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Analysis of reinforced concrete shell elements subjected to membrane forces, bending moments and transverse shear forces encountered in RCCV

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ABSTRACT

This paper presents results obtained by solving a set of equations, in closed form, derived for the conditions normally encountered in the design of a reinforced concrete containment vessel. The results were compared and checked well with those obtained by other approaches.

1. INTRODUCTION

One of the primary functions of a reinforced concrete containment vessel (RCCV) of a nuclear power plant is to resist a design internal pressure that produces membrane forces in both the hoop and meridianal directions in the region away from the discontinuities of the RCCV. In addition to the membrane forces, there are bending moments and transverse (out of plane) shear force in the region adjacent to the discontinuities. The RCCV also has to resist lateral forces such as seismic loads. In addition, the thermal effects introduced by the temperature differential between the inside face and outside face of the RCCV are also significant. In the structural analysis and design of the RCCV, these membrane forces, bending moments and transverse shear force are calculated by finite element analysis using finite element computer program such as NASTRAN [1].

To check against the allowable limits defined in the codes and standards such as ASME Section III, Division 2 in the U.S. [2], the stresses in concrete and reinforcing steels are needed. This step is, in general, calculated by proprietary computer programs developed by the RCCV structural designers. In the case of the RCCV and the reactor building (RB) for the Advanced Boiling Water Reactor (ABWR), Shimizu Corp. applied the SSDP computer program to determine the concrete stress and the reinforcing steel stress after the internal forces were obtained by finite element analysis using NASTRAN. The theoretical basis and the details of SSDP computer program are presented in a companion but separate paper [3] being submitted to SMIRT 14.

2. A METHODOLOGY FOR CONCRETE STRESS AND REINFORCING STEEL STRESS CALCULATION FOR CONDITIONS ENCOUNTERED IN RCCV

As part of an effort to verify or check the results from SSDP, a few cases that are fundamental to the stress state and the design of a RCCV were selected. A typical RCCV is a cylindrical shell structure with main reinforcing steel in both hoop and meridianal directions but with shear ties perpendicular to the main reinforcement. The cases considered were:

Case 1: An element with given thickness and main reinforcing steel in two orthogonal directions is subjected to in-plane forces N_x, N_z and N_{xz}.

Case 2: An element with given thickness, main reinforcing steel and shear tie is subjected to in-plane axial tension and transverse shear.

Case 3: Bending moment is added to Case 2.

Case 4: An element with given thickness, main reinforcing steel and tie bar steel is subjected to tension in both hoop and meridianal directions, a tangential shear force and a transverse shear force.

Case 5: An element with given thickness and main reinforcing steel is subjected to an axial compression, a bending moment and a temperature gradient. The axial direction is allowed to expand freely.

Case 6: Same as Case 5, except the axial direction is not free of constraint. Concrete stress, rebar stress, and cracking direction are to be calculated.

Concrete stress, rebar stresses cracking depth and cracking direction are calculated for all the cases. In addition, for Cases 2, 3, and 4, tie-bar steel stresses and, for Cases 5 and 6 thermal moments are calculated after consideration of concrete cracking.

For Case 1, the methodology for the concrete stress and reinforcing steel stress calculations furnished in Reference 6 was used. For Cases 5 and 6, the approach discussed in References 7, 8, 9 and 10 that deal with the calculation of concrete stress and reinforcing steel stress for conditions encountered in a RCCV was adopted with slight modifications. The details of the methodology for the concrete stress and reinforcing steel stress calculations which can be found in the literature [6] for Case 1 are not presented here. For Cases 2, 3 and 4, the methodology for stress and cracking calculations is not conveniently available and cannot be found in the standard concrete structure design textbooks [4 and 5]. For this reason, the assumptions used for the derivation of the governing equations and procedures of calculations are presented. For simplicity, Case 2 is presented as an example below. The actual calculations for all the above cases were carried out by MathCAD [11].

