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Evaluation of seismic damageability of structures using fragility models and damage risk matrices

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ABSTRACT. The analysis of the damage states of a structure subjected to eartquake actions is considered. The damage levels are quantified in terms of damage indices, and the damage-induced losses in terms of damage factors. Stochastic matrices are defined for describing the damageability of a structure, using fragility models and the DPD approach. Both damage level indices and damage loss factors are take into account, with appropriate distributional assumptions.

1 INTRODUCTION

Large earthquake motions induce a large variety of damages to structures (buildings, lifelines, industrial facilities) and also imply a wide range of losses (material, economic and social ones). Probabilistic approaches to the evaluation of both (earthquake-induced) damages and losses in urban areas have been developed [1,2,3]. The *fragility models* give the possibility to evaluate the *failure probability* of an element / structure / equipment, conditional on certain earthquake motion intensity levels. Such models were mostly devised for NPP structures and equipment, but recent studies also employ them in the damage studies for (civil) buildings [3]. Alternative approaches to condition assessment by probabilistic methods use time-dependent models, as the ones due to B.Ellingwood *et al* [4]. We have also proposed the use of fragility concepts for the seismic risk evaluation of general structures (not only in NPPs) [5,6,7].

A typical fragility model has to estimate the failure probability of an element or a structure subjected to earthquake loads whose intensity is probabilistically described. When the analysis is focused on the *levels of damage* induced by the eartquakes, *specific models* have to be proposed and employed.

The effect of eartquake loads on the damage state / level of induced losses has to be quantified according to a scale of levels along the range(s) of one or more *damage / loss indices*. This discretization process naturally leads to the need of using certain matrices, similar to the DPMs (damage probability matrices) as considered in [8]. In this paper, the authors propose a more general concept of (damage / loss) *risk matrices* with stochastic entries. They are built according to two discretizations on the earthquake intensity parameter taken into account, and on the damage / loss index employed. The idea is not quite new, but we propose a rather natural way to pass from fragility families of curves to the risk matrices in terms of the DPDs (discrete probability distributions) as they were introduced by S.Kaplan *et al* [9,10] and also considered in a couple of our papers [7,11,12]. The DLRMs (damage level risk matrices) are presented in Section 2, and DFRMs (damage fector RMs) are proposed in Section 3.

2 DAMAGE STATES AND SEISMIC FRAGILITY MODELS

The prediction of the lifetime of buildings in a seismic area needs some mathematical models to be built, able to describe (as close as possible) the evolution of the structural damage. Certainly, any such model should take into account as many sources of uncertainty / randomness as possible, but without becoming practically unemployable. The randomness in some relevant structural parameters is taken into account in [8], with certain assumed distributions for them. But it is hard to select the "best" (or - at least - the most appropiate) set of structural parameters for describing the damage states induced by earthquakes in structural components and systems.

To fix the ideas, let us first take into account a single damage state parameter as the story drift ratio δ for a multistory building. It is assumed that the possible values of δ cover an interval $[0,D]$. This interval has to be partitioned into a finite number of subintervals:

$$[0,D) = \bigcup_{k=1}^{\ell} [d_{k-1}, d_k) \text{ with } d_0 = 0. \tag{2.1}$$

Each interval corresponds to a category of structural damage (from, e.g., no damage to severe damage). Obviously, the random damage indicator Δ can take values in one of the subintervals of (2.1) with a specific probability. If the impact of the external actions would not be take into account, such a probability would be given by

$$P\big(\Delta \in [d_{k-1}, d_k)\big) = F_\Delta(d_k) - F_\Delta(d_{k-1}) \text{ for } 1 \le k \le \ell, \tag{2.2}$$

where F_Δ is the *cdf* (cumulative distribution function) of Δ. But such an oversimplifying assumption cannot be accepted. The damage state of the system essentially depends on the actions from the environment, including the impact of earthquake motions. A parameter describing the intensity of the seismic excitation at the site of the system has to be considered, for instance the PGA A as in the most fragility models. The possible values of A also cover an interval $[0,\alpha)$ that has to be partitioned:

$$[0,\alpha) = \bigcup_{j=1}^{m} [a_{j-1}, a_j) \text{ with } a_0 = 0. \tag{2.3}$$

Conditional probabilities have now to be introduced for the damage indicator $\Delta \in [d_{i-1}, d_i)$ provided $A \in [a_{j-1}, a_j)$:

$$P\big(\Delta \in [d_{i-1}, d_i) \mid A \in [a_{j-1}, a_j)\big) = q_{ij}, \tag{2.4}$$

for $1 \le i \le \ell$ and $1 \le j \le m$. The conditional probabilities are the entries of a stochastic matrix

$$Q = [q_{ij}]_{\ell,m} \tag{2.5}$$

that can be called the *damage level risk matrix* (DLRM). Obviously, the entries of Q have that can be called the *damage level risk matrix* (DLRM). Obviously, the entries of Q have to satisfy the condition

$$\sum_{i=1}^{\ell} q_{ij} = 1 \tag{2.6}$$

since - for any earthquake input intensity - the structural component / system has to be in one and only one damage state / damage level, falling in one (and only one) of the intervals $[d_{i-1}, d_i) \subseteq [0, D)$ with $P(\Delta \in [0, D)) = 1$.

