links the total stress increment with the total strain increment on each step of the incremental calculation. The general expression of $[D']$ is:

$$[D'] = [D]\left[I - \frac{\left[\frac{\partial F}{\partial \sigma}\right]^T [D]}{H + \left[\frac{\partial F}{\partial \sigma}\right]^T [D] \frac{\partial F}{\partial \sigma}}\right]$$

For the concrete model, there are many expressions of $[D']$ depending on the damaging type. We have chosen to compute rigorously the main ones and to assume, for the other cases:

$$[D'] = 0$$

Incremental calculation with this model has been validated by 3D representation. For similar analysis, the 2D/3D time calculation ratio is inferior to 1/14.

3 APPLICATION EXAMPLE

Our topic is to understand the behaviour of these structures in their life time considering normal loads with the help of parametric analysis. An example of application of this model that we present concerns the study of a cooling tower affected by the typical following damages:
1. The geometrical imperfections are a combination of many circumferential modes (from 1 to 30) with in this case, four principal modes on harmonics 4, 8, 12 and 20.

2. Many meridional cracks regularly distributed on all the shell on the external surface (no observation of the internal surface). The average length of these cracks is 1 meter.

All these observations (except the average length of the cracks) can be included in the typical damages of cooling towers.

3.1 creep under self weight

Our aim is to study the influence of the creep in the evolutions of these damages in time and effects on the stability of the shell.

Two initial states have been analysed:
1 - a structure with initial geometrical imperfections on circumferential modes 4, 8, 12 and 20. The maximum amplitude of the imperfection is 300 mm.
2 - a structure with initial geometrical imperfection and importants cracks on all the shell (depth = 80% of the thickness; OC = 100 mm.). Reinforcement is considered without damage.

Displacements under self weight are small (maximum 1.5 cm). After twenty years, the creep displacements becomes negligible and after forty years, instantaneous displacements are multiplied by 2.1. The maximum displacements and their evolutions are axisymmetric on the top (axial displacement) and at the lintel of the shell (radial displacements) (Figure 2). Meridional cracks reduce the circumferential stiffness of the shell and so, act mainly on radial displacements depending on the hoop stresses. The maximum influence of cracks is observed on the radial displacement of the lintel (maximum compressive hoop stresses).

There is no crack creation during the creep. Hoop tension stresses above the throat cannot create cracks but can open existing ones.

The influence of the geometrical imperfections, cracks and creep under S.W., on the stability, is analysed in comparison to the perfect shell. All results are presented in table 1. Critical load of the perfect shell under S.W. is almost equal to the security coefficient given by French rules (λ=10). For a structure without cracks, self stiffening effect created by observed geometrical imperfections is confirmed (2). The particular shape of these geometrical imperfections can be compared to axial stringers and contributes to stiffen the shell in the axial direction. For all analysed cases, meridional cracks highly reduce the critical load. For a cracked shell, geometrical imperfections do not stiffen the shell because buckling is no more induced by axial stresses but by circumferential hoop stresses below the throat.

Effect of creep can be totally neglected for the analysis of the stability of the structure because creep displacements are very small comparatively to the cooling tower dimensions. Moreover stresses are almost constants.

3.2 Ultimate strength under self weight and wind

3.2.1 Effect of geometrical imperfections

The behavior is analysed for a shell with geometrical imperfection (cf 3.1) for three maximum amplitudes : 180, 300 and 600 mm.
The maximum radial displacements are compared on the Figure 3.A. The behavior of the cooling tower under the wind pressure can be compared to a pipe under bending. Geometrical imperfections, as meridional stringers, stiffen the shell. This stiffening effect go with a weakening effect because the difference of the load level between the ultimate strength and the crack plateau get smaller as the amplitude of initial imperfections increases. More over, geometrical imperfections influence the first cracking : for the perfect shell, first cracks are horizontal in front of the wind direction and for imperfect shells, first cracks are vertical and come earlier.

3.2.2 Effect of meridional cracks

The behavior of the structure is analysed considering a perfect shell and shells with meridional cracks regularly distributed on all the surface on 40% and 80% of the thickness. OC = 100 mm. Reinforcement is considered without damage.

Results presented on the Figure 3.B seems to be surprising because there is no difference of stiffness of the shell without or with very important meridional cracking until the arrival of the first horizontal crack. This analysis is very helpful for the understanding of the behavior of cooling towers under the wind pressure that can be described with three main stages:

1 - Before horizontal cracking, the structure works mainly in the meridional direction as a pipe under bending. The cracks are parallel to the strain and do not reduce the stiffness of the shell.

2 - The crack plateau corresponds to an important horizontal cracking in front of the wind direction. The structure still works in the meridional direction and there is no effect of the initial meridional cracking.

3 - Horizontal cracking induces large displacements and switches the strain from the meridional direction to the circumferential one. For a shell without initial crack, first meridional crack appears. The three curves presented on the Figure 3.B become different because meridional cracks reduce the circumferential stiffness of the shell.

CONCLUSIONS

For analysis of the mechanical behaviour of cooling towers, 2D approach with Fourier series is convenient because it permits quite the same analysis as 3D models but allows parametric analysis with relatively small calculation time. Particularly, This approach permits to make parametric analysis on initially cracked shells. Moreover, when local loads are disregarded, displacements, stresses, and critical geometries of real shells are modal. To understand their behaviour, the Fourier analysis is very helpful. The limit of this kind of modelling is reached when important damages are created by the loading, as it is the case for example in incremental analysis under S.W. and wind. When local cracking becomes too important, Fourier series are not well adapted to describe the state of stress of the circumference and divergence problems appear.

This kind of modelling permits to determine the influence of numerous parameters on the evolution in time and on the stability of cooling towers (3). The application examples presented on this paper lead to the following results:

- Typical geometrical imperfections of cooling towers increase the stability of these structures but meridional cracks, if they are regularly distributed on all the shell, can be dangerous because the hoop stiffness of the shell is highly reduced.
- Creep analysis proves that the creep under self weight don’t contribute to the observed evolutions of damages and so, don’t change the strength. This results cannot be generalised. For example, creep can be dangerous if the shell presents an important axisymmetric imperfection.

- Typical damages have an influence on the behavior under the wind pressure but do not reduce significantly the ultimate strength of the structure.

REFERENCES


![Figure 1: Concrete model Load surface and flow rule.](image-url)
Figure 2: Maximum displacements during creep under self weight for a shell without and with meridional cracks.

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Loading</th>
<th>Instantaneous λ</th>
<th>λ after 40 years</th>
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<tr>
<td>Perfect shell</td>
<td>PP</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
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Figure 3: Maximum radial displacements under wind pressure. A) influence of geometrical imperfections. B) Influence of meridonnal cracks.

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