The basic assumptions used in the derivation are:

(1) A plane section normal to the axis of the section before bending remains plane after bending. Normal strain $\varepsilon_x(y)$ is linear in y, while the shear strains are assumed to be quadratic in y. The notations and coordinate systems are shown in Figure 1. These two assumptions imply:

$$\varepsilon_I(y) = \varepsilon I + \varepsilon'_I y + \varepsilon''_I y^2$$
$$\varepsilon_{II}(y) = \varepsilon II + \varepsilon'_{II} y + \varepsilon''_{II} y^2 \qquad (1)$$
$$\Gamma(y) = \gamma + \gamma' y + \gamma'' y^2$$

Because of the assumption on $\varepsilon_x(y)$, we have

$$\frac{d^2 \varepsilon_x(y)}{dy^2} = 0 \tag{2}$$

(2) Concrete takes no tensile stress. The section is considered to be cracked wherever the concrete is in tension. Direction II is perpendicular to the crack, while Direction I is parallel to the crack. Along the crack, σc_I is zero. Stress-strain relations are given by:

$$\begin{aligned} \sigma c_I(y) &= 0 \\ \sigma c_{II}(y) &= E_c \varepsilon_{II}(y) \end{aligned} \tag{3}$$

The unknown quantities are: εI, εII, ε''_I, ε''_{II}, γ'' and θ. The equations to be satisfied are:

(i) Equilibrium condition on Nx.

(ii) Equilibrium condition on M.

For Case 2, M=0. Because of the symmetrical arrangement of rebar, ε'_I ,ε'_{II} , γ and γ' in Equation 1 are assumed to be zero. With this set of strain field, equilibrium condition M=0 is identically satisfied.

(iii) Equilibrium condition on Q_x.

(iv) Boundary conditions on the surfaces:

(a) On shear stress: $\tau c(\frac{\pm D}{2}) = 0$

(b) On normal stress in thickness direction: $\sigma c_y(\frac{D}{2}) + p_w \overline{\overline{\sigma s_y}} = 0$

where $\sigma c_y(D/2)$ is the normal stress of concrete on y=D/2, and $\overline{\overline{\sigma s_y}}$ is the average stress in the tie-bar.

(v) Condition on $\varepsilon_x(y)$.

(vi) Equilibrium condition in y-direction (i.e., thickness direction) on the average concrete stress and the average tie-bar stress.

There are six equations for Case 2. After algebraic manipulations of the above equations, a transcendental equation on θ is obtained. After θ (the angle of normal to the cracking surface) is found, the concrete stresses, rebar stress and tie-bar stress can be calculated. For Case 3, there are 10 unknown quantities: εI, εII, ε'_I, ε'_{II} , ε''_I, ε''_{II}, γ', γ''_{12}, y_n and θ. There are 10 equations to solve for these 10 unknown quantities. Similarly, for Case 4, there are 10 unknowns in the strain expressions (ε_1, ε_2, ε_3, ε''_1, ε''_2, ε''_3, γ_{12}, γ'_{12}, γ''_{23}, γ''_{33}) plus two components of direction cosines that determine the cracking surface. There are 12 equations to solve for these unknown quantities. The concrete and steel stresses can be calculated after these unknown quantities are found.

For Cases 5 and 6, the section shown in Figure 2 is subjected to the action of an axial force N, a moment M and a thermal gradient given by $(T_2-T_1)/D$. M_F and N_F are produced by externally applied loads. Equilibrium consideration of internal and external forces gives:

$$\frac{f_c \cdot a \cdot b}{2} - \frac{(d-a)}{a} \cdot \frac{f_c}{E_c} \cdot E_s \cdot A_s = 0 \tag{4}$$

Consideration of moment equilibrium of internal and external forces about the neural axis gives:

$$\frac{N \cdot I}{S} - N \cdot (e'+a) - E_c \cdot I_{cg} \phi_T = 0 \tag{5}$$

In Equations 4 and 5, E_c = Young's modulus of concrete, E_s = Young's modulus of steel, b = the width of beam or unit width of a plate/shell element, A_s = steel area, and f_c = the maximum compressive stress of concrete. The other symbols are defined in Figure 2.