This model is connected with the concept of fragility [9], [10], [5]. In most references, lognormal distributions are assumed for the PGA capacity = the ground acceleration A corresponding to failure: $A = \tilde{A}\varepsilon_R \varepsilon_U$ where $\tilde{A}$ = the median of A and ε_R, ε_U are two random variables with unit medians that represent the inherent randomness about the median and the uncertainty in the median value, respectively. The logarithmic standard deviations of ε_R, ε_U are β_R, β_U. The three parameters $\tilde{A}$, β_R, β_U are enough for expressing the failure probability at a level a of the ground acceleration:

$$P_f(a) = \Phi\left[\frac{\ln(a/\tilde{A})}{\beta_R}\right] \quad \text{with } f_{\tilde{A}}(c) = \frac{1}{\sqrt{2\pi}\, c\,\beta_U} \exp\left[-\frac{1}{2}\left(\frac{\ln c/\tilde{A}}{\beta_U}\right)^2\right]. \tag{2.7}$$

This model makes possible to evaluate the fragility (that is, the failure frequency conditional on a seismic intensity a) of the structure by

$$F(a, p_0) = \Phi\left[\frac{1}{\beta_R}\ln\left(\frac{a}{\tilde{A}\exp(-\beta_U \Phi^{-1}(p_0))}\right)\right], \tag{2.8}$$

where Φ is the standard normal *cdf* and p_0 is the probability that determines a corresponding fragility curve ($p_0 = 0.5$ determines the median curve). This formulation is adapted from [13].

We are going to see how a fragility model could be employed for building a DLRM as the one given (2.5) with the definition (2.4) of its entries q_{ij}. An exact application of expression (2.4) would be (according to the definition of the conditional probabilities)

$$q_{ij} = \frac{P[\{\Delta \in [d_{i-1}, d_i)\} \cap \{A \in [a_{j-1}, a_j)\}]}{P[\{A \in [a_{j-1}, a_j)\}]}. \tag{2.9}$$

Alternatively, a representative (central) value may be taken in each interval of the partition (2.3) : $\hat{a}_j \in [a_{j-1}, a_j)$; then, the probabilities (2.4) of the stochastic matrix Q can be evaluated by

$$\begin{aligned} q_{ij} &= P[\Delta < d_i \mid \hat{a}_j] - P[\Delta < d_{i-1} \mid \hat{a}_j] = \\ &= P[\Delta \geq d_{i-1} \mid \hat{a}_j] - P[\Delta \geq d_i \mid \hat{a}_j]. \end{aligned} \tag{2.10}$$

Let us assume that a double lognormal format is acceptable for the damage index Δ :

$$F_\Delta(a,C) = \Phi\left[\frac{\ln(a/C)}{\beta_R}\right] \quad \text{with } f_C(c) = \frac{1}{\sqrt{2\pi}\, c\,\beta_U}\exp\left[-\frac{1}{2}\left(\frac{\ln(c/\tilde{\Delta})}{\beta_U}\right)^2\right]. \tag{2.11}$$

In (2.11), C is the median capacity of the damage index Δ, while $\tilde{\Delta}$ is the median of C. Then we could express the entries of the DLRM Q as

$$\Phi\left[\frac{1}{\beta_R}\ln\left(\frac{\hat{a}_j}{\tilde{\Delta}\exp[-\beta_U\Phi^{-1}(p_{i-1})]}\right)\right] - \Phi\left[\frac{1}{\beta_R}\ln\left(\frac{\hat{a}_j}{\tilde{\Delta}\exp[-\beta_U\Phi^{-1}(p_i)]}\right)\right], \tag{2.12}$$

where p_i is the probability that determines the damage-fragility curve corresponding to the damage level $d_i \in (0,D)$ $(1 \le i \le \ell)$ (see Eq.(2.8)).