When N_F, M_F, T_1 and T_2 are given, Equation 5 can be solved for a, the location of the neutral axis. After a is found, the concrete stress f_c and the steel stress f_s can be calculated by the following expressions:

$$f_c = \frac{N \cdot a}{S}, \qquad f_s = E_s \frac{d-a}{a} \cdot \frac{f_c}{E_c} \tag{6}$$

The residual axial thermal force is:

$$N_{th} = E_c \cdot \varepsilon_T \cdot (b \cdot a + n \cdot A_s) \tag{7}$$

The residual thermal moment is:

$$M_{th} = E_c \cdot I_{cg} \cdot \phi_T + \frac{E_c \cdot \varepsilon_T \cdot b \cdot a^2}{2} - E_c \cdot n \cdot \varepsilon_T \cdot A_s \cdot (d-a) - N_{th} \cdot (a - \frac{D}{2}) \tag{8}$$

The basic relations given above are for Case (5). They are essentially the same as those found in Reference 8. In Case 6, when the axial direction is constrained, N_T and M_T are given. Equation 5 is replaced by:

$$\frac{N \cdot I}{S} - N_F \cdot (e'+a) - E_c \cdot I_{cg} \phi_T + \frac{E_c \cdot \varepsilon_T \cdot b \cdot a^2}{2} - E_c \cdot \varepsilon_T \cdot n \cdot A_s \cdot (d-a) = 0 \tag{9}$$

where

$$\varepsilon_T = \frac{N_T}{E_c \cdot D}$$

and

$$N = N_F + E_c \varepsilon_T \cdot (b \cdot a + n \cdot A_s)$$

In the above expressions, ε_T is the axial strain due to N_T, the thermal axial force. The residual ratios for axial force and the moment are defined as:

$$\beta_N = \frac{N_{th}}{N_T}$$

$$\beta_M = \frac{M_{th}}{M_T} \qquad (10)$$

In addition to the formulas presented here, more detailed information can be found in Reference 4. The results obtained by applying MathCAD to the selected six cases are summarized together with those calculated by SSDP in Tables 1 through 6. The results obtained by MathCAD agree very well with those obtained by SSDP.

3. CONCLUSIONS

(1) This paper presents a methodology including assumptions and procedures for the concrete stress and reinforcing steel stress calculations which are basic for the RCCV design. The procedures presented here will be supplementary to those in existing literature.

(2) The approach presented here can be applied independently to verify or check computer programs that are used for stress calculations for RCCV design.

(3) The results obtained by applying MathCAD to the set of equations obtained from the methodology presented herein are in good agreement with those obtained from SSDP.

4. REFERENCES

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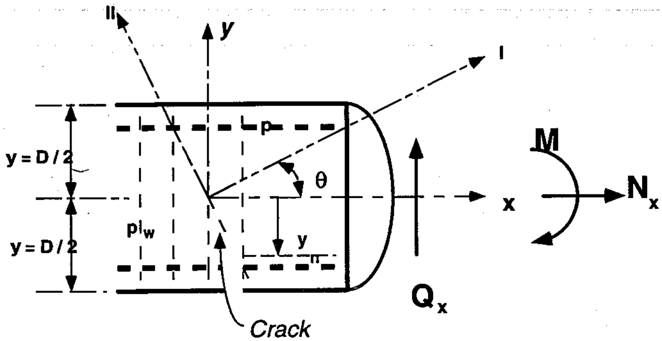
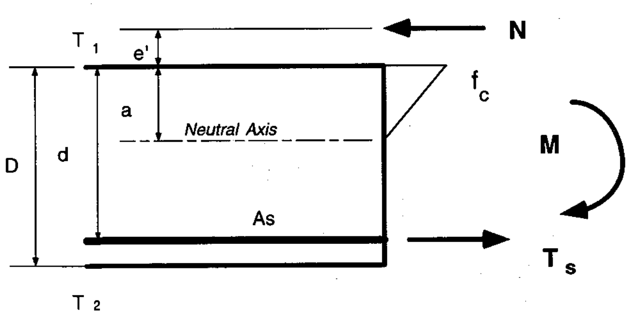