The fragility model can be discretized in terms of DPDs (discrete probability distributions), as shown in |10|. Two families of curves are defined as

$$[\gamma] \equiv \{\langle q_j, \gamma_j(a)\rangle\}, \quad [\Delta] \equiv \{\langle p_i, \Delta_i(a)\rangle\}, \tag{2.13}$$

where Δ is fragility curve of the DI depending on the seismic acceleration, while γ_j is a seismicity curve (also known as a hazard curve). A rather widely accepted analytical expression for γ involves Fréchet's type II extreme value distribution:

$$\gamma(a) = P[A > a] = 1 - \exp[-(\sigma/a)^\kappa], \tag{2.14}$$

where $\sigma =$ the size parameter and $\kappa =$ the shape parameter. Such an expression can be found, e.g., in |14|. Families of hazard curves and fragility curves are presented in Fig.1.

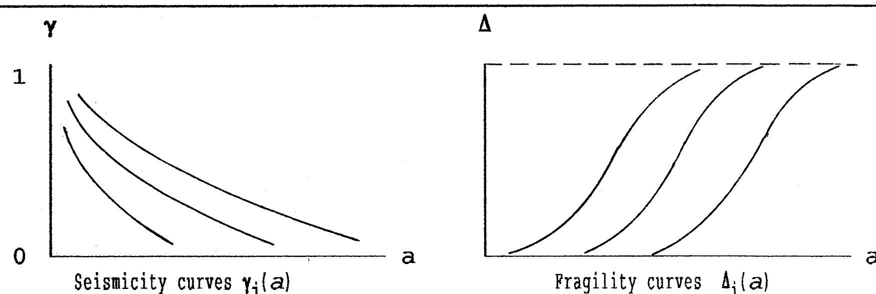

Fig.1 Families of curves γ_j and Δ_i under the DPD approach

The two families of curves can be composed resulting in a "two-dimensional" DPD

$$[\varphi] = \{\langle p_{ij}, \varphi_{ij}\rangle\} \quad \text{with } p_{ij} = p_i q_j \quad \text{and} \quad \varphi_{ij} = -\int_0^\infty \Delta_i(a)\frac{d\gamma_j}{da}\,da. \tag{2.15}$$

Let us recall that fragility curve Δ_i is determined by the probability p_i while a hazard curve γ_j will be entirely determined by specific values on its size and shape parameters (σ_j, κ_j). The derivative under the integral in (2.15) - following from the analytical assumption (2.14) - is the (specific) *pdf* of Fréchet's distribution :

$$f_A^{(j)}(a) = \frac{\kappa_j}{\sigma_j}\left(\frac{\sigma_j}{a}\right)^{\kappa_j+1} \exp\left[\left(\frac{\sigma_j}{a}\right)^{\kappa_j}\right]. \tag{2.16}$$

As regards the probabilities $q_k = P[a_{k-1} < A \le a_k]$, they can be derived from (2.14) by

$$q_k = P[A > a_{k-1}] - P[A > a_k] = \exp[-(\sigma_j/a_k)^{\kappa_j}] - \exp[-(\sigma_j/a_{k-1})^{\kappa_j}]. \tag{2.17}$$

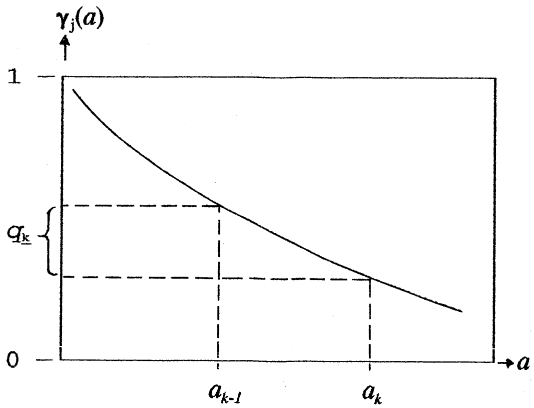

Fig.2 Probabilities of earthquake level intensities

This probability that the earthquake intensity A falls in the interval $[a_{k-1}, a_k)$ is represented in Fig.2. At the same time, the hazard curve γ_j is assumed - under the DPD approach - with its specific probability q_j (not the same with q_k).

The DPDs for the hazard curves are considered as independent from the ones for the fragility curves Δ_i. This independence justifies the expression of $p_{ij} = p_i q_j$ given in Eq.(2.15), as the probability of the fragility φ_{ij}.

3 SEISMIC DAMAGEABILITY QUANTIFICATION

The problem of quantifying the damage level and the losses due to (a certain level of) damage in the structure is clearly very important, both from the engineering and probabilistic points of view. Various damage indicators are met in the literature (e.g., in |8|, |14|, |15|). They should be technically relevant and also characterizable in terms of appropriate probabilistic distributions, as a basis for the fragility models just presented.