A_s = rebar steel area
D = Thickness of Element
p = ratio of rebar steel (horizontal direction) to cross section area = $A_s/D\cdot 1$
p_w = ratio of tie-bar steel (vertical direction) to cross section area
e_I = Principal strain in direction I
e_{II} = Principal strain in direction II
y_n = beginning of crack

Figure 1: Typical Section of an Element subjected to N_x, M, and Q_x



$$n = \frac{E_s}{E_c},$$

$$e' = \frac{M}{N} - \frac{D}{2}, \qquad I = \frac{b \cdot a^3}{3} + n \cdot A_s \cdot (d-a)^2,$$

$$\phi_T = \frac{(T_2 - T_1)\cdot \alpha}{D}, \qquad S = \frac{b \cdot a^2}{2} - n \cdot A_s \cdot (d-a),$$

$$I_{cg} = \frac{b \cdot a^3}{12} + b \cdot a \cdot [\frac{0.5 \cdot b \cdot a^2 + n \cdot A_s \cdot d}{b \cdot a + n \cdot A_s} - \frac{a}{2}]^2 + n \cdot A_s \cdot [d - \frac{0.5 \cdot b \cdot a^2 + n \cdot A_s \cdot d}{b \cdot a + n \cdot A_s}]^2$$

Figure 2: Typical Section of an Element subjected to N_x, M, and temperature T_1 and T_2

Table 1: Comparison of Results for Case 1

Given: N_x = 10000 lb/in., N_z = 5000 lb/in, N_{xz} = 8000 lb/in.,
Es = $29*10^6$ psi, Ec = $3.61*10^6$ psi, t = 100 in.,
As = 1.4 in^2/in. (both direction),

	Results Obtained By SSDP	Results Obtained by MathCAD
Maximum Concrete Stress (psi)	-166	-160.8
Rebar Stress in Direction I (psi)	9846	9879
Rebar Stress in Direction II (psi)	12280	12320
Direction of Principal Strain	47.82°	47.826°
ε_I	$8.075*10^{-4}$	$8.1*10^{-4}$
ε_{II}	$-4.444*10^{-4}$	$-4.454*10^{-4}$

Table 2: Comparison of Results for Case 2

Given: N_x = 1000 lb/in., Q_x = 2500 lb/in., E_s = $29*10^6$ psi,
E_c = $3.61*10^6$ psi, ν_s =0.3, ν_c =0.18, D = 100 in., A_s = 0.7 in^2/in.,
A_{tie} = 0.005 in^2/in

	Results Obtained By SSDP	Results Obtained by MathCAD
Concrete Force (lb/in.)	-166	-160.8
Concrete Stress (psi)	-78.1	-77.9
Rebar Stress (psi)	3050	3065
Tie-bar Stress (psi)	3822	3798
Angle between normal to crack and horizontal axis	52.6°	52.8°
ε_I	$3.258*10^{-4}$	$3.2631*10^{-4}$
ε_{II}	$-2.153*10^{-5}$	$-2.157*10^{-5}$

Table 3: Comparison of Results for Case 3

Given: N_x = 1000 lb/in., Q_x = 2500 lb/in., M = $2.5*10^6$ in-lb/in.,
E_s = $29*10^6$ psi, E_c = $3.61*10^6$ psi, ν_s =0.3, ν_c =0.18, D = 100 in.,
A_s = 0.7 in^2/in.(Note 1), A_{tie} = 0.005 in^2/in