According to |15|, a damage index is a parameter that indicates how close the maximum reponse of a structure is to its maximum ultimate capacity. Such an index based on a fatigue model is

$$DI = \frac{\delta_a - \delta_y}{\delta_u - \delta_y} \, \frac{1}{\left(1 - \frac{E_h}{4(\delta_u - \delta_y)} F_y\right)} \tag{3.1}$$

where δ_a = the maximum experienced deformation, δ_y = the yield deformation capacity, δ_u = the ultimate deformation capacity, F_y = the yield force capacity, and E_h = the cumulative dissipated hysteretic energy. Two of these five parameters are considered as depending on the ground acceleration a : δ_a and E_h. The functional dependence is not explicitly formulated, but the numerical data of Table 2 in [15] describe this dependence. Five PGA values are considered: $0.1g$, $0.2g$, $0.3g$, $0.4g$, $0.5g$. The other three parameters have to be considered as deterministic.

The level of losses due to possible damage states usually quantified in terms of a *damage factor* that can be defined (see, e.g., [3] and Report ATC-13 [16]) as the ratio between the cost of repair and the total cost of the system / structure. Seven classes for such damage states (with respect to the losses) are considered in [16]. These damage states are clearly dependent on the earthquake intensity, and they should be probabilistically described. A (pseudo-)lognormal distribution is used in [3] for describing the seismic fragility of a structure with respect to the earthquake intensity MMI; we keep the notations Δ, δ (as in Section 2) for the damage factor, but we denote by $m \in [0, M]$ the values of the MMI. Thus, the fragility curve for the loss factor can be expressed as

$$P[MMI \geq m \mid a] = \int_0^m \frac{1}{\sqrt{2\pi}\,\mu\,\sigma_{\ln A}} \exp\left[-\frac{1}{2}\left(\frac{\ln\mu - E[\ln\mu]}{\sigma_{\ln A}}\right)^2\right] d\mu\,. \tag{3.2}$$

Certainly, such a model would be not complete without some distributional assumptions on the (damage) loss factor conditional on the parameter MMI $= m$. A Beta distribution is suggested in [3] for this dependence. Therefore, the conditional *pdf* of Δ would be

$$f_\Delta(\delta \mid m) = \frac{1}{B(r,s)}\delta^{r-1}(1-\delta)^{s-1}, \tag{3.3}$$

where B is Euler's Beta function. The density in (3.3) does not explicitly depend on m, but this dependence is assumed in [3] and the estimations are given (Table 4-II, p.4-2) for low, mean and high damage factors. Let us also mention that the simple expression for the Beta *pdf* is obtained from the its general form (with parameters *a, b, u, k*, given by G.I.Schuëller [17]) by replacing $q \to r$, $r \to s$, $a = 0$ and $b = 1$; in other words, the damage loss factor is not measured in percents but it is normalized to $[0, 1]$. Alternatively, we suggest that the two parameters that characterize the *pdf* of (3.3) could be statistically estimated using the expected value and the variance of the Beta distribution:

$$E[\Delta] = \frac{r}{r+s}, \quad Var[\Delta] = \frac{rs}{(r+s)^2(r+s+1)}. \tag{3.4}$$

Then, a *damage factor risk matrix* (DFRM) could be obtained, following the procedure presented in Section 2 with the necessary modifications. As in ATC-13 [16], the damage factors are defined as percentages of replacement value / repair cost with respect to the total value. Seven damage categories can be considered, and (for instance) five earthquake intensity levels (resulting in 7-by-5 DFRMs).

Problems like the short-term and long-term prediction of the damage states of a structure are considered in [14]. The DSP model there presented also takes into account the behaviour in time, including Markov chains. Such an analysis gets beyond the methods we have considered in this contribution, but we suggest that the DLRM's and DFRM's could be adapted for such time-dependent analyses.

4 CONCLUDING REMARKS

The damage / loss states are discretized into a finite number of classes that induce a partition over the interval of possible δ-values. The range of the earthquake intensity parameter is also partitioned, and the classical fragility model is reformulated so that DLRM (damage level risk matrices) and DFRM (damage factor risk matrices) can be defined and used. The discrete probability distribution (DPD) approach is also used, making possible an alternative approach for estimating the probabilities of the stochastic DLR and DFR matrices. We have not given numerical examples to illustrate the notions and models proposed in Section 2 and 3. Numerical data for the fragility models to quantify the earthquake induced dagamges in buildings can be found in the reports ATC-13, ATC-38 [18] and the LAMB Project (which were not directly available to us). We have been mostly interested in mathematical (probabilistic) aspects connected with this approach in terms of fragility. A subsequent investigation has to validate the extent to which the DLRMs and DFRMs could be useful.

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