	Results Obtained By SSDP (Note 1) (Note 2)	Results Obtained by MathCAD
Concrete Stress(psi)	-2071	-1959.2
Rebar Tensile Stress (psi)	44300	44111.3
Rebar compressive Stress (psi)	-9633	-9272.7
Tie-bar Stress (psi)	1044	709.1
Crack Depth (in.)	78.36	77.46
Angle between normal to crack and horizontal axis	14.53°	10.74°
ε_I	$6.162*10^{-4}$	$6.266*10^{-4}$
ε'_I	$2.302*10^{-5}$	$2.2837*10^{-5}$
ε''_I	$3.048*10^{-9}$	$-5.685*10^{-9}$
ε_{II}	$-1.092*10^{-5}$	$-9.784*10^{-6}$
ε'_{II}	$-7.681*10^{-7}$	$-8.21*10^{-7}$
ε''_{II}	$1.973*10^{-8}$	$1.783*10^{-8}$
γ'	$-5.591*10^{-6}$	$-5.4334*10^{-6}$
γ''	$-2.025*10^{-8}$	$-1.34*10^{-8}$

Note 1: Considered 2 ways in SSDP; One way in MathCAD
Note 2: Strain components are calculated by hand.

Table 4: Comparison of Results for Case 4

Given: N_x = 10,000 lb/in., N_z = 5,000 lb/in, N_{zx} = 8,000 lb/in,
Q_x = 2500 lb/in., E_s = $29*10^6$ psi, E_c = $3.61*10^6$ psi, ν_s =0.3,
ν_c =0.18, D = 100 in., A_s = 0.7 in^2/in.(top or bottom),
A_{tie} = 0.005 in^2/in

	Results Obtained By SSDP	Results Obtained by MathCAD
Concrete Stress (psi)	-181.2	-182
Rebar Stress in x-direction (psi)	13550	13572
Rebar Stress in z-direction (psi)	10160	10156
Tie-bar Stress (psi)	6171	6200
Direction cosines of crack	l_3 = 0.644 m_3 = 0.522 n_3 = 0.559	l_3 =0.643 m_3 =0.524 n_3 = 0.558
ε_1	$-4.499*10^{-5}$	$-4.509*10^{-5}$
ε_2	$-8.917*10^{-6}$	$-9.061*10^{-6}$
ε_3	$1.189*10^{-3}$	$1.1932*10^{-3}$
ε''_1	0.0	0.0
ε''_2	$6.8063*10^{-9}$	$6.87077*10^{-9}$
ε''_3	$-1.3418*10^{-7}$	$-1.35267*10^{-7}$
γ_{12}	$-1.88361*10^{-5}$	$-1.905*10^{-5}$
γ''_{12}	$7.5342*10^{-9}$	$7.62*10^{-9}$
γ''_{23}	$-2.1471*10^{-7}$	$-2.158*10^{-7}$
γ''_{33}	$4.6188*10^{-9}$	$4.685*10^{-9}$

Table 5: Comparison of Results for Case 5

Given: N_F = 101,465 lb., M_F = 3,175,000 in-lb, ΔT = 100°F,
b=12in., t=42in.,d=40in., A_s = 1 in^2, E_s = $30*10^6$ psi,
E_c = $3*10^6$ psi, α = $6*10^{-6}$

	Results Obtained By SSDP	Results Obtained by MathCAD
Concrete Stress (psi)	2237	2237
Rebar Stress (psi)	54580	54595
Uncracked Depth (in.)	11.63	11.627
Self-equilibrium Thermal Moment (in-lb)	534,035	534,744
Residual ratio of Thermal Moment β	0.168	0.168

Table 6: Comparison of Results for Case 6

Given: N_F = 101,459 lb., M_F = 3,175,000 in-lb, ΔT = 100°F,
b=12in.,t=42in.,d=40in., A_s = 1 in^2, E_s = $30*10^6$ psi,
E_c = $3*10^6$ psi, α = $6*10^{-6}$ Consider Axial Constraint

	Results Obtained by SSDP	Results Obtained by MathCAD
Concrete Stress (psi)	2900	2899
Rebar Stress (psi)	47116	47130
Uncracked Depth (in.)	15.24	15.23
Thermal Axial Force (lb.)	116,495	116,447
Self-equilibrium Thermal Moment (in-lb)	1,940,650	1,937,672
Residual ratio β_N	0.388	0.388
Residual ratio β_M	0.611	0.61